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Singapore Math - The Most Visual Math?

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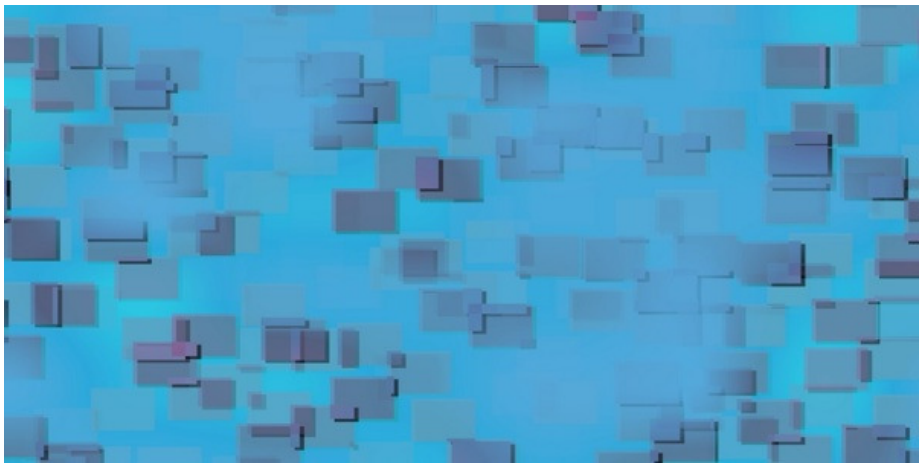


Better For Problem Solving?
Why Are Visualization & Drawing As Important As Numbers?

Plus Bonus Video Below

Editor's Note: *Math-phobia is one of the more challenging issues for our students and for this country's educational make-up and there is much to learn from other leading countries such as Singapore.*

This is the third installment of the 4-part series "Singapore Math Demystified!" from The Daily Riff.
guest posted by Bill Jackson, Math Helping Teacher, Scarsdale, NY Public Schools, one of the highest performing districts in the country, due to the remarkable interest expressed by parents and schools. We are delighted that he is sharing his wealth of knowledge on this subject with us!

If you missed Bill's fascinating journey, from Japan to Germany, discovering and pursuing Singapore Math, see Part 1 - "Why I Became Interested in Singapore Math". For Part 2 - "Can Problem Solving Unravel Our Fear of Math?" . Part 4 "Bringing Singapore Math to your School & Tips for Teachers" . ---C.J. Westerberg

The Singapore Math Model-Drawing Approach

By Bill Jackson

What comes to mind when you think about math word problems? If you are among the lucky few, they may not cause too much anxiety. But for many if not most people, they evoke memories of frustration, failure and dislike of mathematics.

This is strange considering the fact that problem solving has been a major focus of math education in the U.S. since at least 1989 with the publication of the Principles and Standards for School Mathematics by the National Council of Teachers of Mathematics. Since then, virtually every state in the country has developed math standards that include statements like this one from New York: "mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers." U.S. math textbooks even devote entire sections to learning strategies for solving word problems like "guess and check," "work backwards," "make a table," and "act it out." But every day American students are confused, discouraged, and unable to solve them.

Why do word problems cause so much difficulty? Many people struggle with word

problems because words themselves are abstract. In 1977, Australian educator Anne Newman discussed five steps that students need to work through in order to solve a word problem successfully--
(1) reading the problem
(2) comprehending what was read
(3) transforming the words into a mathematical strategy
(4) applying a mathematical procedure and
(5) writing the answer.

Her research showed that over 50% of errors that children make occur in the first three steps--before they even begin to solve the problem!

In order to help children understand word problems, **teachers often focus on key words such as "more" and "times."** This strategy is useful but limited because key words don't help students understand the problem situation (i.e. what is happening in the problem). **Key words can also be misleading because the same word may mean different things in different situations**. Consider the following two examples:

> There are 7 boys and 21 girls in a class. How many **more** girls than boys are there?
> There are 21 girls in a class. There are 3 **times** as many girls as boys. How many boys are in the class?

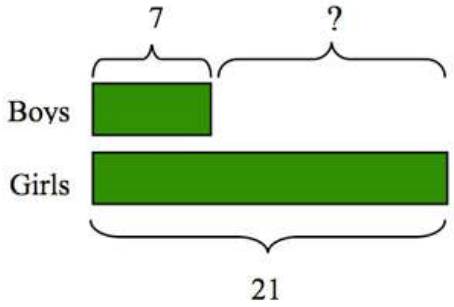
In the first problem, if students focus on the word "more" they may add when they actually need to subtract. In the second problem, if students focus on the word "times," they may multiply when they actually need to divide.

Instead of relying on ambiguous key words, Singapore math textbooks help students to visualize problem situations by turning abstract words into easy to understand pictorial models. By constructing a model we can understand the problem situation more clearly.

Let's think about the above problems again using a "bar model," the most common (but not the only) pictorial model used in Singapore math textbooks.

> There are 7 boys and 21 girls in a class. How many more girls than boys are there?

To illustrate and understand the problem, let's draw two bars. There are more girls than boys, so we'll draw a shorter bar to represent the boys and a longer bar to represent the girls. The question mark indicates what we are trying to find out--the difference between the number of boys from the number of girls.

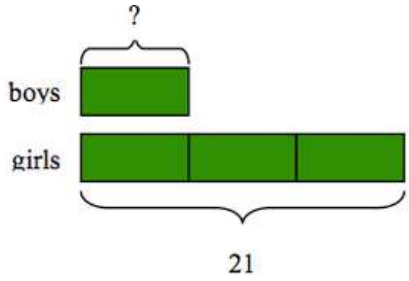


The diagram shows clearly that the answer is less than 21 (so adding the two mounts definitely won't work). By looking at the model it is easy to see that we need to subtract 7 from 21 to find the answer.

Now let's use a bar model to think about the second problem.

There are 21 girls in a class. There are 3 times as many girls as boys. How many boys are in the class?

Again we'll draw two bars to illustrate the situation, one for the boys and another one exactly three times as long for the girls (since there are three times as many girls). We are trying to find the number of boys, so we'll place a question mark above the part representing the boys.



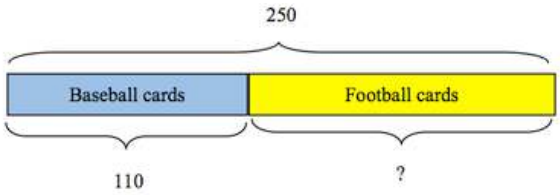
The diagram shows that the answer has to be less than 21 (so multiplying the amounts won't work). By looking at the bar model we can see that 3 units (parts) = 21. Since we want to find 1 unit (the number of boys) we should divide 21 by 3 to get the answer.

According to Singapore's Handbook for Mathematics Teachers in Primary Schools, this model drawing approach is helpful for several reasons:

(1) It "helps pupils visualize situations"
(2) It "creates concrete pictures from abstract situations."
(3) It "satisfies the pupils' learning through seeing and doing."
(4) It "transforms words into recognizable pictures for young minds."

Two basic models are used to represent problem-solving situations in Singapore math--the part-whole model and the comparison model. The particular model that is used depends on the problem situation. The above examples use a "comparison model" because they involve a comparison of two quantities--the number of boys and the number of girls. When the word problem involves a whole and its parts, however, we use a part-whole model as in the following example.

Frank has 250 baseball and football cards altogether. He has 110 baseball card. How many football cards does he have?



In this problem we know the total number of cards (the whole) and the number of baseball cards (one part). We are trying to find the missing part (the football cards). By looking at the diagram we can see that 110 + ? = 250, so to find the answer we should subtract 110 from 250.

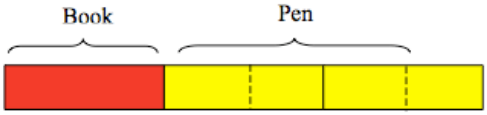
The usefulness of bar models becomes more apparent as students tackle more difficult mathematical concepts. Below is an example of how a bar model can be used to solve a challenging problem from the fifth grade Primary Mathematics textbook involving fractions, a topic many students struggle with.

Meihua spent 1/3 of her money on a book. She spent 3/4 of the remainder on a pen. If the pen cost $6 more than the book, how much money did she spend altogether?

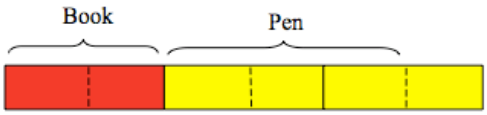
Solving a problem of this level of difficulty would normally require a considerable knowledge of algebra. It can be solved easily by 5th grade students, however, with a bar model.

Let's draw a bar to represent all of Meihua's money. We know that she spent 1/3 of her money on a book, so we'll divide it into thirds. One part (1/3) represents the money she spent on the book. The other two parts represent the remainder of her money.

The problem says that she spent 3/4 of the remainder on a pen. We know that 3/4 is 3 out of 4 equal parts, so let's cut each of the yellow sections in half to make 4 equal parts (the yellow section). Three of these parts represent the money she spent on the pen.

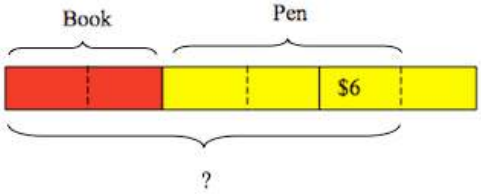


If we make all the parts the same size the problem will be easier to solve because if we can find the value of one unit, all the others will be the same. So let's divide the part representing the book into two equal parts also.



We now have a bar divided into six equal-sized units (parts), three representing the pen and two representing the book. We want to find the amount she spent on both items, which is 5 units.

From the diagram we can see that the difference between the pen and the book is one unit (the 3 pen units - the 2 book units). We know from the problem that this amount is equal to $6 since the pen costs $6 more than the book.



Remember that all the units are the same size and thus have the same value. If we can find the value of one unit, then we will know the value of all of them. So we can use the following method.

1 unit = $6
5 units = 5 x $6 = $30
Meihua spent $30 altogether.

Notice that this problem can be solved with simple multiplication; no algebra or complex procedures involving fractions required.

It takes time to develop model-drawing skills in students, but it is well worth the effort. As children become increasingly proficient at constructing models they gain

confidence in their problem-solving abilities. **Proficiency in the use of model drawing helps students to solve increasingly complex problems as they progress** through elementary and middle school mathematics and eventually make the transition from arithmetic to algebra with greater ease.

Some U.S. mathematics textbooks also incorporate model drawing, but many different models are **typically introduced in a haphazard and unconnected fashion with inadequate development.** The result is that the type of model drawing used in Singapore is essentially non-existent in the U.S. and few American students or even teachers ever learn this powerful problem-solving technique.

Singapore math textbooks, on the other hand, present and develop a few **consistently used models in a clear and systematic way**. The result is that students become adept at **converting abstract word problems into concrete pictures** that are easily translated into mathematical procedures. A recent study reported that Singaporean students are exposed to higher-level, multi-step word problems than are U.S. students, and proficiency in solving these complex problems is a key factor in why they have fared so well on international mathematics assessments.

If children are to learn how to solve word problems effectively, **teachers** need better ways to help them understand them. Pictorial models are an important intermediate step between the difficult transition of reading a word problem to determining the operations and steps necessary to solve it. Singapore's model drawing approach **helps children to get past the words by visualizing and illustrating** word problems with simple diagrams. And as children become better and more confident problem solvers, they become more interested in mathematics.

That's all for now. If you would like more information on solving problems with bar models, I have attached a power point presentation that will help you hone your model drawing skills. Next time I'll share some tips for schools that are interested in using Singapore math.

Until then,
Bill Jackson
Math Helping Teacher
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Part 1: Singapore Math: Can It Solve Our Country's Math-phobia?

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Check out 2min. video from a Singapore math school in Phoenix - Channel 12 - below:

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