Access-orizing ratio and percentage with the unitary method

California Mathematics Council - South
November 15, 2019
Session #343

BEAUTY & BRILLIANCE
CMC-S 2019
60TH ANNIVERSARY

Math Champions
Professional Development, LLC

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@Cassyt Twitter Bio: Passionate about Math, Teacher Trainer/Coach, Treasure Hunter, Learner, Answer to the ultimate question? 42.

Today's Agenda

- Some basic bar models
  (you may call them tape diagrams)
- Some ratio problems
- Some percentage problems
- Q & A

What is a Tape Diagram?

"A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model."

-CCSS Glossary

NAME _____

Lesson 3.1 Understanding Ratios

Ratios can be written based on the number of objects in a set.

There are 2 bottles of soda and 5 bottles of water in the refrigerator. Write the ratio of sodas to waters. $\frac{2}{5}$

Express each ratio as a fraction in simplest form.

a	b
1. There are 2 cubes and 15 spheres in a geometry box. Write the ratio of spheres to cubes. $\frac{15}{2}$	There are 5 cars and 4 vans in a parking lot. Write the ratio of vans to cars. $\frac{4}{5}$
2. There are 5 horses and 15 elephants in a circus. Write the ratio of elephants to horses. $\frac{3}{1}$	There are 16 horses and 14 elephants in a circus. Write the ratio of horses to elephants. $\frac{8}{7}$
3. There are 11 blue marbles and 7 red marbles in a box. Write the ratio of red marbles to blue marbles. _____	There are 12 apples and 15 oranges in a fruit basket. Write the ratio of apples to oranges. _____
4. There are 5 blue marbles and 16 red marbles in a box. Write the ratio of blue marbles to red marbles. _____	There are 12 dogs and 7 cats in a park. Write the ratio of cats to dogs. _____
5. There are 14 cars and 7 vans in a parking lot. Write the ratio of cars to vans. _____	There are 7 blue marbles and 8 red marbles in a bag. Write the ratio of red marbles to blue marbles. _____
6. There are 6 pennies and 10 dimes in a jar. Write the ratio of pennies to dimes. _____	There are 24 butterflies and 16 snails on the ground. Write the ratio of butterflies to snails. _____

CCSS.MATH.CONTENT.6.RP.A.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about … equivalent ratios, tape diagrams… or equations.

CCSS.MATH.CONTENT.6.RP.A.3.C

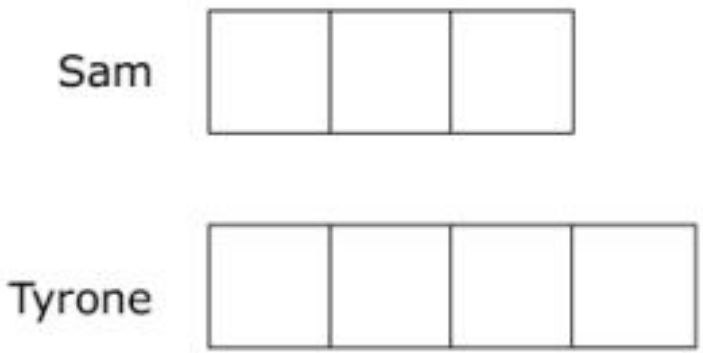
…solve problems involving finding the whole, given a part and the percent.

CCSS.MATH.CONTENT.7.RP.A.3

…solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#12	3	RP	F	2	6.RP.A.3	2, 3

Sam and Tyrone each build model cars. Altogether, Sam and Tyrone have built 42 model cars. The ratio of cars Sam has built to cars Tyrone has built is represented by the tape diagram shown.



Enter the number of cars represented by each square in the tape diagram.

Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#18	1	RP	A	2	6.RP.A.3c	N/A

Ethan correctly answers 80% of the total questions on his history test. He correctly answers 32 questions.

Enter the number of questions on Ethan's history test.

Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#22	1	RP	A	2	6.RP.A.3b	N/A

A bus travels 36 miles in 45 minutes.

Enter the number of miles the bus travels in 60 minutes at this rate.

Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#13	1	RP	A	2	7.RP.A.3	N/A

Sara buys a sweater at a department store. The sweater costs $30. The store is having a 25% off sale on everything in the store.

Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

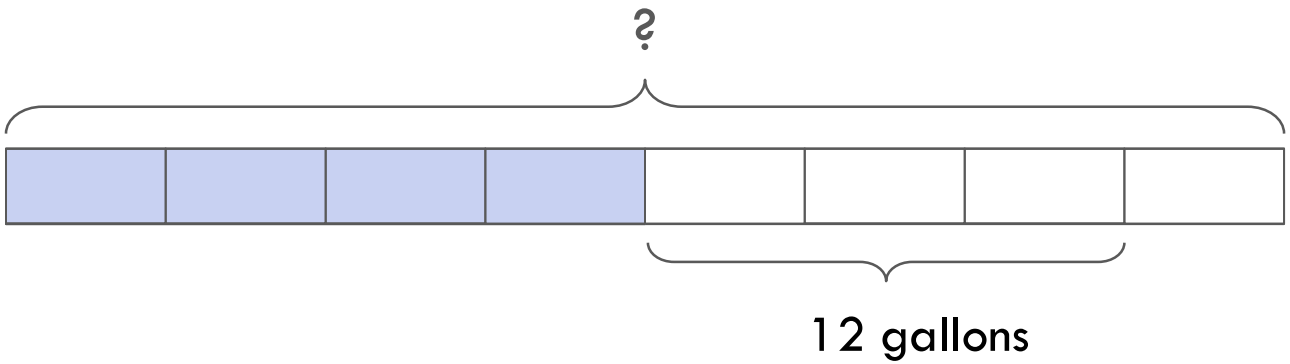
Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#25	2	RP	A	2	7.RP.A.3	1, 7

Lenny bought a motorcycle. He paid 12.5% in tax. The tax added $1437.50 to the price of the motorcycle.

What was the price of the motorcycle, not including the tax?

Part-Whole models

A wading pool is half filled with water. When ▭ more gallons of water are added, the pool is $\frac{7}{8}$ full. How many gallons of water can the wading pool hold?



3 units $\longrightarrow$ 12 gallons

1unit $\longrightarrow$ $12 \div 3 = 4$

8 units $\longrightarrow$ $4 \times 8 = 32$

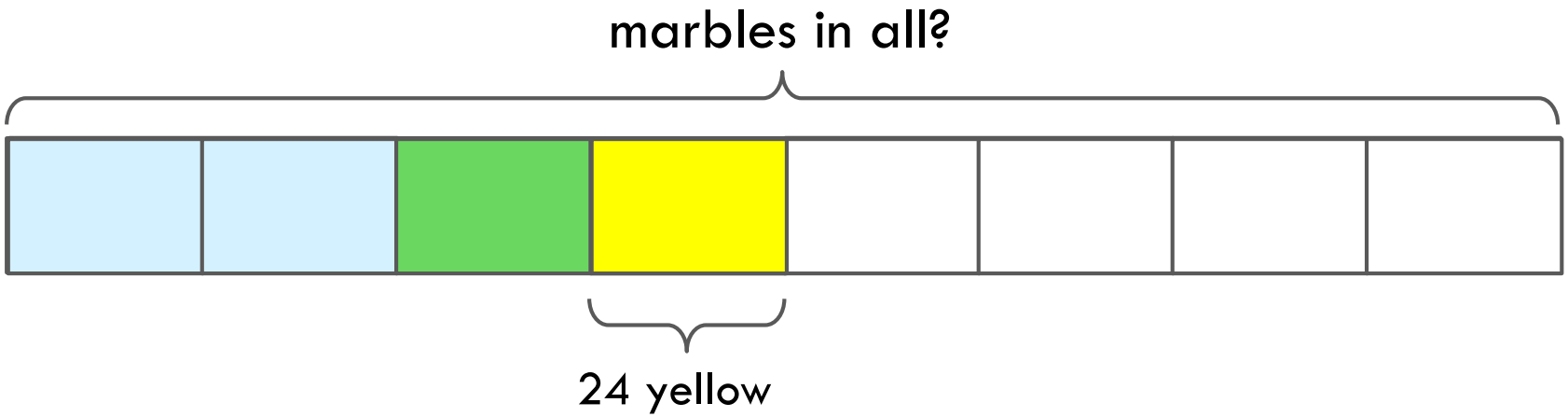
3 units $\longrightarrow$ 12 gallons

1unit $\longrightarrow$ $\frac{12}{3}$

8 units $\longrightarrow$ $\frac{12}{3} \times 8 = 32$

The wading pool can hold 32 gallons.

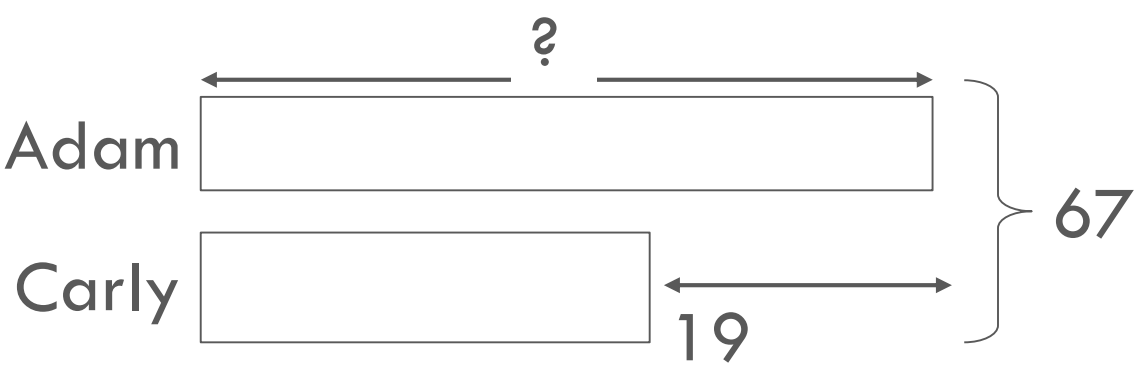
Tyrone bought a bag of marbles. $\frac{1}{4}$ of the marbles were blue, $\frac{1}{8}$ were green, and $\frac{1}{5}$ of the remainder were yellow. If there were 24 yellow marbles, how many marbles did he buy?



He bought 192 marbles.

Comparison Models

Adam and Carly have 67 baseball cards altogether.
If Adam has 19 more baseball cards than Carly, how many baseball cards does Adam have?



Let Carly = x

$x + (x + 19) = 67$

$2x + 19 = 67$

$\quad -19 \quad -19$

$2x = 48$

$x = 48/2 = 24$

Adam $\rightarrow$ 24 + 19

$67 - 19 = 48$

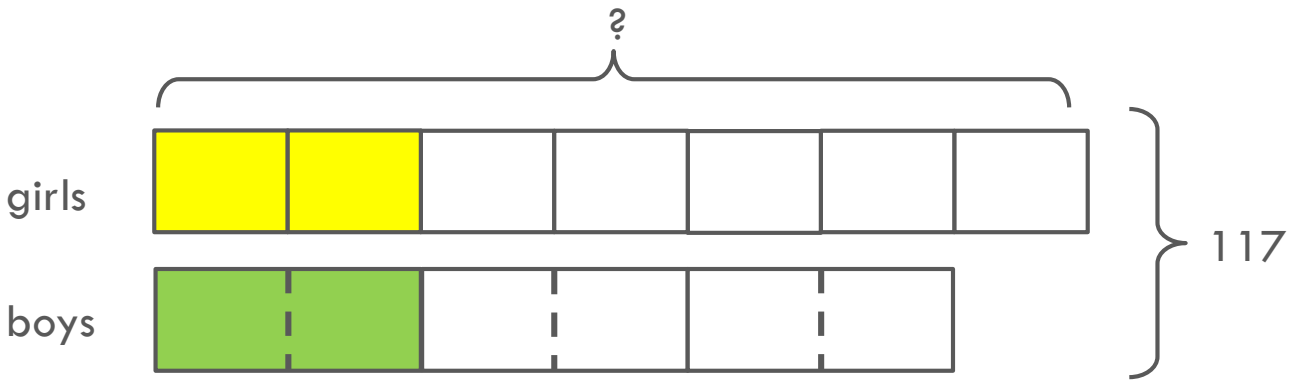
2 units $\rightarrow$ $48 \div 2 = 24$

Adam $\rightarrow$ $24 + 19 = 43$

Adam has 43 baseball cards.

117 children took part in a track meet.

$\frac{2}{7}$ of the number of girls that participated is equal to $\frac{1}{3}$ the number of boys. How many girls took part in the track meet?



13 units ⟶ 117 students

1 unit ⟶ 117 ÷ **13** = 9

7 units ⟶ **7** x 9 = 63 girls

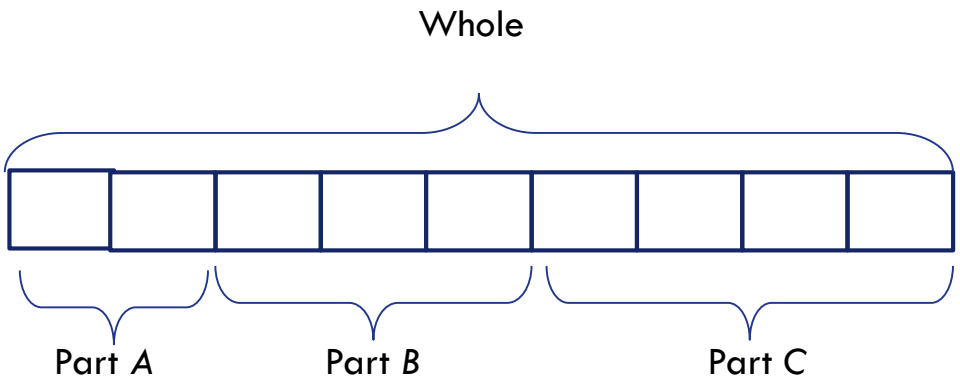
13 units ⟶ 117 students

1 unit ⟶ $\frac{117}{13}$

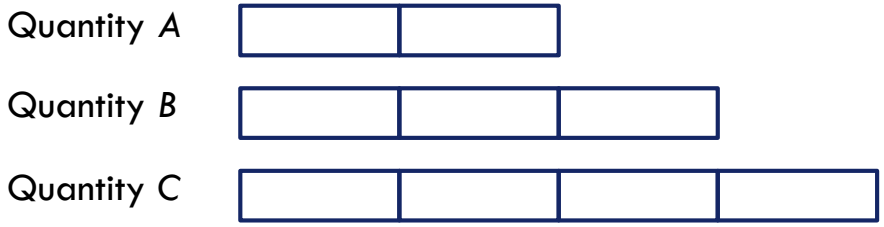
7 units ⟶ 7 x $\frac{117}{13}$ = 63 girls

There were 63 girls at the track meet.

Part-whole model shows a whole divided into three parts A, B, and C in the ratio 2:3:4.



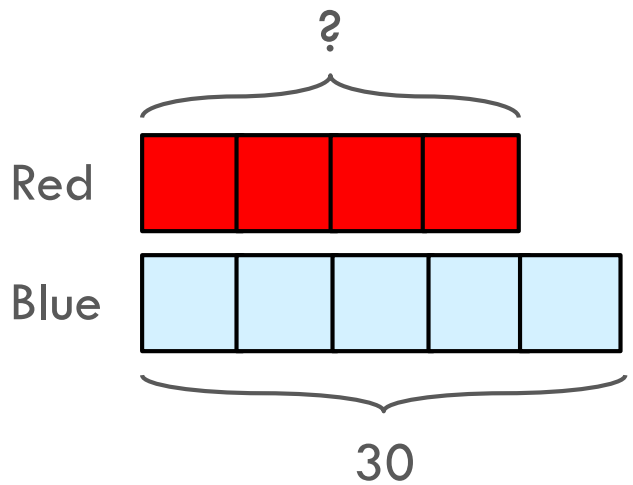
Comparison model shows three quantities A, B, and C which are in ratio 2:3:4.



The ratio 2:3:4 means “2 units to 3 units to 4 units”.

Ratio Models

The ratio of the number of red and blue cars is 4 : 5.
If there are 30 blue cars, how many red cars are there?



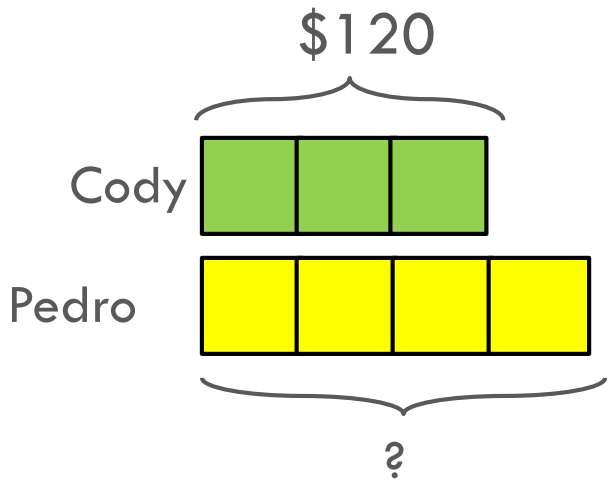
5 units → 30
1 unit → 30 ÷ 5 = 6
4 units → 4 x 6 = 24

5 units → 30
1 unit → $\frac{30}{5}$ = 6
4 units → 4 x 6 = 24

$\frac{4}{5}$ x 30 = 24

There are 24 red cars.

Cody and Pedro share money in the ratio of 3 : 4.
Cody has $120. How much money does Pedro have?



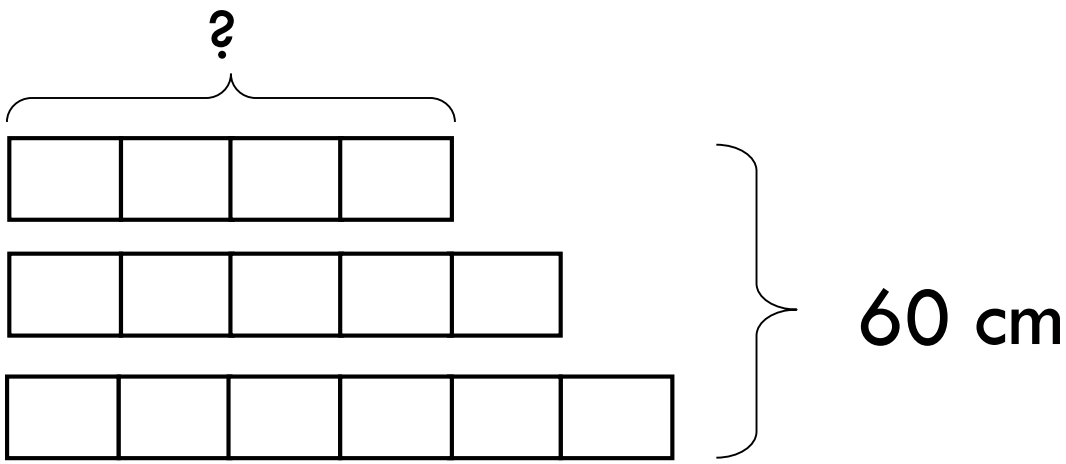
3 units → 120
1 unit → 120 ÷ 3 = $40
4 units → 40 x 4 = $160

3 units → 120
1 unit → $\frac{120}{3}$ = $40
4 units → 40 x 4 = $160

$\frac{4}{3}$ x $120 = $160

Pedro has $160.

The sides of a triangle are in the ratio 4:5:6. If the perimeter of the triangle is 60 cm, find the length of the shortest side.



15 units $\rightarrow$ 60 cm

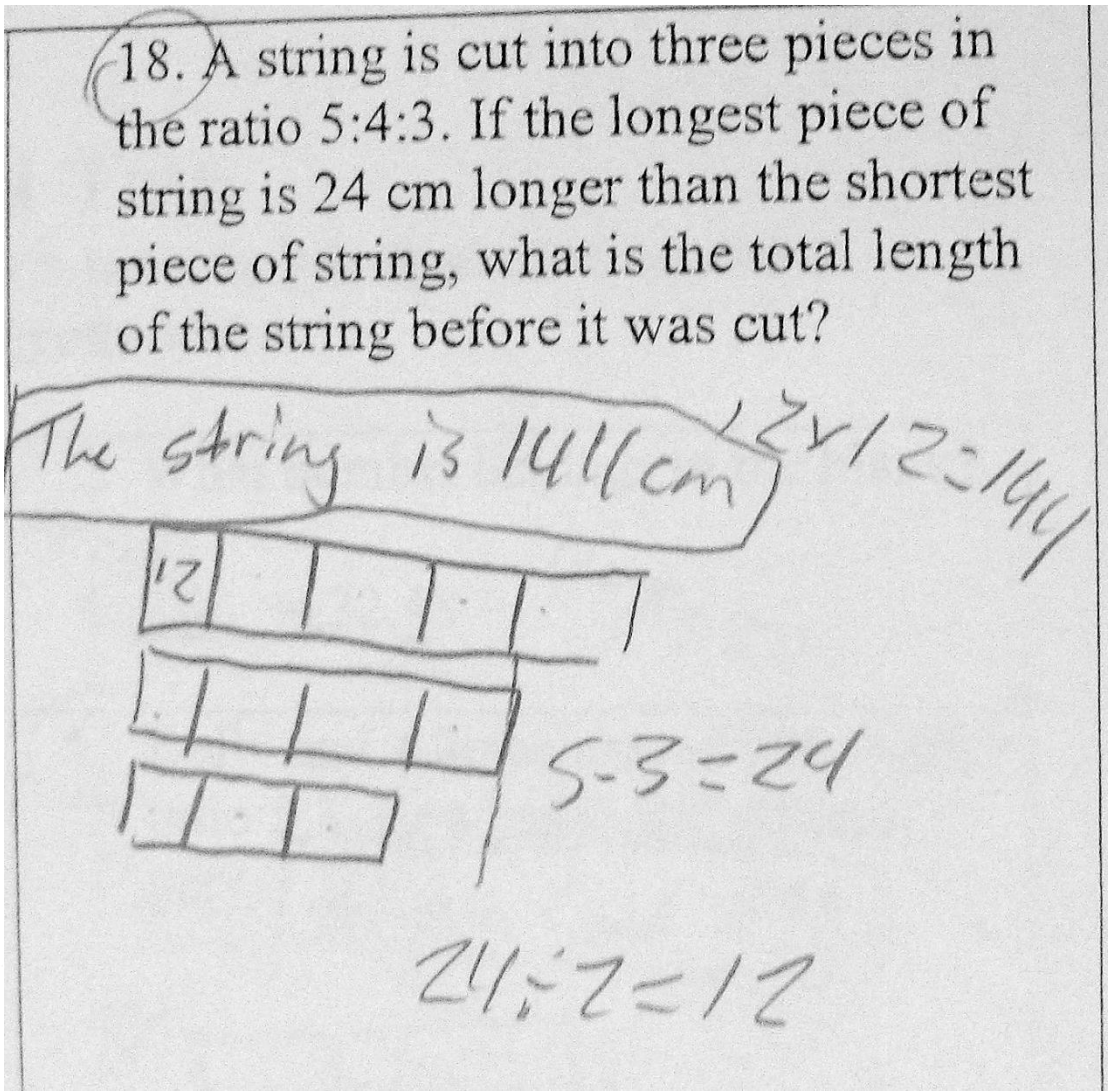
1 unit $\rightarrow$ 60 cm ÷ 15 = 4

4 units $\rightarrow$ 4 x 4 =16

$$60 \times \frac{4}{15} = 16$$

The shortest side is 16 cm long.

A string is cut into three pieces in the ratio of 5:4:3. If the longest piece of string is 24 cm longer than the shortest piece of string, what is the total length of the string before it was cut?


18. A string is cut into three pieces in the ratio 5:4:3. If the longest piece of string is 24 cm longer than the shortest piece of string, what is the total length of the string before it was cut?

The string is 144 cm

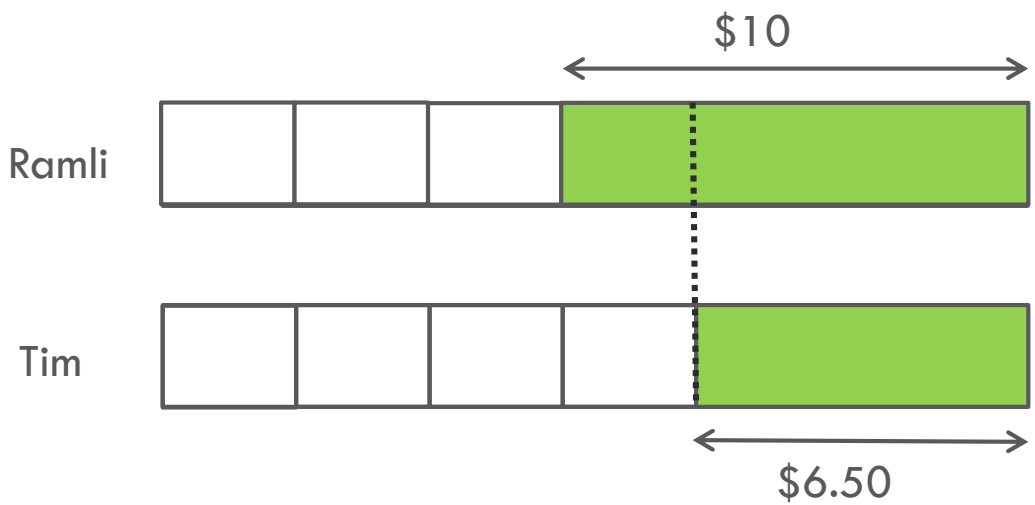
12×12=144

12

5-3=24

24÷2=12

Ramli and Tim had the same amount of money at first. After Ramli spent $10 and Tim spent $6.50, the ratio of Ramli's money to Tim's money was 3 : 4. How much money did they each have at first?



1 unit ⟶ $10 - $6.50 = $3.50

4 units ⟶ $3.50 x 4 = $14

$14 + $6.50 = $20.50

They both had $20.50 at first.

A 42-ounce mix of nuts is 6 parts peanuts to 1 part cashews.

How many ounces of cashews must be added to make the mixture 2 parts cashews to 1 part peanuts?

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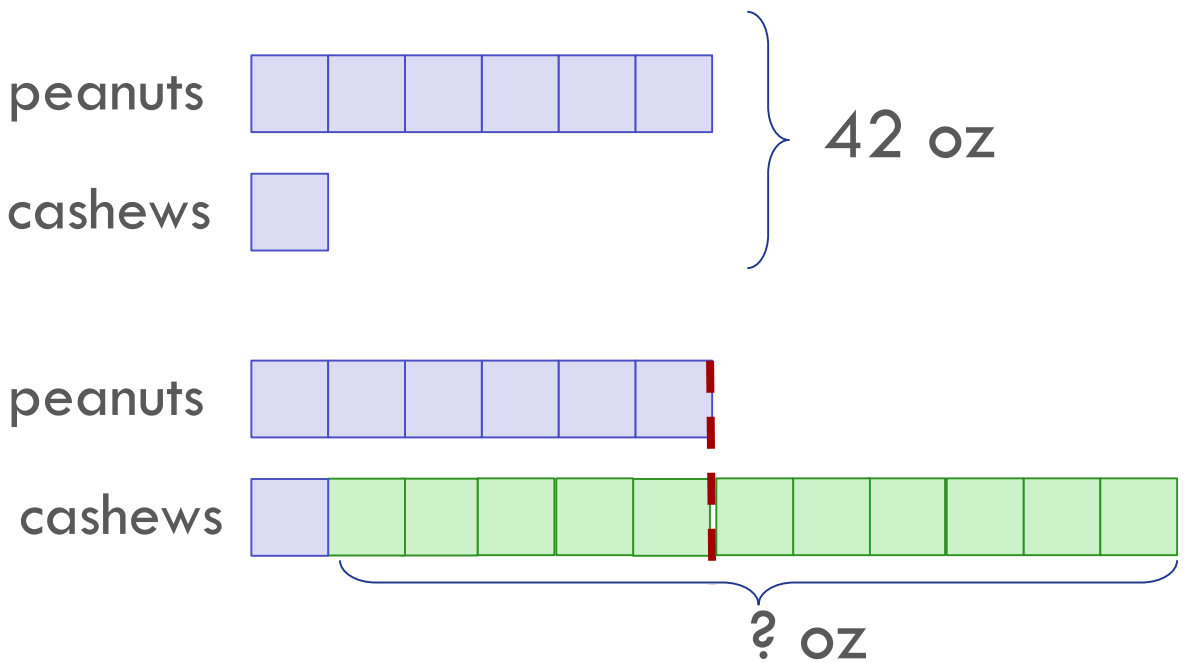


7 units $\rightarrow$ 42 oz

1unit $\rightarrow$ 42 $\div$ 7

= 6

11units $\rightarrow$ 6 x 11

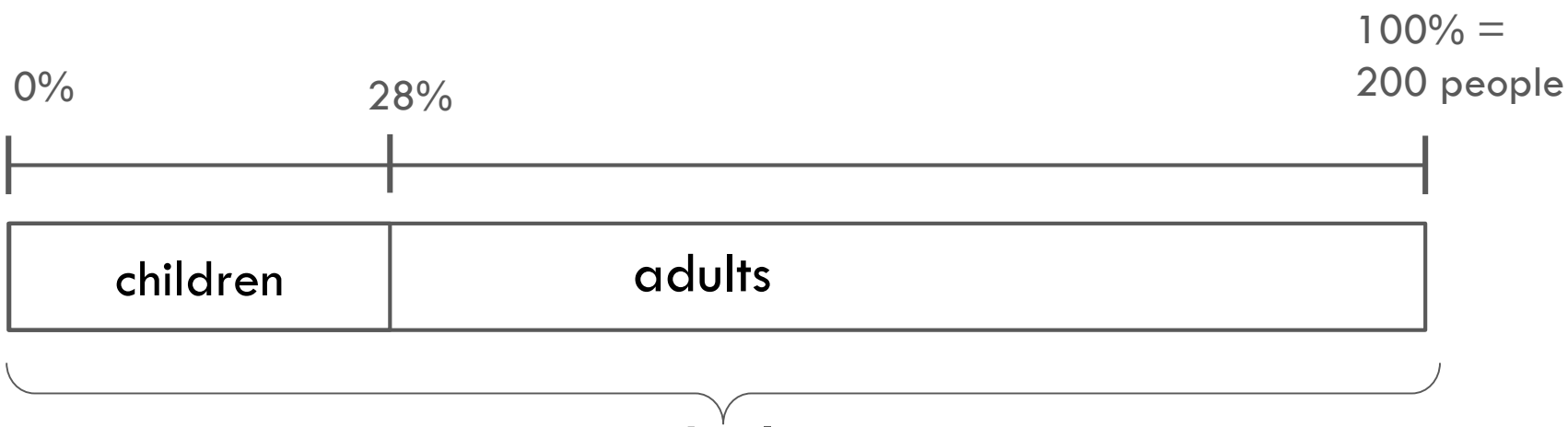
= 66 oz

$\frac{11}{7}$ x 42 = 66

Adding 66 ounces of cashews will make the mixture 2 parts cashews to 1part peanuts.

Percentage Models

There were 200 people at the movie. 28% of them were children and the rest were adults. How many children were at the movie?



$100\% \rightarrow 200$

$1\% \rightarrow \frac{200}{100}$

$28\% \rightarrow \frac{200}{100} \times 28 = 56$

$\frac{28}{100} \times 200 = 28 \times 2 = 56$

$0.28 \times 200 = 56$

There were 56 children at the movies.

Colin bought an iPad, which cost \$324 including 8 % sales tax. How much was the cost of the iPad?

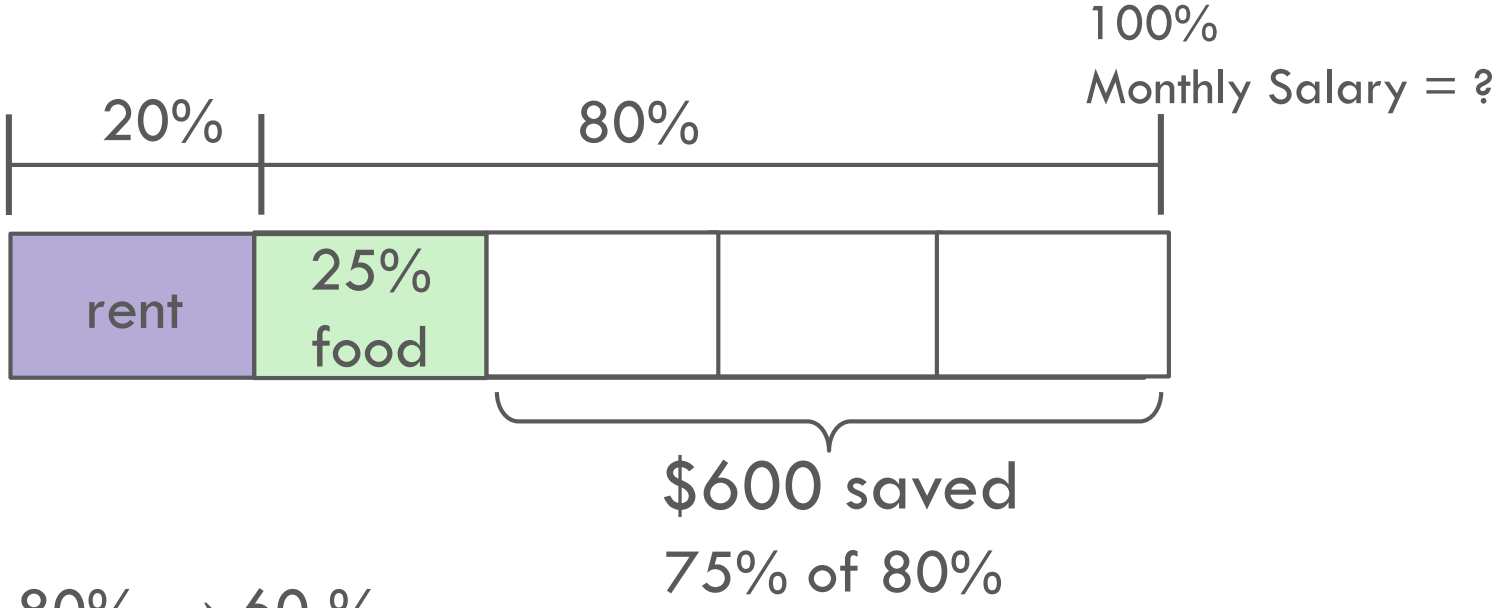
$108\% \rightarrow \$324$

$1\% \rightarrow \frac{324}{108}$

$100\% \rightarrow \frac{324}{108} \times 100 = \$300$

The iPad cost $300.

Rick spent 20% of his monthly salary on rent, 25% of the rest on food and saved the rest. If he saved $600 each month, how much was his monthly income?



75% of 80% → 60 %

60% → $600

1% → $\frac{600}{60}$

100% → $\frac{600}{60}$ x 100

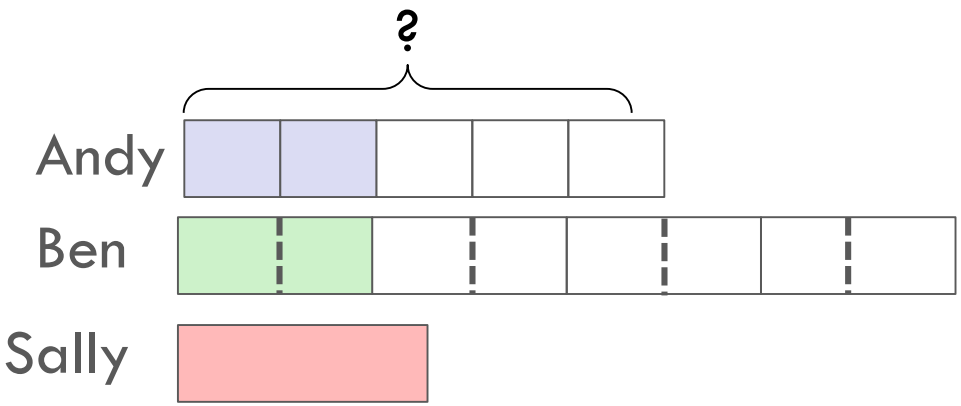
3 units → $600

1 unit → 600 ÷ 3 = 200

5 units → 200 x 5 = 1,000

Rick earned $1,000 a month.

Andy, Ben and Sally share a sum of money. 40% of Andy's money is equal to 25% of Ben's money. Sally has $\frac{1}{3}$ as much money as Ben. If Andy gives 20% of his money to Sally, she will have \$132. How much money does Andy have?



A : B = 5 : 8

B : S = 3 : 1

A : B : S = 15 : 24 : 8

Andy gives 20% of his money to Sally:

20% of 15 units = 3 units

8 units + 3 units → \$132

1 unit → $\frac{132}{11} = 12$

Andy has 15 units → 15 x 12 = \$180

Andy has \$180.

From the ACT COMPASS Math Placement exam:

At Big Al's restaurant, three cheeseburgers and two orders of fries cost \$5.60. Four cheeseburgers and three orders of fries cost \$7.80.

How much does a single cheeseburger and a single order of fries each cost separately?

$$3C + 2F = \$5.60$$
$$4C + 3F = \$7.80$$

$$C + F = ?$$

3 cheeseburgers and 2 orders of fries cost $5.60.
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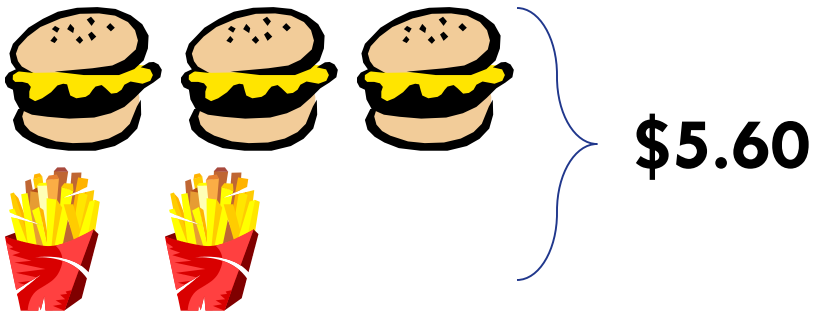
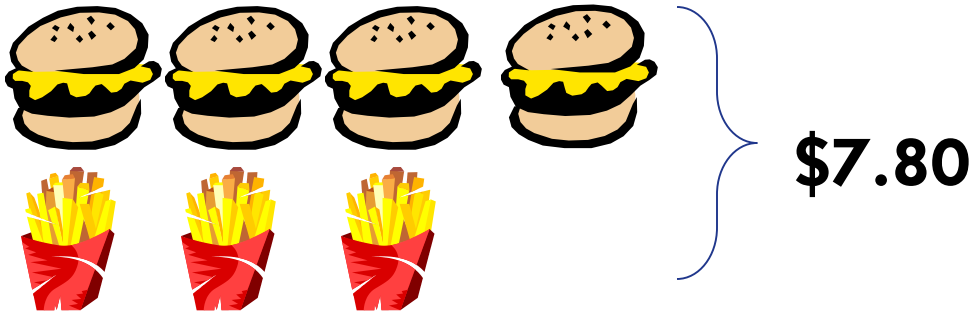
How much does a single cheeseburger and a single order of fries each cost separately?





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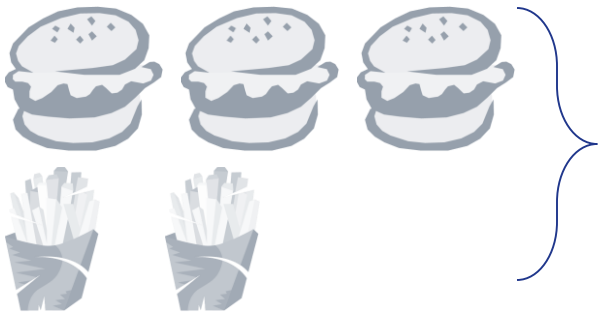
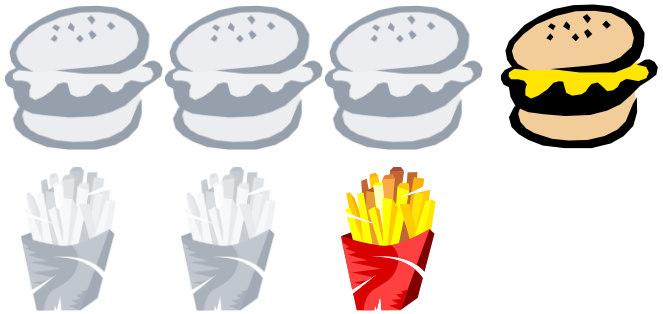


$5.60

1 cheeseburger and 1order of fries costs:

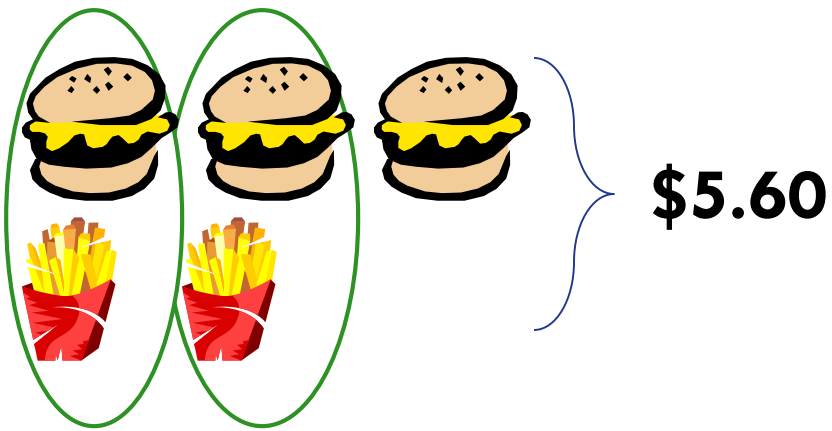


$7.80

$7.80 - $5.60 = **$2.20**

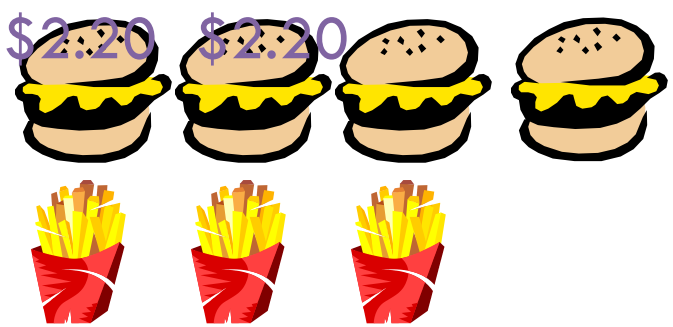
3 cheeseburgers and 2 orders of fries cost $5.60.
4 cheeseburgers and 3 orders of fries cost $7.80.

How much does a single cheeseburger and a single order of fries each cost separately?



$5.60

1 cheeseburger costs:
$5.60 - $2.20 - $2.20 = **$1.20**



$7.80

3 cheeseburgers and 2 orders of fries cost $5.60.
4 cheeseburgers and 3 orders of fries cost $7.80.

How much does a single cheeseburger and a single order of fries each cost separately?

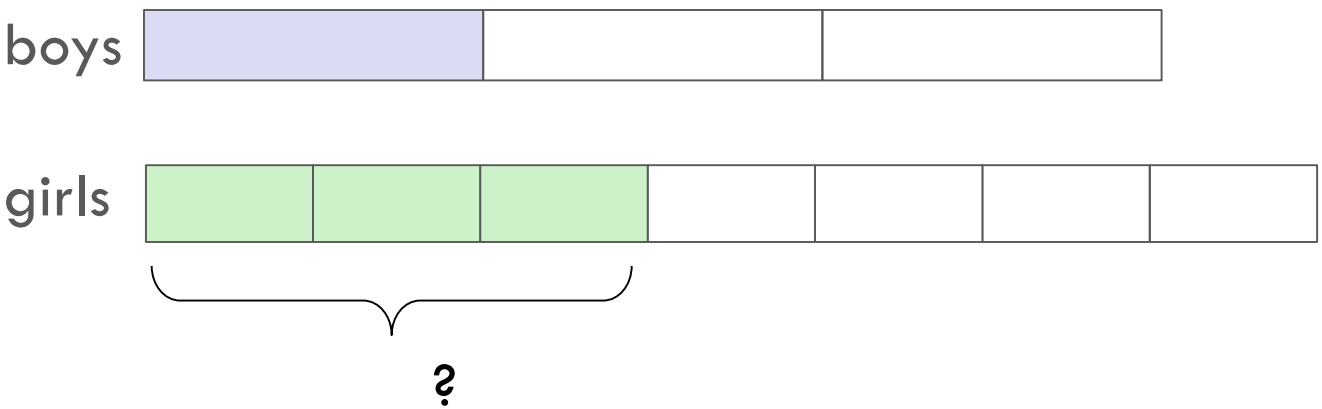
1 cheese burger and
1order of fries costs:

A single cheeseburger costs **$1.20**
A single order of fries costs **$1.00**

$7.80 - $5.60 = **$2.20**

From Jana's session this morning:

94 children are in a reading club. 1/3 of the boys and 3/7 of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?

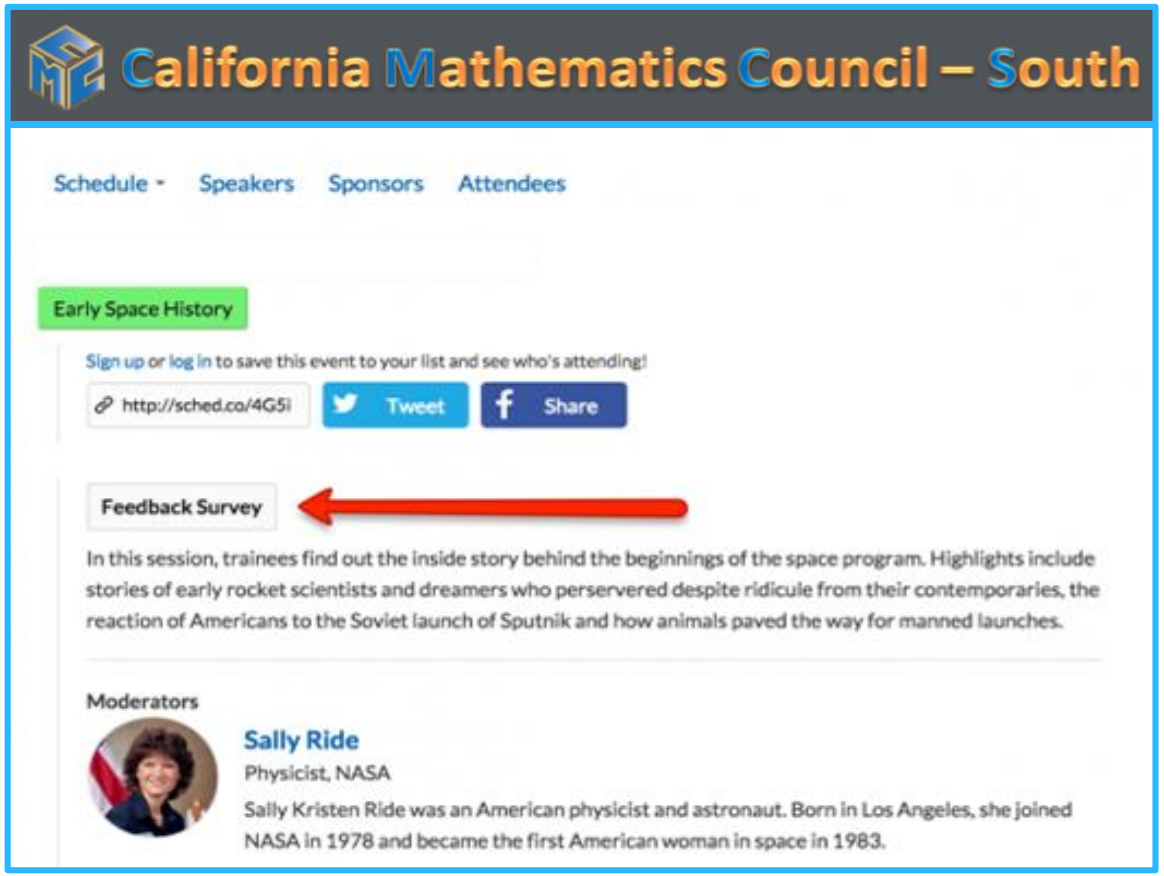


Questions?

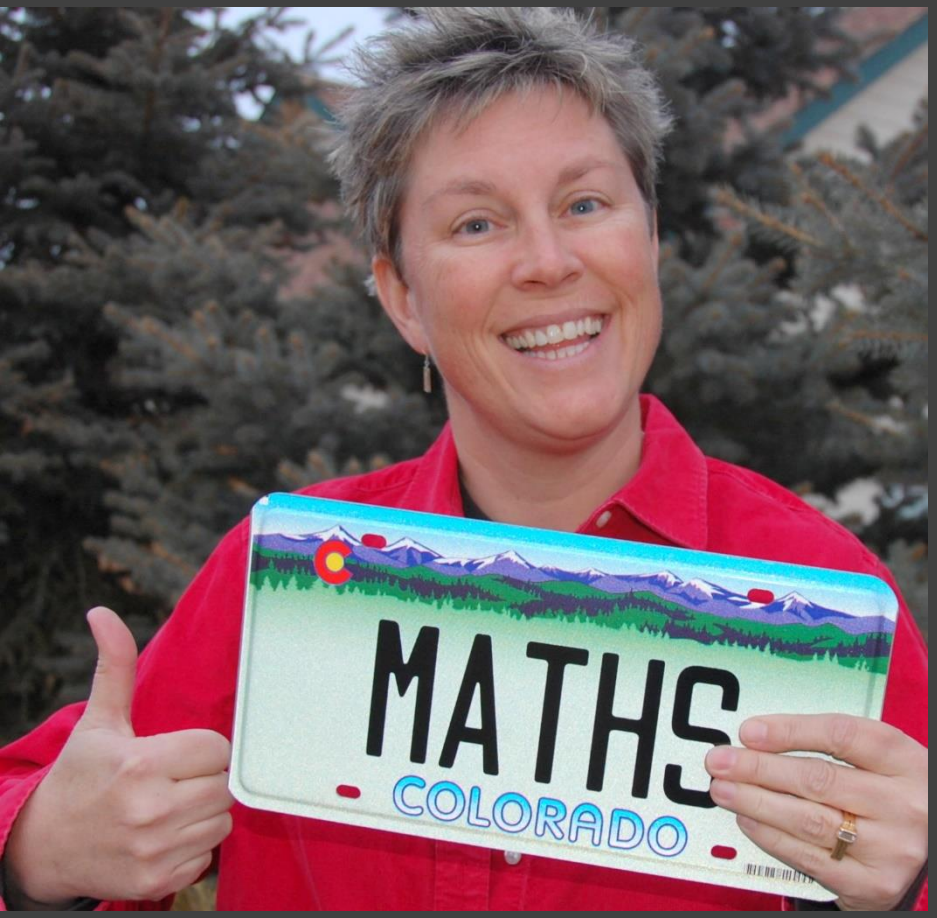
Preso/Handouts:

https://singaporemathsource.com/resources/handouts/

Speaker Evaluation



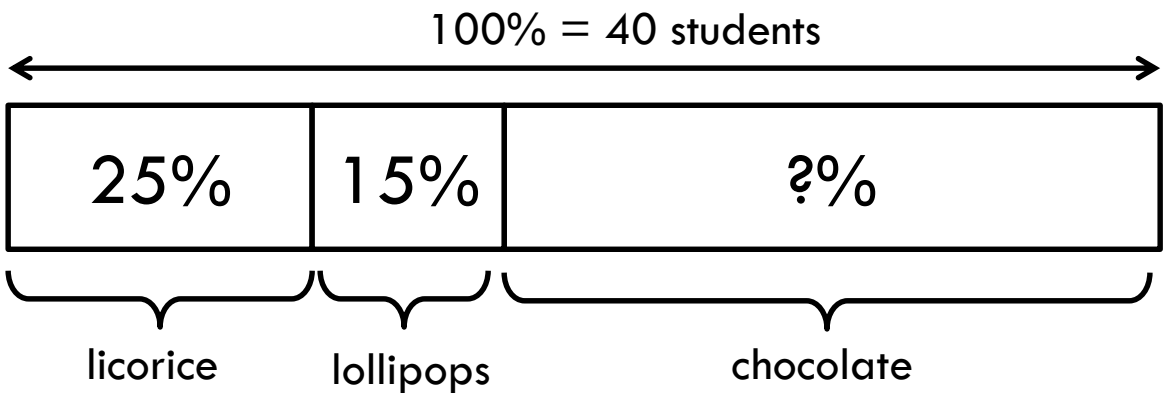
In the app
Go to this session's page, #343, then click the "Feedback Survey" button



Thanks for coming and have an outstanding conference!

Cassy@MathChampions.com

A class of 40 students chose their favorite candy. 25% of the students chose licorice, 15% chose lollipops and the rest chose chocolate. How many students chose chocolate as their favorite candy?



100 % - 25% - 15% = 60%

100% → 40 students

10% → 40 ÷ 10 = 4 students

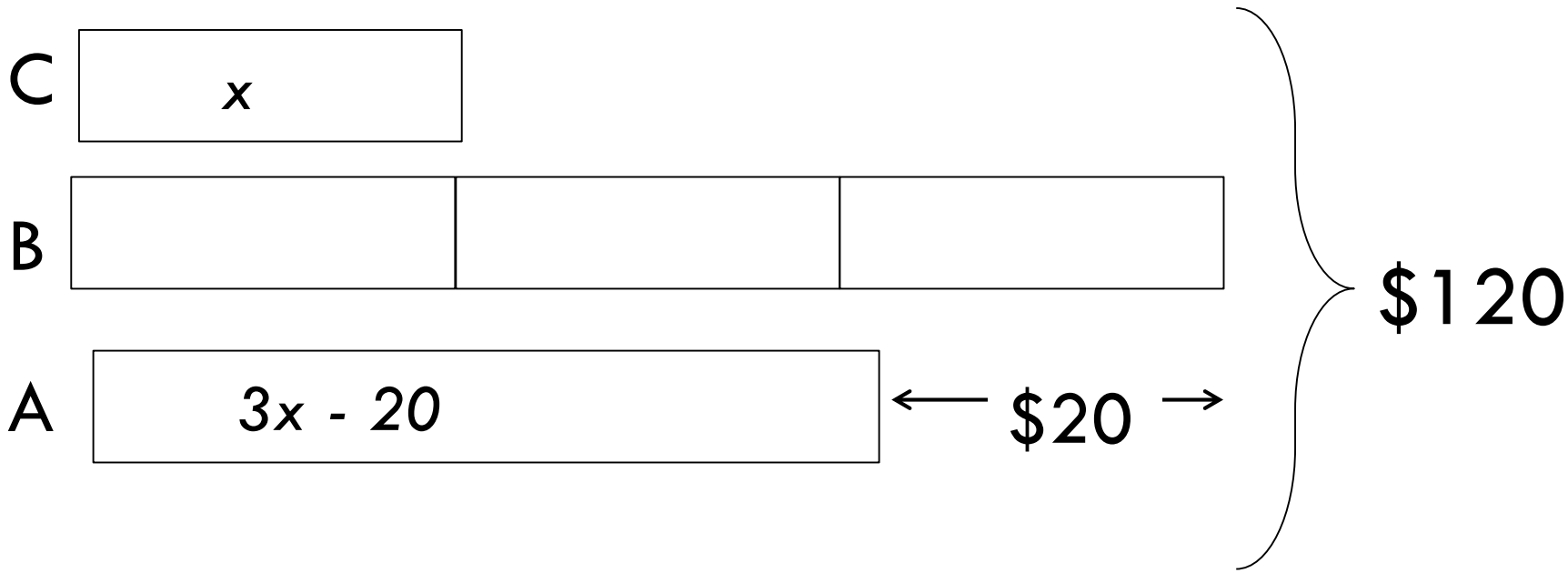
60% → 4 x 6 = 24 students.

.60 x 40 students

$\frac{60}{100}$ x 40 students

$\frac{60}{100} = \frac{?}{40}$ students

$120 is shared among 3 friends, A, B, and C. If A receives $20 less than B, and B receives 3 times as much money as C, how much does C receive?



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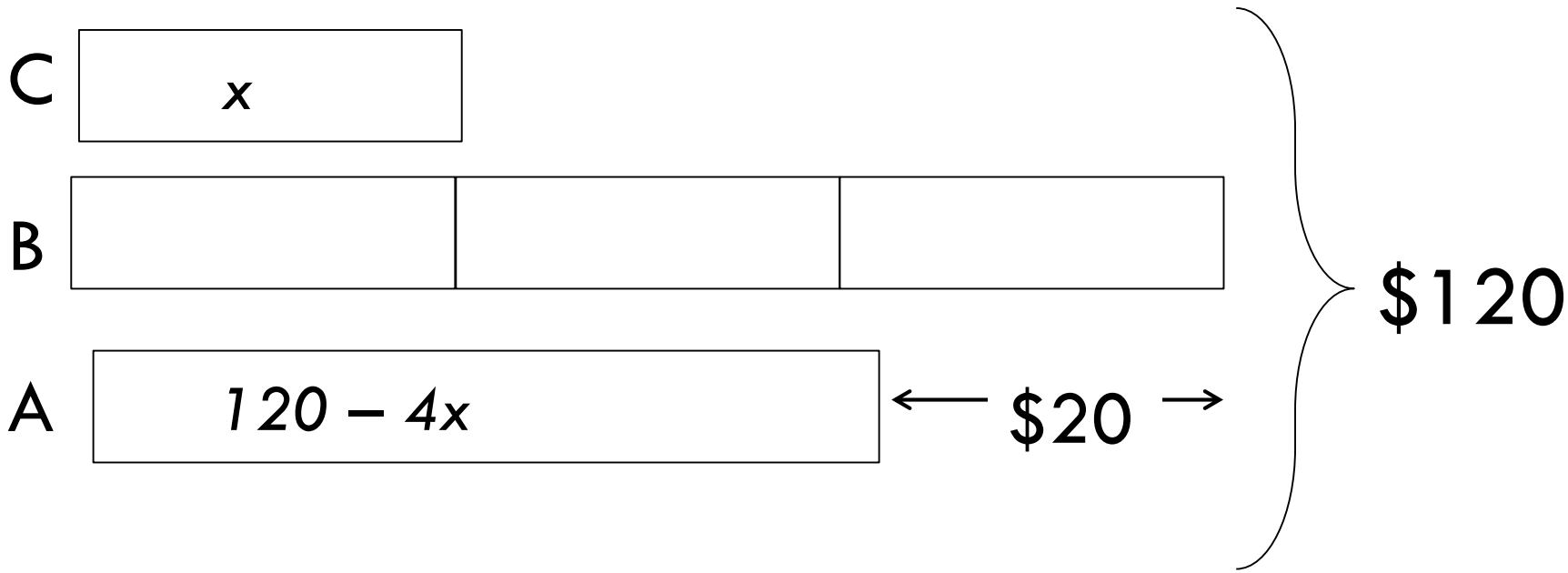
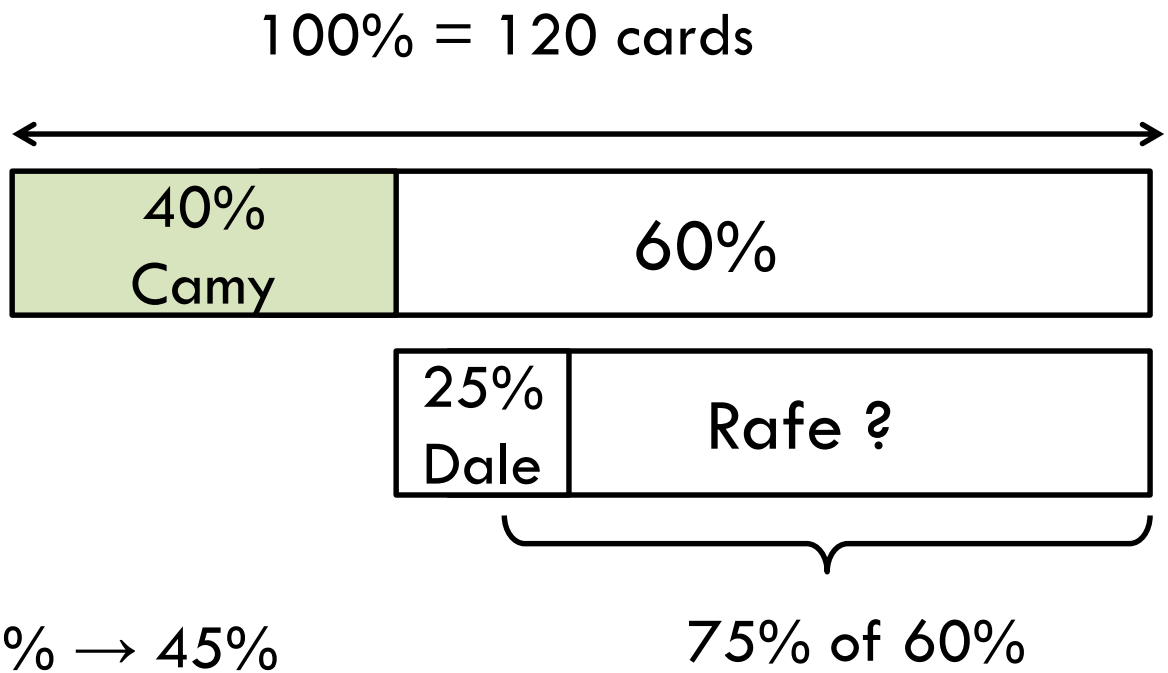


Camy, Dale and Rafe have 120 Pokémon cards altogether. If 40% of the cards are Camy's and 25% of the remainder are Dale's, how many Pokémon cards does Rafe have?



75% of 60% → 45%

100% → 120 cards

1% → 120/100

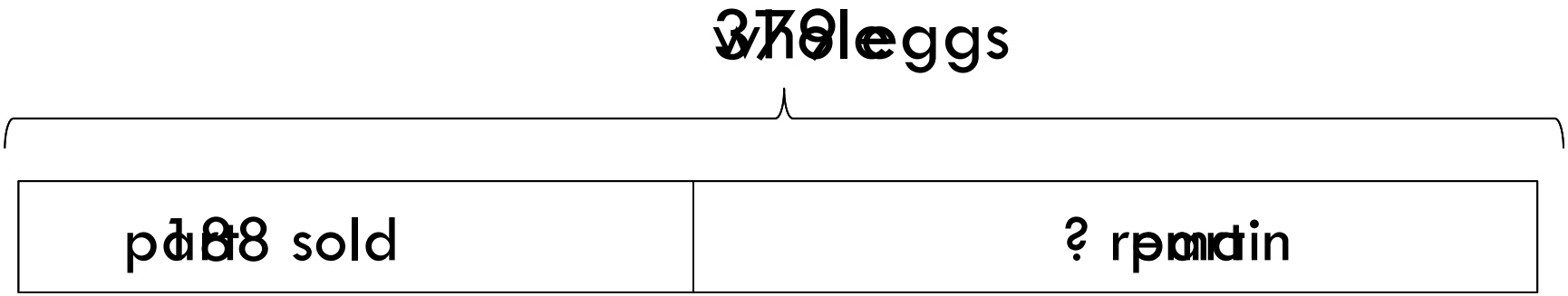
45% → 120/100 x 45 = 54 cards

Rafe had 54 Pokémon cards.

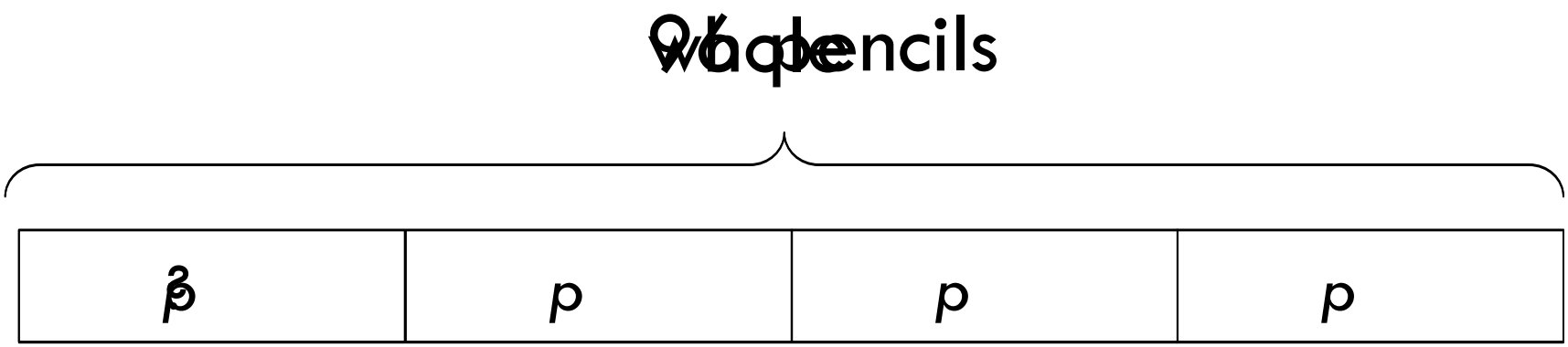
Part-Whole models

Jamie had 379 eggs. She sold 188 of them.

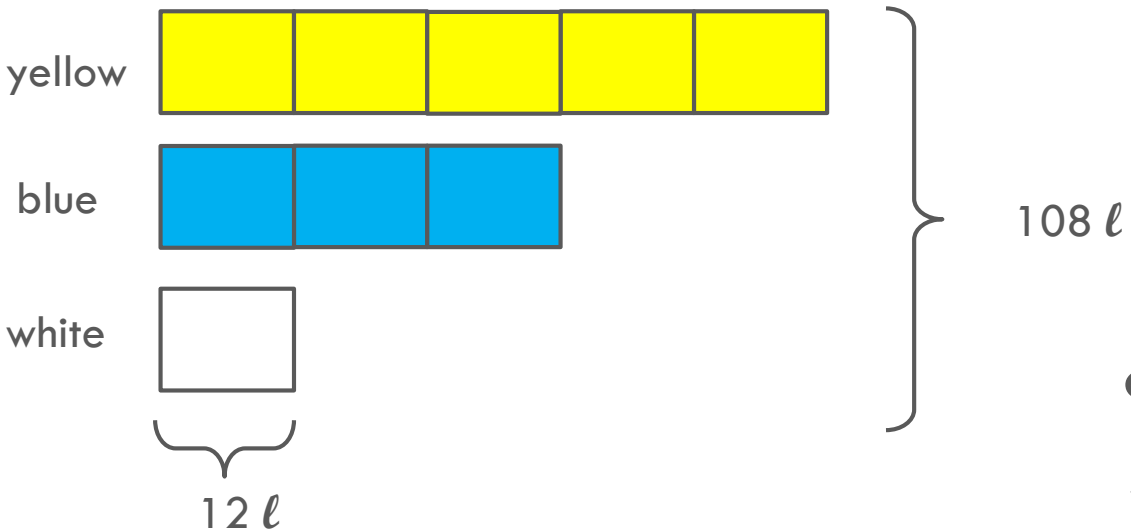
How many eggs did she have left?



Silvia shared 96 pencils among each of her four friends. How many pencils did Silvia have at first?



Mr. Johnson mixed yellow paint and blue paint and white paint in the ratio of 5 : 3 : 1 to make a light green paint for a painting job. If his final mixture for the painting job was 108 liters, how many liters of each color of paint did he use?



9 units → 108 ℓ

1 unit → 108 ℓ ÷ 9 = 12 ℓ

3 units → 12 ℓ x 3 = 36 ℓ

5 units → 12 ℓ x 5 = 60 ℓ