Word Problems and Bar Models

For FAIR School

Cassy Turner
Cassy@SingaporeMathSource.com
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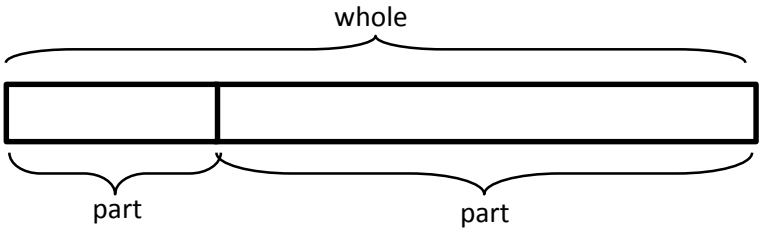
PART – WHOLE MODELS

In this model, the two quantities are considered the parts of the whole. The model provides a visual representation of the part-whole (or part-part-whole) relationship. Either part or the whole could be the unknown quantity.

Amy has 3 apples and 5 bananas. How many pieces of fruit does she have altogether?



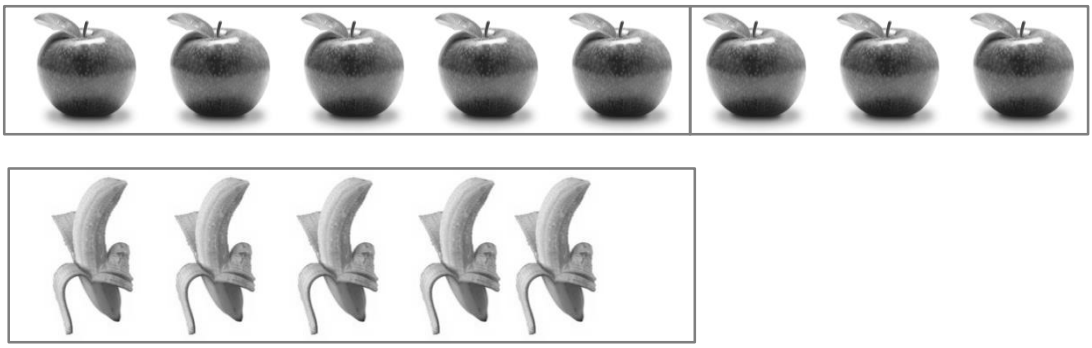
COMPARISON MODELS

Amy has 3 more apples than bananas. If Amy has 8 apples, how many bananas does she have?

To start, students use concrete manipulatives to match or compare the quantities of apples and bananas:

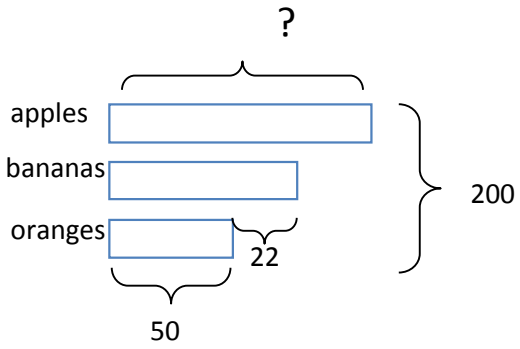
There are 8 apples. There are 3 more apples than bananas.

Next, students will use a pictorial model:



By **Grade 3** students will progress to the abstract comparison model.

This model provides a visual representation of the relationships between the three quantities.

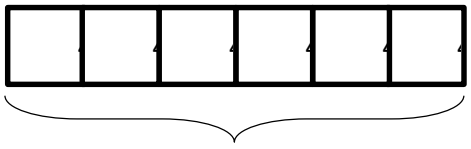


MULTIPLICATION AND DIVISION

Part-Whole Models for Multiplication and Division

Each chair has 4 legs. How many legs do 6 chairs have altogether?

At first, students begin with concrete representations when discussing equal groups.



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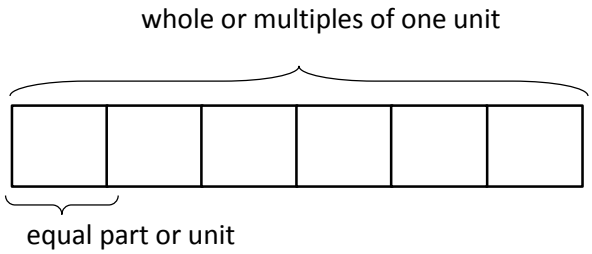
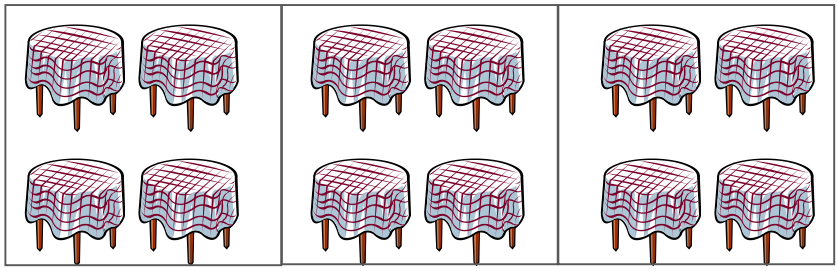
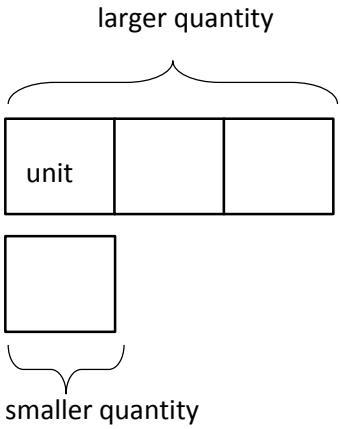
In the third grade and beyond, students will use a pictorial model that is more abstract. The part-whole relationship consists of equal parts.



COMPARISON MODELS FOR MULTIPLICATION AND DIVISION

There are 4 stools. There are 3 times as many tables as stools. How many tables are there?





The pictorial representation is introduced in second and third grade, and then progresses to the more abstract model in third grade. The comparison model for multiplication and division provides a visual representation of the relationships between the three quantities.

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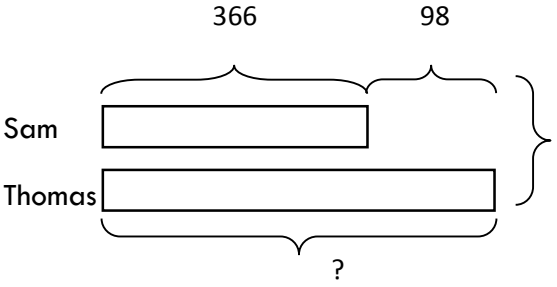
MULTI-STEP PROBLEMS

Multi-step problems combine 2 different operations - the most interesting cannot be classified as a +, - , x, or ÷ problem.

Part of the challenge of multi-step problems is keeping the work organized. When introducing bar model drawing, you may want students to begin by organizing their workspace. Be sure to give students plenty of space to work! One way to introduce two step problems is to have students divide their workspace in half and list the steps individually. Students may begin by drawing a separate model for each step.

Sam had 366 stamps in his collection. He had 98 fewer stamps than Thomas. How many stamps do they have altogether?

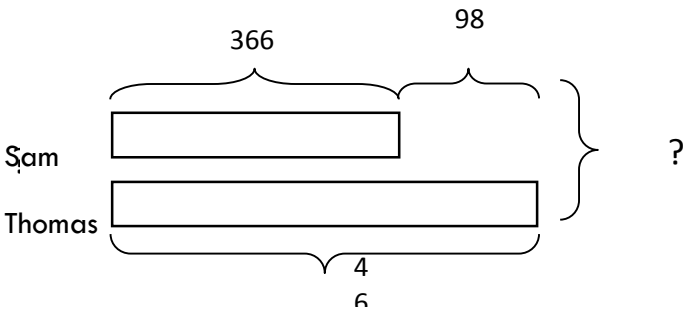
Step 1: Find how many stamps Thomas had.



366 + 98 = 464 stamps.

Thomas has 464 stamps.

Step 2: Find how many stamps altogether.



366
+ 464
830

They had 830 stamps altogether.

* Note: It is perfectly possible to solve this problem in a single step without finding the number of Thomas' stamps by adding: 366 + 366 + 98.

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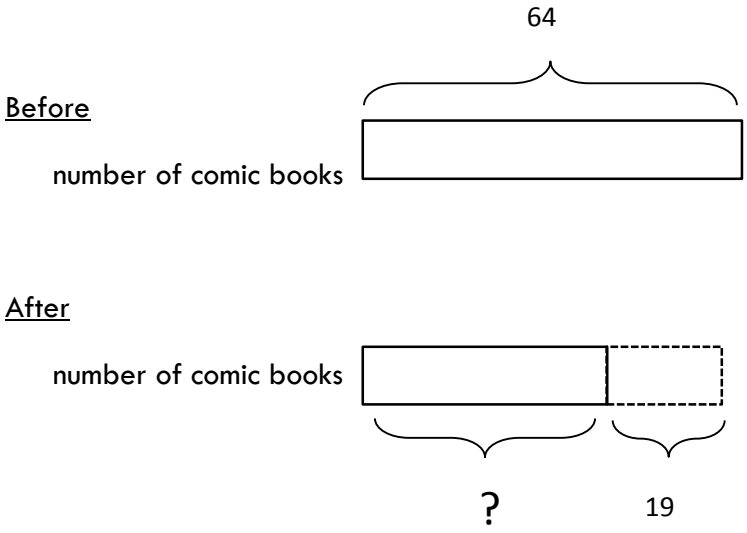
BEFORE AND AFTER MODELS

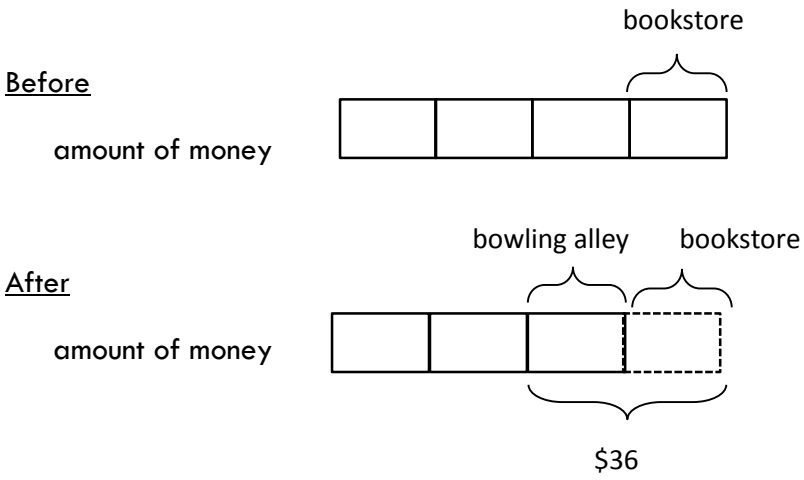
A change scenario can require a more complex bar model. Change problems are written as a *before* and *after* situation:

Julio had 64 comic books. He gave 19 to his little brother. How many comic books did Julio have left?



64 – 19 = 45 **Julio had 45 comic books left.**

Tara spent $\frac{1}{4}$ of her money at the bookstore. She spent $\frac{1}{3}$ of the money she had left at bowling alley. Altogether, Tara spent $36. How much money did she have at first?



2 ☐ → $36

1 ☐ → $36 ÷ 2 = $18

3 ☐ → $18 x 4 = $72

Tara had $72 at first.

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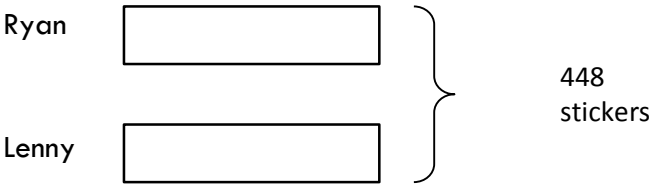
After Ryan gave 128 stickers to Lenny, they had the same number of stickers. If they had 448 stickers altogether, how many stickers did Ryan have at first?

For this problem, it's best to **begin at the end.** A dotted line or unit can represent a shift or movement of a bar:

After



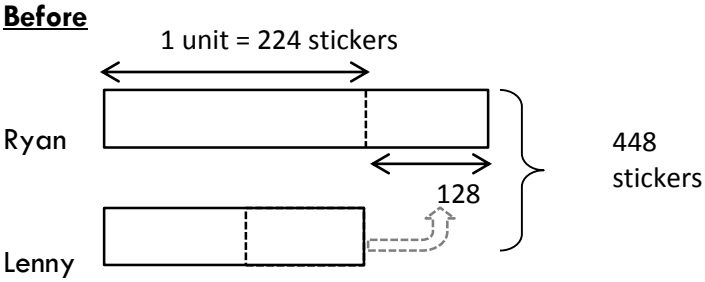
From this model: 2 □ → 448 stickers

1 □ → 448 ÷ 2

= 224 stickers

They each had 224 stickers at the end. Since the question asks, "How many did Ryan have at first?" we can put what we know into a *before* model:

Before

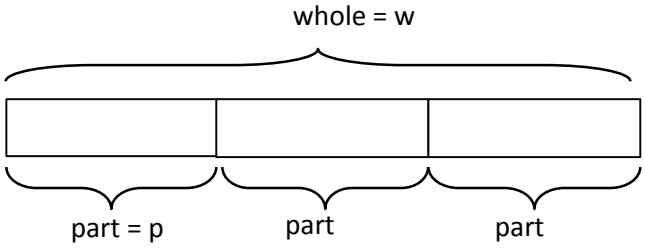


The number of stickers Ryan had at first → 224 + 128 = 352 stickers.

Ryan had 352 stickers at first.

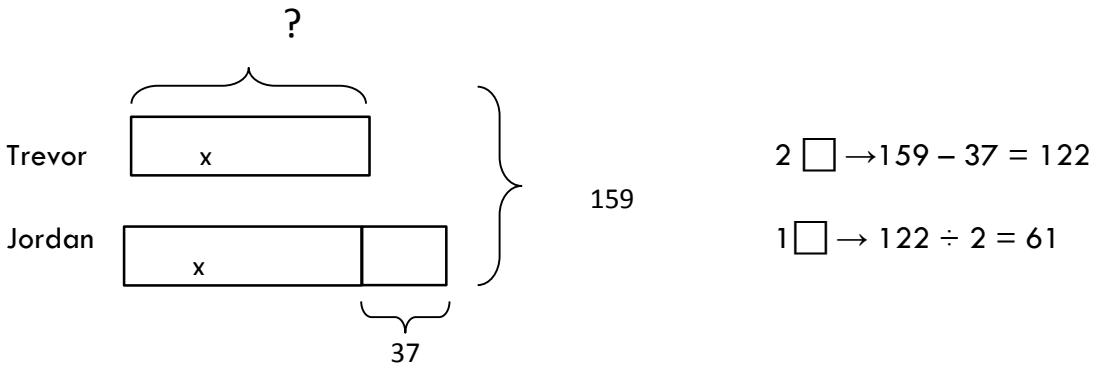
THE MODEL METHOD AND ALGEBRA

To solve many word problems, the goal is to find the unit. The unit can be considered the equivalent of finding x in algebraic problems.



$3p = w$

Trevor has 37 baseball cards less than Jordan. Altogether, the boys have 159 baseball cards. How many baseball cards does Trevor have?



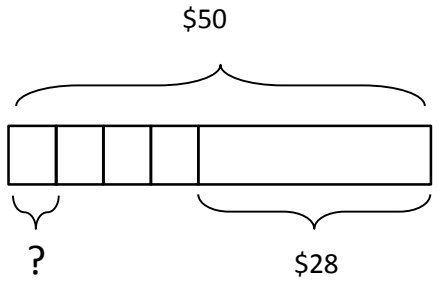
Algebraic equation for the problem:

$x + (x + 37) = 159$

$2x + 37 = 159$

Trevor has 61 baseball cards.

Giri bought 4 kg of grapes. She paid the cashier with a $50 bill and received $28 in change. Find the cost of 1kg of grapes.



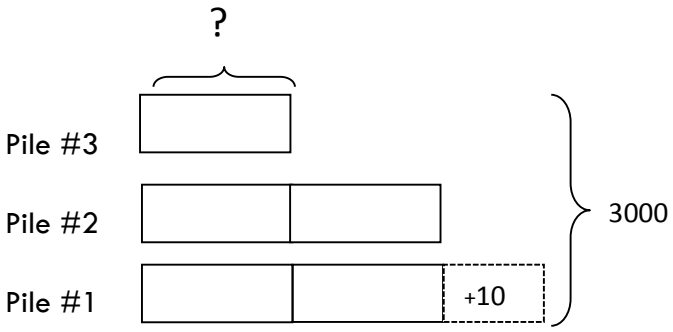
Algebraic equation for the problem: $4x + \$28 = \$50$

$\$50 - \$28 = \$22$

4 ☐ → $22

1 ☐ → $22 ÷ 4 = $5.50 **1kg of grapes cost $5.50.**

3,000 exercise books are arranged in 3 piles. The first pile has 10 more books than the second pile. The number of books in the second pile is twice the number of books in the third pile. How many books are there in the third pile?



Algebraic equation for the problem:

$x + 2x + (2x + 10) = 3000$

$5x + 10 = 3000$

5 ☐ → 2990

1 ☐ → 2990 ÷ 5 = 598 **The third pile has 598 books.**

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WORD PROBLEMS WITH BAR MODELING

A man sold 230 balloons at a fair in the morning. He sold another 86 balloons in the evening. How many balloons did he sell in all?

134 girls and 119 boys took part in an art competition. How many more girls than boys were there?

A bottle of hot sauce weighs 570 g. The empty bottle weighs 315 g. How many grams of sauce are in the bottle?

Jill bought a set of pens for $2.99, a notepad for $1.60 and a dictionary for $4.95. How much did she spend altogether?

Kendra made 284 chicken sandwiches for a party. She made 4 times as many chicken sandwiches as egg salad sandwiches. How many egg salad sandwiches did she make?

A class of 32 students was divided into groups of 4. How many groups of students were there?

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Hannah's pitcher held 16 liters of juice. She poured the juice equally into containers that held 0.8 of a liter. How many containers did she fill?

Sam and Tom had 366 stamps in their stamp collection. Sam had 98 fewer stamps than Tom. How many stamps did Tom have?

Tina, Jean and Ann shared 144 shells. Tina had 3 times as many shells as Jean. Jean had twice as many shells as Ann. How many shells did Tina have?

After giving away $\frac{3}{5}$ of her shell collection, Jane had 24 shells left. How many shells did she give away?

A wading pool is half filled with water. When 12 more gallons of water are added, the pool is $\frac{7}{8}$ full. How many gallons of water can the wading pool hold?

James bought a bag of sourballs. $\frac{1}{4}$ of the sourballs were cherry, $\frac{1}{8}$ were apple and $\frac{1}{5}$ of the remainder were blueberry. If there were 24 blueberry sourballs, how many sourballs did he buy?

On a shopping trip, Joonha spends $\frac{1}{3}$ of his money at the Sport Court. He then spends $\frac{1}{3}$ of the money he has left at June's Tunes. Finally, he spends his remaining $40 on a baseball glove. How many dollars did Joonha have at first?

150 people attended a movie. 40% of them were students. How many students attended the movie?

Ahmad had $400; He spent 31% of his money on an ipod and 37% on a bike.
What percentage of his money was left?
How much money was left?

The ratio of the number of hamburgers to the number of veggie burgers to the number of hot dogs sold at the fair is 4 : 1 : 5. If there were 4732 hamburgers sold at the fair, how many hot dogs were sold?

Mr. Johnson mixed yellow paint and blue paint and white paint in the ratio of 5 : 3 : 1 to make a light green paint for a painting job. If his final mixture for the painting job was 108 liters, how many liters of each color of paint did he use?

The ratio of the number of birds to the number of beasts is 3 to 7. How many beasts were there when the birds numbered 750?

Tamika took 4 h to drive from Town Y to Town Z at an average speed of 75 mph. On her way back, she drove at an average speed of 60 mph. How long did she take to drive back from Town Z to Town Y?

A cyclist traveled $\frac{4}{5}$ of his journey at an average speed of 20 km/h and completed the remaining 60 km in 2 h 45 min. How long did he take to complete his journey?

Talise folded 545 metal lids to make cones for jingle dresses for herself and her younger sister. Her dress had 185 more cones than her sister's dress. How many cones are on each dress?