Middle School Mathematics Comparisons for Singapore Mathematics, Connected Mathematics Program, and Mathematics in Context

(Including Comparisons with the NCTM Principles and Standards 2000)

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1 Introduction

In the mid-nineties, the mathematical community in the United States was severely shaken by the results of the Third International Math and Science Study (TIMSS). The test performance of our students dropped from mediocre at the elementary level through lackluster at the middle school level and down to truly distressing at the high school level. Considerable scrutiny followed, and is still following. Several clear mandates were generated by the study. One was to look closely at mathematics in Singapore, since of the forty-one nations' students tested, Singapore's scored at the very top. Exploration revealed a series of textbooks with a particularly nice development of the mathematical structure. This led to the obvious question: How do Singapore's texts compare with the books which have been designed since the early nineties in the United States, books that represent the direction of the current thrust in American mathematical education? This study is an effort to supply some data and general information toward such a comparison. We were asked to compare the Singapore mathematics materials at the middle school level with two American curricula. We chose two highly regarded and widely used curricula that were both designed to meet the 1989 NCTM Curriculum Standards (http://www.nctm.org/standards): the Connected Mathematics Program (CMP) and Mathematics in Context (MIC).

Before launching into a description of what we have done, it seems wise to mention some of the limitations imposed by the situation, lest we appear to be claiming to have achieved things which the circumstances preclude. To begin with, we are in some sense comparing the incomparable. The Singapore series is intended to be used in the way that textbooks have traditionally been used, under the authority of a traditional teacher. The two curricula we compare it with are based on a quite different philosophy of how learning occurs, and as a result are differently designed. This led to some tough decisions in designing the study, as well as to some places within the study where a caveat is necessary.

We have decided in this study to perform two different kinds of comparisons. First we compare each curriculum with the National Council of Teachers of Mathematics (NCTM) Principles and Standards 2000. Secondly we compare the three curricula with each other, not confined by the NCTM Principles and Standards. The first is admittedly not a fair comparison for the Singapore curriculum, as the two American curricula were designed from the ground up to meet the 1989 NCTM Curriculum Standards, while Singapore's curriculum was designed with Singapore's GCE (General Certificate of Education) examinations in mind. Nevertheless, if a school district in the United States would like to evaluate the Singapore curriculum (or any curriculum) for adoption, then it is important that we provide the information on how it fares against a set of adopted American standards. For example, any curriculum which omits a particular mathematical strand, or which assumes that the teachers are mathematics specialists, or which stereotypes females is probably not viewed by most as appropriate for adoption in American schools. In the second kind of comparison, we draw upon our expertise as mathematicians to evaluate the accuracy, depth, and breadth of the mathematics of one curriculum against another. For example, we will point out cases where two curricula satisfy an NCTM standard although the level of mathematics in one of them is one or two grade levels lower than the other. We also comment on the level and quality of teacher and parent support the curriculum provides, and discuss the difficulties of implementation.

Other studies of American middle school curricula have been done using a different set of criteria. Project 2061 (http://www.project2061.org/matheval/index.htm) is a notable example. The findings of that project placed CMP and MIC among the top of the thirteen curricula studied. Other critiques of curricula from university mathematicians and parents can be found at the Mathematically Correct website at (http://www.mathematicallycorrect.com). We feel, however, that the 2000 NCTM Principles and Standards are widely known by the various communities that will read our report, are readily available (see http://www.nctm.org), and present a more comprehensive structure and balanced perspective. For example, Project 2061 uses six benchmark questions to evaluate twelve curricula as to content and a large number of pedagogical issues. We use 72 questions to compare the curricula against the 10 overarching standards, distributed as: number (14), algebra (9), geometry (12), measurement (9), data and probability (10), problem solving (4), reasoning and proof (4), communication (4), connection (3), and representation (3). In addition, we use 13 questions to compare the curricula with the 6 principles, distributed as: equity (4), curriculum (3), teaching (1), learning (2), assessment (2), and technology (1).

Furthermore, the 2000 NCTM Principles and Standards represent a tremendous amount of thoughtful effort from a community with a wide variety of expertises. The choices they have made are ones to which we can comfortably subscribe, whether or not we agree with them in every detail. To reinvent that particular wheel was not the goal of this study. The recent article in the October 2000 issue of the *Notices of the American Mathematical Society* further supports the choice of the NCTM Principles and Standards 2000 for our study.

Another point to consider is our expertise, both what it is and what it is not. The group that created this comparison consists entirely of people who combine high-level training in mathematics with an interest in education, but who have neither direct experience of teaching in the American K-12 classroom nor the training for it. We have tried, therefore, to distinguish consistently between the statements that we make with genuine authority based on our grounding in mathematics and those statements that we sometimes permit ourselves based on a certain amount of related experience and a lot of observation and study of how teaching and learning can best occur in K-8 classrooms.

The specific structure by which we have proceeded is the following: one of the three faculty members in our group spent many weeks going through every chapter or module of the three curricula being compared, deciding how they met each of the criteria encapsulated in the questions we extracted from the Principles and Standards, and assigning to each question a corresponding score with an explanatory note. When she had finished, each of the three graduate students in the group chose one of the curricula and went over it in the same kind of detail, finding in particular the areas where they questioned the score or the discussion of it. Meanwhile the other two faculty members did a less detailed but more overarching study of the entire set of scores and discussions. There ensued a series of meetings by pairs and trios to clarify or negotiate differences of opinion with regard to the scoring and discussion. In this portion of the study we felt most strongly that we were relying on the knowledge we share in the field of mathematics.

At the same time, each of the graduate students wrote a summary of his impressions of the curriculum in which he had specialized. Each of these students has spent months working in various capacities (generally within classrooms) with elementary teachers who were beginning to teach from the 5th and 6th grade Everyday Mathematics curriculum. In addition, the graduate student who specialized in the Singapore curriculum traveled to Singapore for the purpose of observing classes and talking with teachers and educational administrators. To finish, we wrote a summary of our joint impressions and conclusions that includes comparisons of the curricula with the NCTM standards and directly with each other.

Before we describe the structure of the report itself, we need to make very clear and explicit one of our working hypotheses. The question of what a student learns depends heavily on the teacher. Different styles of teaching bring out different kinds of learning and produce different possibilities for non-learning. On none of these implementation issues can we possibly base our report. We have therefore adopted the universal assumption for each of the curricula that the lessons go exactly as planned, and that the students learn all of what each lesson is set up to offer.

The rest of the report is as follows. Chapter 2 contains brief individual summaries of backgrounds and impressions of each of the three curricula. Chapters 3 and 4 contain the comparisons with the 2000 NCTM Standards and the 2000 NCTM Principles, respectively. The "bullets" that describe the mathematical requirements of each standard (see, for example, the head of the Number Standard at http://standards.nctm.org/document/chapter6/numb.htm) are used as our questions. We assign a score between 0 and 3 (0 being the lowest) to each question and then give reasons for the score. We include a short summary for each curriculum after each of the ten overarching standards. The same type of analysis is done for each of the six principles. We include a short summary for each curriculum at the end of Chapter 4. Most of the questions for Chapter 4 were taken from the *Guidebook to Examine School Curricula – TIMMS as a Starting Point to Examine Curricula, Attaining Excellence Report*, U.S. Department of Education, Office of Educational Research and Improvement (available at http://timss.enc.org/topics/timss/kit). Other questions were made by the project team to cover issues in the 2000 NCTM Principles that were not addressed in the *Guidebook* above. Chapter 5 contains direct comparisons of the curricula using the information gained from Chapters 3 and 4 in conjunction with our joint expertise as mathematicians and applied mathematicians. Chapter 6 contains our conclusions.

Appendices A, B, and C are stand-alone reports on the Singapore curriculum, CMP, and MIC, respectively. Their content consists of the corresponding material from Chapters 2, 3, and 4, sorted by curriculum.

2 Background and General Impressions

2.1 The Singapore Curriculum

Before we can make objective comparisons between American and Singapore mathematics curricula, we must first point out some differences between the educational systems in both countries. Although the purpose of this report is not to critique either educational system or to offer explanations for the TIMSS results, neglecting to pursue this larger perspective gives the impression that the math curriculum bears the entire responsibility for students' education. Instead, we recognize that the mathematics curriculum is only one component of a child's education, which includes among other components teachers, parents, peers, government, and culture.

One member of our team had the opportunity to travel to Singapore in February 2000 to find out more about their education system. During this visit, Singapore's Ministry of Education (MOE) graciously allowed him to talk to its Curriculum Planning and Development Division (CPDD), and to visit several secondary schools. Here is what he learned:

In Singapore, students spend 6 years in primary school (corresponding to American grades 1–6), followed by 4 years in secondary school (corresponding to American grades 7–10). At the end of Primary 6, students' abilities are assessed using the Primary School Leaving Examination (PSLE) and students assigned to one of three streams: Special, Express, and Normal (broken further into two sub-streams, Normal-Academic and Normal-Technical).

The differences between the three streams are largely in their educational goals. The Special and Express streams prepare students to take the General Certificate of Education (GCE) "O" Level exam at the end of their fourth year of secondary school, [1] whereas the Normal stream prepares students for the GCE "N" Level exam. (If students in the Normal stream do well on the "N" exam, they are allowed to study an extra year to prepare for the "O" level exam.) Students who pass the "O" Level exam are allowed to pursue "pre-university" education (corresponding to American grades 11–12); students who pass the "N" Level exam can apply to technical and vocational schools. Which exam a student passes (and prepares for), therefore, is extremely important because it determines the careers the student can pursue. In effect, by the end of the sixth grade, Singapore's education system streams students by their academic ability, predisposing them towards certain vocations. Not surprisingly, we observed that, in general, Singapore's students are more motivated to excel academically than their American counterparts. It is not uncommon for Singapore students to stay after school for enrichment lessons, attend supplemental programs, or hire tutors at a much higher extent than in the United States.

[1] This pace represents a one-year acceleration of the traditional British system, on which it is based. In the British system, the CGE "O" Level exam is taken at the end of the fifth year of secondary school.

Naturally, the three streams also differ in their difficulty and amount of coursework. Typically, students in the Special, Express and Normal-Academic streams spend 2.5 to 3 hours a week on math in the classroom. According to the CPDD, students in the Normal-Technical stream spend 4 to 5 hours a week, as they need more attention.

Only three subjects are included on the Primary School Leaving Examination (PSLE): language, math and science. Teachers at the primary level typically teach all three subjects and serve as the "form teacher" (or "homeroom teacher"), who is responsible for the overall care of a child. Secondary teachers usually teach two subjects (commonly math and computer science or math and language). Singapore's educational system at the secondary level, then, is largely administered by teachers who are content-area specialists.

The most striking difference between the educational systems of both countries, however, is in governmental support of education. For example, the government encourages every teacher to attend at least 100 hours of training each year, and has developed an intranet called the "Teachers' Network," which enables teachers to share ideas with one another. Largely due to the fact that Singapore has a small geographic area, its government has a much greater ability to provide uniform educational experiences for its students than the United States government.[2] Given that all teachers receive their training from the National Institute of Education, the uniformity in teaching styles that we observed in the four classrooms during our February visit becomes less surprising.

It is this uniformity of their education system that allows us even to speak of a "Singapore mathematics curriculum," because there is only one mathematics curriculum (developed by the CPDD) used by all public primary schools. Nevertheless, we must carefully define what we mean by "Singapore mathematics curriculum" in this report. Because the middle school grades (6–8) in the United States correspond to the last year of primary school (Primary 6) and the first two years of secondary school (Secondary 1–2), we must consider both primary and secondary Singapore math curricula. While there is only one mathematics curriculum for Singapore primary schools, there are a handful of choices at the secondary level. The CPDD only sets guidelines for secondary math curricula—independent publishers are responsible for developing the teaching materials. The mathematics department of each secondary school is free to select from the available curricula. Furthermore, there are different mathematics curricula for the different streams. So to simplify matters, we decided to look at the curricula specifically designed for the Express stream, since most students (about 60%) enter this stream. This decision narrowed our focus to two math curricula, the *New Elementary Mathematics* series published by Pan Pacific, and *Syllabus D Mathematics* series published by Shing Lee. From the interviews with our CPDD hosts and the secondary teachers we observed during the February visit, we noticed that the problem solving strategies and problems

[2] Mr. Soh Cheow Kian, assistant director of the CPDD's sciences group, told us that he believes the main differences between Singapore and the United States are teaching approaches and government support.

in the Shing Lee texts are more popular and favorably viewed.

To summarize, when we refer in this report to Singapore's mathematics curriculum at the grade levels corresponding to the American middle school grades, we refer specifically to the Primary 4–6[3] materials (*(4A)* and *(4B)*; *(5A)* and *(5B)*; *(6A)* and *(6B)*), and the Secondary 1–2 materials published by Shing Lee (*(SL1)* and *(SL2)*). In our discussion, we will also occasionally refer to the Secondary 1–2 materials by Pan Pacific. The Secondary 3–4 materials by Shing Lee (*(SL3)* and *(SL4)*) corresponding to ninth and tenth grades, respectively, are also referred to occasionally to supply the reader with further information, but are not part of our study. We examined the student texts, the student workbooks and the Teacher's Guides for the *(4A)* - *(6B)* levels and the student texts, the student workbooks, and the Teacher's Resource Manual for the *(SL1)* and *(SL2)* levels.

The most important fact to remember about Singapore's math curriculum is that it was designed to be used in Singapore, not the United States. Singapore's math curriculum for primary schools is designed to prepare students for the PSLE, and likewise its secondary school curriculum for the GCE Examinations. Obviously, these curricula are not likely to fare well if we compare them against American standards, because they weren't designed to meet them. Nevertheless, there is some opinion[4] that America should start using Singapore's math curriculum. As a first step towards serious consideration of this viewpoint, we must determine how well Singapore's curriculum meets the standards for mathematics education developed by the some of the best teachers in our country.

In addition, we believe that there is a fundamental difference in what a math curriculum is expected to do in each country. We believe that the designers of Singapore's secondary math curricula expect their materials to serve as reference books, with lots of practice questions, for their students. Through our observations and interviews, we noticed that Singapore secondary teachers share this conception of their math curricula's function because they have become adept at collaborating with their peers and drawing on multiple sources of materials. Singapore's secondary math curricula do not provide many instructional hints or suggestions for assessment, but that is because there is little expectation for those types of materials. (Singapore's primary math curriculum, however, comes with excellent teachers' guides, which give the teacher instructional help, outline the ways the mathematical ideas connect throughout the curriculum, and provide articulation across the primary grades. This could be because the primary teachers are not mathematics specialists, and there is a perceived need at this level.)

In contrast, our great diversity of teacher support structures and teacher expertise levels in the U. S. prompts curriculum designers to create curricula that try to be many things to many people. For better or worse, American teachers have come to expect

[3]Students typically spend the last 30% of Primary 6 preparing for the PSLE, so the amount of math content contained in the Primary 6 mathematics book is lower than in the books from previous levels. To compensate, we carefully looked through the Primary 4 and 5 textbooks as well.

[4]One example is Cheryl Corley's report on NPR's *Morning Edition*, June 8, 2000.

well-designed, thorough instructional materials from curriculum makers. Therefore, we believe that many American school districts will find the lack of substantial printed guidance for teachers to be a significant deterrent against the adoption of Singapore's secondary math curricula.

One of the most attractive features of Singapore's math curricula is that the student books contain many problems that are worked out explicitly. Often, the examples that are worked out demonstrate the necessary steps in great detail, although rarely does one find alternative methods of solving problems. In addition, the Singapore math curricula have many practice problems for students. Singapore's primary math curriculum and the Shing Lee secondary curriculum's workbooks contain additional practice for the student.

The presentation in the Primary 4–6 books includes concrete examples, pictorial representations, cartoons of children explaining how to think about the topic, and finally the general case. The problems are short word problems that assume a grade-appropriate reading level. The presentation in the Secondary *(SL1)* and *(SL2)* books follow the form of a standard mathematics text. The topic is introduced, terms are defined, examples are given to illustrate the recommended strategies, and word problems are given for students to practice. The word problems and presentation assume a grade-appropriate reading level. The textbooks are careful to indicate new mathematical terminology by displaying it in bold or italics in the text, but they do not encourage students to actively use this terminology in the problems they do.

The materials that we studied do not involve technology in a significant way. Besides the use of calculators, the Department of Educational Technology (CDIS) computer software accompanies the Primary 4–6 materials, but we did not have access to this software. The student materials for Primary 6 do not include references to CDIS, but the teacher's manual frequently gives cross-references to it.

The quality and accuracy of the mathematics in Singapore's textbooks is high. There are a few notable examples where the mathematics in Singapore's textbooks is at a level of difficulty that exceeds NCTM expectations for the middle grades. In Primary 6, students solve complicated word problems involving proportions, without using algebra. In Secondary 1 and 2, students get a lot of practice manipulating algebraic expressions, and perform some difficult triangle congruence proofs. However, Singapore's curriculum does not introduce students to statistics until Secondary 2, and does not include any probability in the equivalent of the American primary or middle school grades.

While the mathematics in Singapore's curriculum may be considered rigorous, we noticed that it does not often engage students in higher-order thinking skills. When we examine the types of tasks that the Singapore curriculum asks students to do, we see that Singapore's students are rarely, if ever, asked to analyze, reflect, critique, develop, synthesize, or explain. The vast majority of student tasks in the Singapore curriculum is based on computation, which primarily reinforces only the recall of facts and procedures. This bias towards certain modes of thinking may be appropriate for an environment in which students' careers depend on the results of a standardized test, but we feel it discourages students from becoming independent learners.

We also point out some unbalanced gender references in Singapore textbooks. In particular, the textbooks published by Shing Lee and Pan Pacific provide unnecessarily stereotypical depictions of men and women in their word problems, cartoons, and textbook prose.

Furthermore, we believe that Singapore's curriculum does not adequately recognize that students have a wide range of learning styles. For example, Singapore's curriculum does not recognize that some students learn better through guided discovery than a direct presentation of concepts and procedures. Singapore's curriculum seems to be based on the view that the teacher is the primary disseminator of information in the classroom.

To summarize, our overall opinion of Singapore's mathematics curriculum is that its educational objectives are not well aligned with those in the NCTM standards. Singapore's mathematics curriculum does an excellent job of helping students acquire mathematical facts and procedures, through the many worked-out examples and large numbers of practice problems in its textbooks. However, the NCTM has identified other goals of mathematics education that are largely missing in Singapore's curriculum, most notably reasoning, communication, and connections (a larger conception of mathematics as a coherent whole that interacts with our world). Furthermore, Singapore's secondary math curricula provide very little support for teachers.

2.2 The Connected Mathematics Program

The Connected Mathematics Project (CMP) was developed by Glenda Lappan and others at Michigan State University and funded by the National Science Foundation. The 1998 edition was published by Dale Seymour Publishers. The curriculum was developed to be in line with the pedagogy and content in the 1989 National Council of Teachers of Mathematics (NCTM) standards, namely the *Curriculum and Evaluation Standards for School Mathematics*, the *Professional Standards for Teaching Mathematics*, and the *Assessment Standards for School Mathematics*. More information on CMP can be found at their website (http://www.math.msu.edu/cmp/index.html).

CMP focuses on mathematical content in the number, geometry, measurement, algebra, statistics and probability strands. The "Getting to Know CMP" guide, which comes with each grade's books, stresses that CMP students use the processes of counting, visualizing, comparing, estimating, measuring, modeling, reasoning, playing and using tools. Each of the grade levels has eight modular units. Some of these units have titles such as *How Likely Is It?* (probability) and *Growing, Growing, Growing* (exponential growth) which allow the teacher, students, and parents to get a sense of the mathematical content. A Teacher's Guide and student book is provided for each of the units.

The Teacher's Guides contain all the pages of the student book numbered in a manner consistent with the teacher pages, a "Teaching the Investigation" section, different types of assessments, additional problems, samples of student work, articulation information for the teacher, blackline masters, and form letters to parents in both English and Spanish describing the purpose of each unit and how they can best support their child's mathematical development at home. The "Teaching the Investigation" sections are the heart of the CMP curriculum. They give the teacher guidance on how to teach the lesson, an explanation of the mathematics in the lesson, and specific questions to ask students to make sure the important mathematical points are brought out during class. In the absence of a mentor teacher in each building, the assistance provided by the "Teaching the Investigation" sections could be very valuable to teachers that are not yet comfortable with the mathematics or with the discovery method of teaching. Even though CMP provides enough guidance to support a novice teacher, an experienced teacher can use his or her own creativity to supplement lessons and to meet the individual needs of students.

A CMP lesson, called an *Investigation*, is organized in three parts: *Launch*, *Explore*, and *Summarize*. *Launch* is the lesson introduction; it includes definitions, explanation of relevant concepts and other background material. *Explore* encourages students to work individually, then in pairs or groups, while the teacher circulates through the classroom and listens to students. *Summarize* allows for groups to share their findings followed by a class discussion. The typical lesson concludes with a "Mathematical Reflections" section, enabling students to reflect on their own learning. CMP emphasizes a discovery-based approach to learning that encourages students to select, adapt, and analyze problem solving strategies in order to develop mathematical understanding and become autonomous learners.

The curriculum provides numerous projects and problems for students. "ACE" problems appear at the end of each Investigation. These are grouped into Applications, Connections and Extensions. The Application problems reinforce the ideas currently being studied. The Connection problems integrate these ideas with strands that have been previously taught. The Extension problems are often the most challenging and carry the ideas forward. The Teacher's Guides also contain a *Question Bank* with additional problems that can be assigned to students that require more practice. Most of the twenty-four books in the curriculum have a Unit Project that requires the students to use the mathematics they have learned in pursuit of a larger goal. Students also have journals in which they can write their thoughts and record their work. The curriculum suggests that teachers should check these journals for completeness and not for correctness, so students are free to express their thinking. By examining these records, both correct and incorrect, the teacher is better able to assist students and assess the progress of his or her class.

The curriculum is constructed around five instructional themes: teaching for understanding ("big ideas"), connections, investigations, representations and technology. For example, CMP presents mathematics in a coherent way with an emphasis on connections among the mathematical ideas - thus the title of the curriculum. Students are urged to use multiple representations. For example, in the section *Solving Linear Equations* from the 8^{th} grade book *Say It With Symbols*, students are asked to compare graphical, tabular, and symbolic representations of a linear function. Regarding the technology theme, CMP 6^{th} grade students are required to have a standard "four-function" calculator and 7^{th} and 8^{th} grade students utilize graphing calculators with statistical capabilities. Some computer programs are included with the curriculum for enhancing probability and geometry lessons. We find that in most cases these technologies did not replace pencil and paper arithmetic, but since students themselves choose when to use calculators, a dependence on the calculator for problems of too low a level could develop.

Turning to the mathematical content of CMP, we find that most of the concepts presented in the number strand are a review for students that have gone through, for example, the Everyday Mathematics curriculum for grades K-6. The number strand is arguably the most basic and fundamental mathematics strand and much of the presentation in CMP is below the level articulated in the 2000 NCTM number standard for grades 6-8. Specifically, we find that CMP students are not expected to compute fluently, flexibly and efficiently with fractions, decimals and percents as late as 8^{th} grade. Standard algorithms for computations with fractions (e.g. $\frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$, $\quad \frac{a}{b} \div \frac{a}{c} = \frac{c}{b}$) are often not used. We understand that the developers of CMP are aware of the absence of material on division of fractions and probably will correct this in the next edition. Conversion of fractions to decimals is discussed only in simple cases such as for fractions with denominators of ten, and CMP lacks a discussion of repeated decimals. A discussion of long division is also missing. Such a discussion could make the conversion of fractions like $\frac{1}{7}$ to decimal form a simple procedure and would tie in with a discussion of rational numbers and repeated decimals. Long division is also a basis for the division of algebraic polynomials that students will see in high school. Multiplication of fractions is discussed in 7^{th} grade but mostly in simple cases. This is an area where multiplication algorithms could be exploited to solidify the concept of place value.

CMP does a good job of helping students discover the mathematical connections and patterns in the algebra strand, but falls short in a follow-through with more substantial statements, generalizations, formulas or algorithms. For example, in *Growing, Growing, Growing* exponents are discussed, but the exponential laws are not explicitly written down for the students even after they are discovered. In one exercise students discover that $2^6 = (2^2)^3$, but they need more practice to reach the generalization that $(a^n)^m = a^{nm}$. There is no discussion of negative and fractional exponents except when students explore exponential functions using graphing calculators. As a result, students miss an opportunity to revisit square roots and cube roots. In the 8^{th} grade unit *Frogs, Fleas and Painted Cubes*, students are required to be able to recognize that the same equation can be modeled in more than one way, but CMP misses the opportunity to discuss the quadratic formula or the process of completing the square.

Many mathematicians and educators believe that when using a curriculum that relies on discovery learning, such as CMP, teachers must understand the material even better than when teaching from a more traditional curriculum. Moreover, since students often work in pairs or in groups, teachers must be effective in establishing a classroom where all students participate in the mathematical work. Also, in order for students to effectively discover the mathematics, more time needs to be devoted to the lessons than in a traditional curriculum. The recommended minimum of 45 minute-long classes seems insufficient.

In conclusion, CMP corresponds well to the 2000 NCTM Principles and Standards, with the notable exception of the number standard. We feel that CMP's overwhelming emphasis on conceptual development neglects standard computational methods and techniques. In our opinion, concepts and computations often positively reinforce one another. While we understand that CMP seems to be motivated by the criticism that traditional curricula produce students that can compute but lack conceptual understanding, there is a danger here of producing students with conceptual understanding but limited computational skills. CMP admits that "because the curriculum does not emphasize arithmetic computations done by hand, some CMP students may not do as well on parts of the standardized tests assessing computational skills as students in classes that spend most of their time on practicing such skills." This statement implies we have still not achieved a balance between teaching fundamental ideas and computational methods.

2.3 The Mathematics in Context Curriculum

The study team reviewed the Mathematics in Context (MIC) middle school curriculum, published by the Encyclopaedia Britannica Educational Corporation and funded, in part, by the National Science Foundation (NSF). MIC was developed by the Wisconsin Center for Educational Research, at the University of Wisconsin-Madison, and by the Freudenthal Institute of the University of Utrecht, the Netherlands. Information on MIC can be found at its website (http://www.ebmic.com/ebec/index.htm). The curriculum was developed to be in line with the pedagogy and content in the 1989 National Council of Teachers of Mathematics (NCTM) standards, namely the *Curriculum and Evaluation Standards for School Mathematics*, the *Professional Standards for Teaching Mathematics*, and the *Assessment Standards for School Mathematics*.

An important factor to keep in mind when reading this curriculum is its division into four parts, one each for grades *(5/6)*, *(6/7)*, *(7/8)* and *(8/9)*. This allows for some flexibility in how it would be used in a school, but must be kept in mind when comparing MIC to a standard three year curriculum. A sample curriculum ("Plan B") is included for those using Mathematics in Context as a three-year, rather than as a four-year, program. (In assessing MIC, we first look for evidence within the Plan B materials. If we need to extend the search beyond these materials, we note this in the report.) Each of the four grade levels is further divided into 10 units, contained in a separate book per unit. Each unit belongs to one of four topical strands: algebra, geometry, statistics, or number. The books are color-coded by grade-level and by strand, making organization simple. There are both student books and Teacher Guides for each unit. We reviewed only the Teachers' Guides; however, since the Teachers' Guides contain the student pages, we know both what the instructor and what the student would see. A notable exception to this is that the Teachers' Guides lacked the "Try This!" activities, a flaw which should definitely be corrected in future editions. The Teachers' Guides also contain assessments, blackline masters, and a form letter to parents describing the purpose of each unit and how they can best support their child's mathematical development at home. An overview of the MIC curriculum is given in the Teacher Resource and Implementation Guide.

Pedagogically, the program should be very simple - in theory - for teachers, even novices, to implement. Since the curriculum is broken up into units composed of bite-size lessons, and since the teacher is provided with a clear explanation of how each unit fits into the curriculum's overall scheme, there can be little confusion about how to structure the program. Alongside each lesson are comments about the underlying mathematical concepts in the lesson ("About the Mathematics"), as well as how to plan and to actually teach the lesson. A nice feature is that these comments occur in the margins of the Teachers' Guides, the bulk of which are occupied by replicas of the actual student pages, which should make the books very easy to use for educators. On the other hand, these comments often contain some useful mathematical facts and language that could be, but most likely wouldn't be, communicated to the students; in particular, high-end students could benefit from these insights if they were available to them. In addition, the lack of a glossary hides mathematical terminology from the students, a language which they should be beginning to negotiate by the middle grades. Exposure to the precise terminology of mathematics is crucial to students at this stage, not only as a means of exemplifying the rigor of mathematics, but as a way to communicate their discoveries and hypotheses in a common language, rather than the idiosyncratic terms that a particular student or class may develop. The glossaries in the Teachers' Guides should be made part of the student books. This would also be helpful to parents trying to help their child with his or her homework.

A problem, created as a by-product of the teacher-friendly units, is that the curriculum lacks coherence. The units are designed as stand-alone topics, or at best as continuations in a particular strand. Previously studied topics are not integrated well, and interaction between the strands is minimal. Students will come out of the program seeing mathematics as coming in unit-size chunks, rather than as part of a larger whole. This could easily be ameliorated, though, through exercises that emphasize connection across the curriculum.

Our subjective evaluation of the Mathematics in Context curriculum is mixed. The curriculum is very good at teaching basic conceptual points: what statistics are and how they can be used or misused; how different representations can be used to answer different types of problems effectively; what multiplication and division mean; how fractions, decimals, and percents are related; and so on. The students often develop these ideas to some extent on their own. Many of the lessons are to be used for individual or small group explorations and then brought into focus by the teacher; certainly students will get a chance to feel that they really own the material. Additionally, a "Letter to the Student" opens each book, explaining to the student what they can expect to gain from the unit, imparting a sense of educational control and responsibility to the student (as well as his or her parents). The work is almost always tied to some real context that the students can understand, and efforts are made to give examples that would interest students regardless of their gender, race, or class. However, it is our impression that the writing, examples, and pictures may be aiming a little low for students of this age group.

Our central criticism of the Mathematics in Context curriculum concerns its failure to meet elements of the 2000 NCTM number standards. Because MIC is so fixated on conceptual underpinnings, computational methods and efficiency are slighted. Formal algorithms for, say, dividing fractions are neither taught nor discovered by the students. The students are presented with the simplest numerical problems, and harder calculations are performed using calculators. Students would come out of the curriculum very calculator-dependent, and we find it hard to believe that students could divide 3.67 by .02 efficiently, even if they could explain what it meant, in abstraction, to perform that division. To us, this represents a radical change from the old "drill-and-kill" curricula, in which calculation itself was over-emphasized. The pendulum has, apparently, swung to the other side, and we feel that a return to some middle ground emphasizing both conceptual knowledge and computational efficiency is warranted. In addition, much of the time spent on the number strand is aimed too low. Students should have an understanding of equivalent fractions, decimals, and percents before the middle grades; multiplication of a decimal by a power of ten should be accomplished before grades *(6/7)*. Exponents are only used with base ten. Thus, teachers using Mathematics in Context would need to greatly augment the number strand.

There are more limited difficulties with the geometry and measurement strands. Discussion of cones and cylinders should be more prominent, and there is no discussion of density. Much more depth is needed in coordinate geometry. On the stronger side is the algebra strand, which we found fully met the standards.

Another concern, touched on above, is that high-end students may not find this curriculum very challenging or stimulating. The language and examples are fairly simple, so that such a student may feel she is being talked down to. Options for the teacher to scale-up the curriculum for advanced students are limited. The teacher could set up more advanced units for the student, which is the obvious intent of the *(5/6)*, *(6/7)*, *(7/8)*, *(8/9)* construction of the curriculum; yet even at a more advanced conceptual level, the student would still only be exposed to simple examples. Most likely the teacher would have to reach outside the curriculum to find supplemental materials.

In fact, the level of mathematics in MIC is often too low for students, particularly in the *(8/9)* books. For example, in "Going the Distance," the last unit in the geometry sequence, formulae for the area of a circle, the area of a general triangle, and the circumference of a circle are developed, as is the Pythagorean Theorem. (Some of this material is being covered for the second time, since it is worked with in "Reallotment," a *(6/7)* unit.) In "Graphing Equations," the first of the units in the *(8/9)* algebra strand, considerable time is spent graphing lines, given a linear equation. These topics are all covered by sixth grade in the Everyday Mathematics curriculum.

The deliberate emphasis of the program is on encouraging students to think about and to discuss mathematics, particularly in a problem-solving context, rather than on a direct transmission of a set body of material. Students are expected to be, and encouraged to be, active participants in developing their mathematical knowledge, rather than passive recipients of information. The underlying idea is that students today will need to use mathematics to solve real world problems; thus, presenting mathematics in a realistic setting will be both useful and interesting to students. It is hoped that this problem-solving, conceptual approach will provide students with a deeper understanding of middle school mathematics, as opposed to the superficial background they may have obtained from most curricula used in the United States in the past. As opposed to the lecture-drill format, the structure of the lessons in MIC emphasizes discovery, and teachers using MIC should be comfortable with this mode of instruction and the significant amount of time it requires. Additionally, such a curriculum will require teachers to have a deeper understanding of the mathematics than when using a more traditional approach.

In conclusion, Mathematics in Context corresponds well to the 2000 NCTM Principles and Standards, with the notable exception of the number standard. Going a bit beyond these standards, however, we feel that MIC places an over-emphasis on learning by discovery, which prevents the curriculum from reaching more advanced material by grades *(8/9)*. The curriculum, while covering basic concepts well, does not reach the level of mathematics we would hope to see by the end of the middle grades. Advanced students, in particular, would feel unchallenged.

3 Comparisons with the 2000 NCTM Standards

3.1 Methodology

In this chapter, we use the 2000 NCTM Standards as a guide to gather information about the three curricula. We then use this information (evidence) to score each curriculum on how well it compares with these standards. A brief summary for each curriculum is given for each of the ten overarching standards. In Chapter 4, we use the 2000 NCTM Principles to compare each curriculum in a similar way with the six principles. We now describe the source of our questions, the meaning of our scoring system, the criteria we use in presenting the evidence, and the purpose of the brief summaries.

The Questions:

The questions for each of the ten overarching 2000 NCTM Standards were taken verbatim from the "bullets" listed as the *Expectations* for what grades 6-8 students should be able to do. These can be found in the *Principles and Standards for School Mathematics* section of the NCTM web page, http://www.nctm.org. We formulated each bullet as a question, and as a result, the number of questions we examine is not the same for each overarching standard. The overarching standards and the number of questions used for each are: Number(14), Algebra(9), Geometry(12), Measurement(9), Data Analysis and Probability(10), Problem Solving(4), Reasoning and Proof(4), Communication(4), Connections(3), and Representation(3).

Most of the questions for each of the six 2000 NCTM Principles were taken from the *Guidebook to Examine School Curricula – TIMMS as a Starting Point to Examine Curricula, Attaining Excellence Report*, US Dept. of Education, Office of Educational Research and Improvement. This report is available at http://timss.enc.org/topics/timss/kit. The project team composed other questions after reading in detail the *Principles for School Mathematics* document, available at the NCTM site. These questions were posed in the same format as those from the *Guidebook* above. These principles and the number of questions used for each are: Equity(4), Curriculum(3), Teaching(1), Learning(2), Assessment(2), and Technology(1).

The reasons we chose the 2000 NCTM Principles and Standards as our guide are threefold. First, these standards are the most recent version available from NCTM. The second reason is the availability and knowledge of these standards by the various communities that may benefit from our report. The third reason, as we mentioned in the Introduction (p. 2), is that these standards represent a tremendous amount of thoughtful effort from a community with a wide variety of expertise, and we can comfortably subscribe to them, whether or not we agree with them in every detail.

The Evidence:

Each book involved in the study was examined and references gathered to compare the curricular materials with the standard. In reporting this evidence, we seek to meet the following two goals:

1. List as much evidence as necessary to support our conclusion about how fully the standard is met.

2. List evidence for each grade to show how well the curriculum builds up the concepts across the grades to meet the standard. For CMP, our goal is to list evidence at all three grades *(6)*, *(7)*, and *(8)* for each question. For MIC, where the grades were *(5/6)*, *(6/7)*, *(7/8)*, and *(8/9)* our goal is to list evidence that includes *(5/6)* or *(6/7)*, *(6/7)* or *(7/8)*, and *(7/8)* or *(8/9)*. For Singapore, our goal is to list evidence that includes *(6A)* (or *(6B)*), *(SL1)*, and *(SL2)*.

With these goals in mind, it is important to know the following before reading the report:

1. For the Number, Algebra, Geometry, Measurement, and Data and Probability Standards, we are evaluating whether the entire middle grades curriculum meets the standard. Every grade level does not necessarily have to address every question. The 2000 NCTM Principles and Standards document makes this point clear. If the standard is met before 6th grade, we note that as well.

2. For the Problem Solving, Reasoning and Proof, Communication, Connections, and Representation Standards, and the six Principles, we are evaluating whether the standard (or principle) is frequently met. The 2000 NCTM Principles and Standards document also makes this point clear. The expectation is that evidence of the standard (or principle) should be seen frequently in each grade.

3. If evidence is not supplied for a grade, it is because we were not able to find any.

4. In answering different questions, we may supply different amounts of evidence. It is possible that one of the first five standards is met totally in one book at one grade level. If this happens, we still make an effort to report what has happened at grades below or above to show the entire conceptual story across the grades in the study. On the other hand, if the materials only meet part of the standard, we still report the conceptual story that is found in the materials.

The Scoring:

The questions relating to the ten NCTM overarching standards were scored with the numbers 3, 2, 1, and 0. The meaning of each is given below.

- **Score of 3: The standard is fully met.** By the end of grade *(8)* for CMP, *(8/9)* for MIC, and *(SL2)* for Singapore, the answer to the question is an unqualified yes.

- **Score of 2: The standard is adequately met.** By the end of grade *(8)* for CMP, *(8/9)* for MIC, and *(SL2)* for Singapore, the answer to the question is not an unqualified yes. Evidence has been found for meeting parts of the standard. A judgment has been made that the curriculum's material, though not fully meeting the standard, is adequate for use.

- **Score of 1: The standard is not adequately met.** By the end of grade *(8)* for CMP, *(8/9)* for MIC, and *(SL2)* for Singapore, the answer to the question is not an unqualified yes (same as for a score of 2). Although evidence has been found for meeting parts of the standard, a judgment has been made that the curriculum's material related to this standard is not adequate for use.

- **Score of 0: The standard is not met.** By the end of grade *(8)* for CMP, *(8/9)* for MIC, and *(SL2)* for Singapore, no evidence was found for meeting the standard.

The questions relating to the six 2000 NCTM Principles are also scored with the numbers 3, 2, 1, and 0. The meaning for each of these scores is given directly below each question. The score of 2 also carries the connotation that the material does not fully meet the standard, but is adequate for use, while a score of 1 implies that the material related to this standard is not adequate for use.

The Summaries:

After each overarching standard and the principles are scored, we tabulate the scores for each question for each curriculum. We then give a short written summary for each curriculum. The purposes of this written summary are twofold:

1. To highlight the main deficiencies that led to the scores less than 3.

2. To point out, in some cases, facts worth mentioning that the evidence revealed, but that the scoring did not reflect.

Note: In what follows for CMP, IV1 refers to Investigation 1, IV2 to Investigation 2, etc.

3.2 Number Standard

3.2.1 Number Standard Question 1.

Does the curriculum enable all students to work flexibly with fractions, decimals, and percents to solve problems?

Singapore: Score: 3
Evidence:

- *(6A).* In Ch. 1, 2, and 3, students go between these concepts flexibly to solve problems. They solve multi-step word problems. Two step word problems were solved in *(5A)* and *(5B)*. They see fractions, decimals, and percents using pie charts, fraction strips, and fraction disks. They divide fractions by fractions concretely as well as use the invert and multiply algorithm. They solve problems with mixed fractions using addition, subtraction, multiplication, and division including parentheses. They work fluently using the proper order of operations.

- *(6A).* On p. 8, problem 3, the Teacher's Guide suggests that students fold paper to see the concepts concretely.

- *(SL1).* On p. 20, the Teacher's Manual suggests that teachers work quickly through Chapter 4 on Fractions and Decimals since the material was covered in depth in lower grades.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 2
Evidence:

- *(6) Bits and Pieces I.* This is a beginning unit focused on developing the meaning of fractions, decimals, and percents and in which situations to apply each. Students are not expected to compute flexibly with fractions, decimals, and percents yet. There is more emphasis on deciding *when* it is appropriate to represent information using each. The extent of solving problems is dividing a rectangular pan of brownies into 20 pieces.

- *(6) Bits and Pieces II.* In IV4, students add and subtract fractions with like and unlike denominators for simple cases. They develop their own algorithm and on p. 48 are asked to explain how it works. In IV1 and IV2, students solve problems with sales tax, tips, and discounts. They really don't need to know how to divide since problems are cast as multiplication ones. For example, given the sales price and the percentage discount, the original price is not to be calculated. Instead the sales price is asked for, given the percentage discount and the original price. In IV5, students multiply fractions using the area model, and in doing so discover the standard algorithm. In IV6, students add, subtract, and multiply decimal numbers, but do not divide them.

- *(7) Comparing and Scaling.* In IV2 and IV3, comparisons are made using percents and ratios, respectively.
- *(8) Looking for Pythagoras.* Irrational numbers are introduced. Students change repeating decimals to fractions on p. 56.

Discussion: This standard is adequately met. According to the standards, students should have learned how to recognize equivalent forms of fractions, decimals, and percents in the benchmark cases in grades 3-5 and more flexibility and fluency is expected in grades 6-8. The discussion of this standard for grades 6-8 mentions that students should be *facile* in working with fractions, decimals and percents. These units adequately meet this standard by setting a firm conceptual foundation, but for grades 6-8, much more facility should be expected. There is no evidence that by the end of 8th grade students could divide 23/45 by 1/6, multiply .43 by .27, or write (1+4/5)/9 as a single fraction.

Mathematics in Context: Score: 2
Evidence:

- *(5/6) Sum of Parts.* This unit is an informal introduction to fractions. No flexibility is expected yet.
- *(5/6) Per Sense.* This unit is an introduction to percent. Students work with easy benchmark percents (1 and 10) to develop concepts with a percent bar.
- *(6/7) Made to Measure.* The initial meaning of place value is introduced through money and the metric system. Decimals and fractions are used with money as well.
- *(6/7) Fraction Times.* Ratio tables, pie charts, and bar charts are used to add and subtract fractions with unlike denominators. This is still very informal with easy numbers. Students go between the three concepts of fractions, decimals, and percents. Multiplication of fractions is introduced informally, though the students are not made aware that this is what they are doing. There is no division of fractions yet. A decimal is found from a fraction with the aid of a calculator for fractions with any complexity. This is moving very slowly toward algorithms and students cannot work flexibly yet to convert say 23/85 to a decimal without the aid of a calculator. In *Number Tools, Vol. 2*, Sections J and K work with these concepts, but still there is not much practice.
- *(6/7) More or Less.* Students multiply decimal numbers using a calculator. Interesting problems are included for finding percentage increase and decrease. Benchmark percents are converted to fractions and decimals. Students solve discount, tax, photo reduction, and interest problems.
- *(6/7) Rates and Ratios.* Students learn to multiply a decimal number by a power of ten.

- *(7/8) Cereal Numbers.* Students work with percentage of ingredients, serving sizes and price comparisons.

- *(7/8) Powers of Ten.* Students multiply and divide decimals by 10, 100, and other powers of 10 using the 10-machine.

Discussion: This standard is adequately met. According to the standards, students should have learned how to recognize equivalent forms of fractions, decimals, and percents in the benchmark cases in grades 3-5 and indeed the first 3 units above are 5th grade units and are being reviewed here because Plan B recommended they be used at the 6th grade level and because they are listed as *(5/6)* which includes 6th grade. Multiplying or dividing a decimal by 10 should be a quick review for grade *(6/7)*. The standard for grade 6-8 mentions that students should be *facile* in working with fractions, decimals, and percents. For grades 6-8, much more facility should be expected. For example, there is no evidence that by the end of 8th grade, students could multiply .43 by .27, divide 23/45 without the aid of a ratio table, or write (1+4/5)/9 as a single fraction.

3.2.2 Number Standard Question 2.

Does the curriculum enable all students to compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line?

Singapore: Score: 3
Evidence:

- *(5A) and (5B).* Students compare and order fractions and decimals and convert between these two forms. Decimals and fractions are located on a number line.

- *(6A).* In Ch. 3, students use the double number line to order and compare percents and decimals.

- *(SL1).* On p. 20, the Teacher's Manual suggests that teachers work quickly through Ch. 4 on Fractions and Decimals since the material was covered in depth in lower grades.

- *(SL1).* On pp. 30–31, students learn to compute the highest common factor and the least common multiple. They also learn to express repeating decimals as fractions.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces I.* Students are beginning the study of comparisons with fractions, decimals, and percents. They are not efficient in their use yet. No algorithms are analyzed yet. On p. 23, students compare fractions by making fraction strips and locate fractions on a number line using these strips. The treatment is concrete to build a firm base of understanding. On p. 23, they compare fractions to a benchmark fraction. They aren't required to compare them in general without using a calculator. In IV4, they order and compare decimals to place value 10^{-4}. On p. 58, prob 3, students estimate a "good" fraction to represent a decimal. On p. 81 they locate fractions, decimals, and percents on a number line.

- *(6) Bits and Pieces II.* In IV3, students learn fraction benchmarks. On p. 33, they locate fractions on a number line in relation to these benchmarks. On p. 36, problem 36, they order decimals and on p. 74, problem 11, they locate decimals on a number line.
- *(7) Comparing and Scaling.* In IV2 and IV3, comparisons are made using percents and ratios, respectively.
- *(8) Looking for Pythagoras.* Students compare the square root of 10 to the nearest fraction on p. 63.

Discussion: This standard is fully met. We note that students are not comparing general fractions by finding the common denominator with pencil and paper without the aid of a calculator. The score of 3 reflects our interpretation that the NCTM Standards say a calculator is a valid tool for accomplishing these tasks for more general fractions.

Mathematics in Context: Score: 3
Evidence:

- *Number Tools, Vol. 2.* On p. 47, fractions and decimals are placed on a number line.
- *(5/6) Sum of Parts.* On p. 74, students start to order fractions.
- *(5/6) Per Sense.* Students compare fractions with the percent bar. No comparison with a decimal or multiplying fractions is done. A denominator of 100 is used.
- *(6/7) Made to Measure.* On p. 60 decimals are ordered using a number line and units of measure are converted. Money sums are estimated on p. 64.
- *(6/7) Fraction Times.* Fraction bars are used to compare fractions with unlike denominators.
- *(6/7) More or Less.* A rubber band device is constructed to study whole integer percentages on the number line.
- *(6/7) Rates and Ratios.* Comparisons are made using part:part, part:whole, percent, and ratio.
- *(7/8) Cereal Numbers.* Absolute and relative comparisons are made using fractions.

Discussion: This standard is fully met.

3.2.3 Number Standard Question 3.

Does the curriculum enable all students to develop meaning for percents greater than 100 and less than 1?

Singapore: Score: 3
Evidence:

- *(6A).* On p. 36, one girl's savings is computed as 125% that of another girl's savings.
- *(6A).* In Ch. 3, the selling price is 120% of the cost price.
- *(6A).* On p. 38, the concept of a fraction of a percent is introduced in terms of a fraction. For example 12.5% = 1/8 and terminating decimals such as .275 are written as percents (27.5%).
- *(SL1).* On p. 235, the meaning of a percent as the number out of 100 leads to writing .4% = .4/100 = .004.
- *(SL2).* On p. 7, problem 3, students work with 3/5% yearly tax to calculate the total tax collected on a house valued at $98,000.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces I.* On p. 74, students work with 12.5% discount. On p. 79, 1/2 of a square on a 100 grid is shaded to show a percent less than 1. On p. 11, the terminology "exceeding the goal" was used and the fraction $1\frac{1}{4}$ was used.
- *(6) Bits and Pieces II.* On p. 13, problem 14, students find 3 ways to represent 120% of a dollar.
- *(7) Data Around Us.* On p. 51, 1178% was used when talking about exponential growth problems over time.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Per Sense.* One example of 200 percent is found.
- *(6/7) More or Less.* Examples of percents over 200 occur on p. 59, p. 69, and from p. 82 onward. No evidence is found that students work with percents less than 1.
- *(7/8) Cereal Numbers.* On p. 20, in problems 6 and 7, students work problems involving .2% and 1.4%.

Discussion: This standard is fully met.

3.2.4 Number Standard Question 4.

Does the curriculum enable all students to understand and use ratios and proportions to represent quantitative relationships?

Singapore: Score: 3
Evidence:

- *(6A).* Ch. 2 gives a thorough conceptual treatment of ratio and proportion. Pictures (bars) are used to solve word problems.

- *(6A).* The unitary method is introduced in Ch. 3 to solve proportion problems involving percent. As an example, if 75% corresponds to 42, then 1% corresponds to 42/75, and then 100% would correspond to 56. On p. 46, the CDIS software is referred to for more work with this concept. (We did not have access to this CDIS software, but it appears to be referenced throughout the curriculum.)

- *(6B).* Ch. 5 has many extension problems for abler students that use pictures with the unitary method to solve complicated problems involving proportion *without algebra*. Some of these problems are worked out for the teacher and for the students. Even though these problems are not from real contexts, they are at a level that trains the mind in thinking skills. An example is given in *(6B)*'s Teacher's Guide on p. 71: *The ratio of Andrew's money to Paul's was 3:1 at first. After Andrew spent $25, Andrew had $5 more than Paul. How much money did Paul have at first?*

- *(SL1).* On p. 219, direct proportion is revisited.

- *(SL1).* On p. 222, inverse proportion is introduced.

- *(SL2).* On p. 222, ratios and proportions are used to find area and volumes and missing side lengths for similar figures and solids. Students are proficient in working with these concepts.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Stretching and Shrinking.* In IV5, students find the missing side lengths in similar triangles.

- *(7) Comparing and Scaling.* In IV3, p. 27, students use part-to-part and part-to-whole comparisons. In IV4, they compare by finding rates such as mi/gal, $dollars/hr$, and use proportional reasoning with these unit rates to solve problems. They use rates and proportions with understanding.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* The ratio table is used in recipes with fractions. For example, 1/2 cup serves 12 implies 1 cup serves 24, 3 cups serve 72, etc.

- *(6/7) Fraction Times.* Equivalent fractions are done using the ratio table.

- *(6/7) Rates and Ratios.* Miles/gal, percent smokers, and ratio of smokers to non-smokers examples are given.

- *(7/8) Cereal Numbers.* Students divide fractions to compute serving sizes using the ratio table. The fractions involved are fairly simple.

- *(7/8) Powers of Ten.* The magnification that is seen through a student-build viewer is related to a ratio.

- *(8/9) Triangles and Patchwork.* Ratios in similar triangles are investigated.

Discussion: This standard is fully met.

3.2.5 Number Standard Question 5.

Does the curriculum enable all students to develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation?

Singapore: Score: 3
Evidence:

- *(5A and 5B).* Students compare and order numbers within 100,000. They read and write numbers up to 10 million. A firm foundation is set for large numbers up to 6 digits in terms of place value. These books have an extensive conceptual presentation of place value.

- *(SL1).* On p. 36, exponential notation is introduced such as 19^3.

- *(SL1).* On p. 37, the y^x key on the calculator is introduced.

- *(SL1).* On p. 121, the rules for determining the number of significant digits are given, but no scientific notation is introduced.

- *(SL2).* On p. 20, the formal rules for exponents are outlined. This includes negative exponents for bases other than 10. Many problems are done with these rules.

- *(SL2).* On p. 26, scientific notation is introduced. Students work many problems using this notation.

- *(SL2).* On p. 29, exponential notation on a calculator is introduced and students do many non-contextual problems on a calculator.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 49, exponential notation with prime numbers is given, e.g. 5^4.

- *(7) Data Around Us.* Students use the contexts of natural disasters (exponential growth problems) to study large numbers. These numbers are compared in IV3 and rounded on p. 28. On p. 41, scientific and calculator notations are introduced. Students work problems using these notations. On p. 45, connections between prime factorization and exponential notation are made.

- *(8) Growing, Growing, Growing.* In IV1, functions of the form $y = a^x$ are studied. In IV4, exponential decay of the form $y = a^{-x}$ is studied. Both exponential growth and decay are seen from tables, graphs and equations. Graphing calculators are used on p. 50.

Discussion: This standard is fully met.

Mathematics in Context: Score: 2
Evidence:

- *(6/7) Made to Measure.* Large numbers are rounded to the nearest million.

- *(7/8) Cereal Numbers.* Large integers are compared.

- *(7/8) Powers of Ten.* Students build a viewer (for magnifying objects) to investigate the effect of successive doubling and multiplication by 10. Rule of exponents with a base of 10 are given: $10^a * 10^b = 10^{a+b}$ and $10^a/10^b = 10^{a-b}$. On p. 48, positive exponents are used with other bases (e.g. 2^3 or 3^5), but negative exponents are only used with base 10. Students build a scale model of the solar system and then investigate different scales that must be used to fit it into the classroom. Scientific notation and calculator notation are used.

Discussion: This standard is adequately met. Exponential notation with negative exponents is only used with base 10.

3.2.6 Number Standard Question 6.

Does the curriculum enable all students to use factors, multiples, prime factorization, and relatively prime numbers to solve problems?

Singapore: Score: 3
Evidence:

- *(4A, 5A)*. Students list all factors of a whole number up to 150. They can list common multiples of two or three one-digit numbers and the multiples of a given one-digit number.
- *(4A, 5A)*. Students list common factors of two whole numbers.
- *(up through 6B)*. No prime factorization and no relatively prime numbers are discussed.
- *(SL1)*. On p. 26, the formal tests of divisibility are given.
- *(SL1)*. On p. 23, factor, multiple, and prime are defined.
- *(SL1)*. On p. 27, prime factorization is introduced using a factor tree representation.
- *(SL1)*. In the Student Workbook, 89 problems are provided using the concepts of factors, multiples, and prime factorization. Students find the highest common factor of pairs and triples of numbers. They do not encounter the term relatively prime.
- *(SL2)*. On p. 57, students find the HCF (highest common factor) and the LCF (lowest common factor) of algebraic expressions.

Discussion: This standard is fully met. We note that students work extensively with the common factors of numbers, and work with the procedures that would determine whether a number is relatively prime, even though they do not use this language explicitly. (We note that on p. 15 in Pan Pacific book for Secondary 1, students examine whether a triple of numbers are relatively prime, even though this language is not used.)

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time*. The entire unit is about factors, multiples, prime factorization, and relatively prime numbers. In IV1, computing factors is practiced using the Factor game. In IV2, multiples are studied. In IV3, factor pairs are studied. In IV4, students find common factors and multiples. In IV5 and IV6 students use the Prime Factorization Theorem. On p. 48, students discover this theorem. On p. 51, they work with relatively prime numbers. In IV6, the popular locker problem (the locker doors are opened and closed in particular patterns related to factors and primes) is solved using the concepts developed in the unit. Students work with the GCF and the LCM on p. 50.

Discussion: This standard is fully met.

Mathematics in Context: Score: 2
Evidence:

- *(8/9) Reflections on Number* (Not in Plan B). Prime factorizations, multiples, and factor trees are used to solve problems. This is the only unit in this curriculum that addresses this standard.

Discussion: This standard is adequately met. There is no mention in the curriculum of finding common factors of two numbers, a prerequisite for studying relatively prime numbers.

3.2.7 Number Standard Question 7.

Does the curriculum enable all students to develop meaning for integers and represent and compare quantities with them?

Singapore: Score: 3
Evidence:

- *(4A, 5A)*. Students compare and order whole numbers within 100,000. They work many problems and have a very solid foundation.

- *(up through 6B)*. Students do not work with negative numbers.

- *(SL1)*. In Ch. 1 and Ch. 2, the concepts above are solidified for whole numbers. On p. 24, *Just for Fun* is a puzzle that requires knowledge of prime numbers and adjacent whole numbers.

- *(SL1)*. In Ch. 5, negative integers are introduced and the meaning of the addition, subtraction, multiplication, and division operations with these numbers is investigated. Pictures are used to take the student from the concrete very quickly to working on the abstract level with these concepts.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Accentuate the Negative.* Understanding of addition, subtraction, multiplication and division with negative and positive integers is developed. This understanding is developed using the number line, red and black chips, games in the contexts of negative temperatures, bank accounts, profit, and deficit.

- *(7) Data About Us.* Understanding is further developed for large positive integers by measuring them in terms of benchmarks that have meaning to students.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *Number Tools, Vol. 1*: In Sections A and E, students work with a number line, but there is no negative side to this line.

- *(6/7) Operations.* Students add and subtract positive and negative numbers and divide and multiply positive and negative numbers in a formal way. Contexts of temperatures and time zone data are used. Numbers are represented on a coordinate grid.

- *(7/8) Cereal Numbers.* Students compare large positive numbers.

- *(7/8) Powers of Ten.* Students work with large positive numbers in the context of filling a large volume with items of smaller volume. (How many boxes will fit in your room?)

- *(8/9) Reflections on Number.* (Not in Plan B.) Students work with odd and even positive integers.

Discussion: This standard is fully met.

3.2.8 Number Standard Question 8.

Does the curriculum enable all students to understand meaning and effects of arithmetic operations with fractions, decimals, and integers?

Singapore: Score: 3
Evidence:

- *(6A).* In Ch. 1, students do a large number of practice problems that emphasize the order of operations using fractions, decimals, and integers. They evaluate expressions with and without brackets. They work with general numbers. They divide fractions by fractions.

- *(SL1).* In Ch. 1, students work problems involving integers that emphasize the commutative, associative, and distributive laws. In Ch. 4, they convert fractions to decimals and vice versa. They order fractions. They simplify calculations using the commutative, associative, and distributive laws.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 1
Evidence:

- *(6) Bits and Pieces I.* Only the simpliest arithmetic is used.

- *(7) Bits and Pieces II.* Students add, subtract and multiply fractions. They multiply decimals by whole numbers. No division is done with fractions and decimals. The curriculum does not include division of fractions.

- *(7) Accentuate the Negative.* Students do operations with addition, subtraction, multiplication, and division of integers. Division is just seen as the inverse of multiplication and problems are cast that way. The complexity of the problems is of the level $-3.4+? = -5.6$.

Discussion: This standard is not adequately met. The arithmetic operations of dividing fractions, multiplying decimal numbers, or multiplying general fractions are studied only for simple cases and not to an extent that would ensure students understand the "meaning and effects" of these operations. Specifically, the meaning of division is not dealt with in enough detail. It is conceivable that students can complete a middle-grades mathematics program without having seen the effects of dividing fractions and decimals.

Mathematics in Context: Score: 1
Evidence:

- *(5/6) Sum of Parts.* Multiplication of fractions as repeated addition of fractions is presented.

- *(6/7) Made for Measure.* The meaning of decimals is presented but students do minimal computation.

- *(6/7) Fraction Times.* Students use fraction bars to add and subtract fractions. They use repeated addition to multiply a fraction by a whole number. The curriculum includes very little practice with arithmetic operations that involve fractions and decimals.

- *(6/7) More or Less.* The meaning of a percentage as a decimal and a fraction is presented. For example $12\% = .12$ and $75\% = 3/4$.

- *(6/7) Rates and Ratios.* Fractions are related to averages and ratios and used as comparisions. Students do not divide fractions or multiply or divide decimals with more complicated numbers. For example, $4.5/.9$ represents the level of complexity of decimal division found in the curriculum.

- *(6/7) Operations.* Students add, subtract, multiply and divide positive and negative numbers. Chips are used to show concretely with a clear progression to formal definitions.

- *(7/8) Cereal Numbers.* Students multiply fractions using an area model picture, and divide them by using a ratio table. No formal algorithms are presented yet.

- *(7/8) Powers of Ten.* Whole numbers and decimals are multiplied by 10 and divided by 10 and powers of 10.

- *(8/9) Reflections on Number.* (Not in Plan B). The standard algorithm for multiplying integers with multiple digits is given.

Discussion: This standard is not adequately met. The arithmetic operations of dividing fractions, multiplying decimal numbers, or multiplying general fractions are studied only for simple cases and not to an extent that would ensure students understand the "meaning and effects" of these operations.

3.2.9 Number Standard Question 9.

Does the curriculum enable all students to use the associative, commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals?

Singapore: Score: 3
Evidence:

- *(6A).* In Ch. 1, the rules for order of operations are given.
- *(6B).* In Ch. 2, students simplify expressions that require these properties. An example on p. 19 shows $6a - 6b = 6(a - b)$ and $3e + 6 + e - 3 = 4e + 3$. These properties are not referred to by name.
- *(SL1).* On p. 102, the associative, commutative, and distributive properties are used by name to perform operations with whole and real numbers.
- *(SL1).* On p. 124, problem 4a, students have an opportunity to use the distributive property to calculate more efficiently.
- *(SL1).* In Ch. 1, students work problems involving integers that emphasize the commutative, associative, and distributive laws. In Ch. 4, they convert fractions to decimals and vice versa. They order fractions. They simplify calculations using the commutative, associative, and distributive laws.
- *(SL2).* On p. 34, these properties are used explicitly by name when factoring expressions with algebraic symbols and expanding them.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 1
Evidence:

- *(6) Bits and Pieces II.* There are no computations complicated enough that simplification is needed.
- *(7) All Units.* There are no computations complicated enough that simplification is needed.
- *(8) Frogs, Fleas, and Painted Cubes.* On p. 84f, the teacher is told what the order of operations are. On p. 39, the factoring of quadratic equations leads to a discussion of the distributive property. This is the first such discussion in the curriculum.

- *(8) Say It With Symbols.* IV1 is entitled "Order of Operations." On p. 34, students see the distributive property and on p. 35 the commutative property of addition and multiplication. On p. 45, students must insert parentheses to make the statement a valid equation. This is building toward the associative property, but this property is never mentioned.

- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In IV4, the commutative property of addition is given for numbers in order that a discussion can go forward about whether there is a commutative property for symmetry rotations.

Discussion: This standard is not adequately met. The associative property is not mentioned.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Fraction Times.* $4 \times 3.26 = 3 \times 4 + .20 \times 4 + .06 \times 4$ illustrates the distributive property. Very little practice is done to reinforce this idea.

- *(6/7) Operations.* Parentheses are used for operations involving integers.

- *(6/7) More or Less.* At this time in the curriculum, the distributive property is used as the algorithm for multiplying decimals (as illustrated above). This will help students understand the standard algorithm once it is taught.

- *(7/8) Building Formulas.* The associative, commutative, and distributive properties are mentioned by name to the teacher but not to the students. The students do work with these laws.

- *(8/9) Reflections on Number.* (Not in Plan B.) These laws are not stated explicitly, but are mentioned to the teacher. Examples include $42 \times 32 = (40+2) \times (30+2)$. The standard algorithm is then given and these strategies for multiplication compared to it.

Discussion: This standard is fully met.

3.2.10 Number Standard Question 10.

Does the curriculum enable all students to understand the use of inverse relationships of addition and subtraction, multiplication and division, and squaring and finding the square roots to simplify computations and solve problems?

Singapore: Score: 3
Evidence:

- *(5A) and (5B).* Addition and subtraction are seen as inverse operations.

- *(6A).* In Ch. 1, division by 4 is introduced as multiplication by 1/4.

- *(SL1).* On p. 32, square, square root and perfect squares are introduced. Students see squaring and taking the square root as inverse operations. In the Class Investigation, p. 34, students explain how one would proceed to determine whether or not a number is a perfect square.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces II.* The inverse relationship of addition and subtraction is made clear. Division of fractions and decimals are not present.

- *(7) Accentuate the Negative.* In IV3, subtraction is seen explicitly as the inverse of addition. In IV4, p. 59, division is presented as "undoing" multiplication.

- *(8) Looking for Pythagoras.* On p. 19, square root is introduced. In IV4, students use the Pythagorean Theorem to solve problems using right triangles.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Expressions and Formulas.* The inverse operations of addition (subtraction) and multiplication (division) are made explicit by the use of reverse arrow language.

- *(6/7) Operations.* Addition and subtraction, multiplication and division are seen as inverse operations.

- *(7/8) Powers of Ten.* The inverse properties of multiplication and division are investigated in this unit by using an imaginary device called the 10's machine. Students imagine if the handle of the machine is turned forward (backward) once, the number in the machine is multiplied (divided) by ten.

- *(7/8) Building Formulas.* Section C, p. 64 deals with the inverse operations of square root and squaring.

Discussion: This standard is fully met.

3.2.11 Number Standard Question 11.

Does the curriculum enable all students to select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply selected methods?

Singapore: Score: 1
Evidence:

- *(4B)*. In Ch. 3, students estimate and then calculate the exact answer to problems involving decimals. They are told which methods to select.
- *(5A)*. Students round numbers.
- *(5B)*. Students approximate and estimate answers with decimals.
- *(5B)*. On p. 11, students do mental calculations such as 4.203×20.
- *(up through 6B)*. No calculator usage is found. The calculator usage begins in *SL1* which corresponds to 7th grade.
- *(SL1)*. Many, many problems are done using the calculator. For example, see p. 23 of the Workbook for *SL1*.

Discussion: This standard is not adequately met. Students do a large amount of practice with mental computation, estimation, pencil and paper exact computation, and calculators (in grades 7 and 8). The curriculum tells the student what method or tool to apply to each problem. Since the students do not have to make that level of decision, the *select* part of this standard is not met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces I.* Students use benchmark fractions to compare and estimate values of other fractions. Not much computation is done, rather a firm understanding of the concepts is built here. For example, in IV4, students try to figure out how to add fractions with unlike denominators before any algorithms are given. This gives them a chance to struggle with the concepts. On p. 58, prob 3, students estimate a "good" fraction to represent a given decimal number.
- *(6) Bits and Pieces II.* Students estimate by comparing to benchmarks in IV3. On p. 37, estimation is done without the aid of a calculator. On p. 24, students do 15 easy problems involving percents. For these problems, they are allowed to use a calculator, but only a few should require one. For example 5% of 40 could be done without a calculator. (The teacher must be careful that an over-dependence on the calculator is not developed and that mental strategies do not suffer.)
- *(7) Data Around Us.* On p. 47, mental calculation of $15{,}000 \times 50{,}000$ is done.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *Number Tools, Vol. 1.* In Section D, the calculator is used for whole number division and for adding decimals.

- *(6/7) Made to Measure.* Students estimate sums of decimal numbers.
- *(6/7) Fraction Times.* Students use the calculator for converting fractions to decimals or use the ratio table for simple cases.
- *(6/7) More or Less.* Students select from the ratio table, the double number line, conversion to a decimal, the calculator, estimation, the 10% rule, or the 1% rule. Students still haven't been introduced to the standard algorithm from even a conceptual (not mechanical) standpoint.
- *(7/8) Powers of Ten.* Students use a calculator for multiplication with large numbers and decimals. They use pencil and paper for multiplication and division involving powers of 10.

Discussion: This standard is fully met.

3.2.12 Number Standard Question 12.

Does the curriculum enable all students to develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use?

Singapore: Score: 1
Evidence:

- *(5A).* Students are asked to calculate the mixed fraction subtraction 1 3/8 - 7/12 early in 5th grade.
- *(6A) and (6B).* Students develop fluency in applying these algorithms to solve a large number of word problems. They are required to compute fluently.
- *(6A).* The Teacher's Guide, Ch. 3, p. 6, encourages teachers to have students try different methods and gives the teacher a couple of methods for some problems.
- *(SL1).* Students use the associative, commutative, and distributive laws to compute with fractions and decimals in Ch. 4 and integers in Ch. 5.
- *(SL1).* On p. 84, an algorithm is given for dividing two decimals that tells how to shift the decimal point, but not why, nor is the student asked to explain why it works.

Discussion: This standard is not adequately met. Students are fluent in using what is developed in the book, but are not asked by the curriculum to *develop* or *analyze* algorithms.

Connected Mathematics Program: Score: 1
Evidence:

- *(6) Bits and Pieces I.* Students find equivalent fractions in simple cases. They convert decimals to fractions (out of 100). They are not yet fluent in computing with fractions and decimals as evidenced by the simplicity of the problems they do.

- *(6) Bits and Pieces II.* Students do develop strategies and probably arrive at the standard ones. They must explain to the teacher and to other students their strategy and test it out on several problems. The class discusses the strategies to make sure they are correct. The missing component here is is the fluency. They can not divide fractions yet, or decimals, or work with general numbers away from their benchmarks.

- *(7) All units.* Very few problems require more than simple computation. Not enough problems are included that require the use of the associative, commutative, and distributive properties.

- *(8) All units.* No associative property is evident. No division of fractions is found. Not many simplification strategies are needed that would actually test the conceptual knowledge of the properties above. Not much fluency of use is required.

Discussion: This standard is not adequately met. Students develop and analyze algorithms, but they are not fluent in the use of these algorithms except in simple cases. Not enough work is done with multiplying decimal numbers (say .23x.37) from even a conceptual point of view that reinforces place value and use of the distributive law. These units are at a lower mathematical level than the standards say should be present in grades 6-8.

Mathematics in Context: Score: 1
Evidence:

- *(6/7) Fraction Times.* The strategies work only in simple cases. Students are not yet *fluent*.

- *(6/7) More or Less.* Simplified problems are used in this unit.

- *(6/7) Rates and Ratios.* Reasoning is used with numbers that are easy to work with. Students must depend on the calculator for anything that is not extremely simple.

- *(7/8) Cereal Numbers.* The area model is used for multiplication of fractions and the ratio table for division of fractions. No fluency is expected for more general cases.

- *(7/8) Powers of Ten.* Fluency is seen here in multiplying by 10 and dividing by 10. According to the Standards, this should have been accomplished as early as 3rd or 4th grade.

- *(8/9) Reflections on Number.* (Not in Plan B.) This is the point in the curriculum where the standard algorithm for multiplying *integers* and dividing them is introduced for the first time. The mathematical terms of divisor and dividend are introduced. Without this unit, students will not be exposed to these algorithms. This unit does not have very many practice problems for those students that learn to solidify concepts through practice. The ratio table is relied upon to explain how things work and to relate to work that students have done with this representation throughout the curriculum. The standard algorithm is analyzed in this unit.

Discussion: This standard is not adequately met. Algorithms are developed and analyzed, but students are not fluent in the use of these algorithms except in simple cases. Very little work is done with multiplying decimal numbers (say .23x.37) from even a conceptual point of view that reinforces place value and the use of the distributive law. The units above are mathematically correct and teach for conceptual understanding, but are at a lower mathematical level than the standards say should be present in grades 6–8.

3.2.13 Number Standard Question 13.

Does the curriculum enable all students to develop and use strategies to estimate the results of rational number computations and judge reasonableness of results?

Singapore: Score: 3
Evidence:

- *(4A to 5B).* Students estimate the results of multiplication and division problems with rational numbers.
- *(SL1).* Ch. 6 is entirely about estimation. Students work with general numbers and are able to judge the reasonableness of the results.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces II.* In IV3, students estimate the results of fraction and decimal computations. On p. 42, the reasonableness is judged using the vicinity of a benchmark. No instructions are given in the student manual about asking if it seems reasonable other than comparing to a benchmark. On p. 42a, the quote "The primary objective is for them to use benchmarks for making sense of fractions and decimals as quantitites." On p. 42b, benchmarks are refined to more decimal places.
- *(7) Comparing and Scaling.* On p. 46 students are asked, "How confident are you that your estimate is accurate? Explain."

- *(7) Stretching and Shrinking.* In problem 4.1 on p. 42, students must say if their estimate is an overestimate or an underestimate. On p. 62 they are asked to explain if they think their height estimates are reasonable.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Fraction Times.* Students round numbers in order to estimate the result of decimal additions.

- *(6/7) More or Less.* Students estimate answers and question the reasonableness of the answer.

- *(7/8) Cereal Numbers.* Student work with easier numbers that are close to get a ballpark idea of the range of an answer.

Discussion: This standard is fully met.

3.2.14 Number Standard Question 14.

Does the curriculum enable all students to develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios?

Singapore: Score: 1
Evidence:

- *(6A).* In Ch. 2 and Ch. 3, ratio tables and the unitary method are used for problems involving proportion. A visual and conceptual introduction using bar charts is used. Many word problems are solved that require sophisticated thinking skills. Proportions with three things (a:b:c) are done as well.

- *(6B).* In Ch. 5, many extension problems for abler students are included on this topic.

- *(SL1).* Ch. 11 revisits proportion and ratio in light of equivalent ratios, e.g. on p. 206, 30:15 is the same as 2:1. Ch. 16 is entirely about scaling using proportions and ratios. The method for solving such problems is given to the student.

- *(SL2).* On p. 291, students are asked to compare these concepts to what they learned in Ch. 10.

- *(SL2).* On pp. 294–296, students discuss observations in class of enlarging pictures using scaling. On p. 297, students use the strategy given in the book to perform successive transformations of rotating and enlarging.

Discussion: This standard is not adequately met. Students work fairly complicated problems involving proportions. They do discuss the methods they use, but it is not evident in the curriculum that they *develop* or *analyze* the methods. They are given problems to solve and told which method to use to solve the problems.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Covering and Surrounding.* Students use the idea of scale to design a park based on area specifications.

- *(7) Stretching and Shrinking.* The entire unit is about scaling 2D figures to produce similar figures. The idea of proportion is used to find the missing side lengths in similar triangles.

- *(7) Comparing and Scaling.* On p. 40, problem 4.2b, students use the rate table to solve problems using proportions.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* This book uses recipes to introduce the ratio table.

- *(6/7) Rates and Ratios.* Section E gives a good treatment on scaling and finding equivalent ratios.

- *(7/8) Cereal Numbers.* Students use ratio tables to find equivalent ratios to calculate serving sizes. This is essentially division of fractions.

- *(7/8) Powers of Ten.* In Section E, scaling factors are used to make a model of the solar system.

- *(8/9) Triangles and Patchwork.* Multipliers are found that relate the corresponding sides of similar triangles.

Discussion: This standard is fully met.

3.2.15 Number Standard Summary

Question	Singapore	CMP	Math-in-Context
Number 1.	3	2	2
Number 2.	3	3	3
Number 3.	3	3	3
Number 4.	3	3	3
Number 5.	3	3	2
Number 6.	3	3	2
Number 7.	3	3	3
Number 8.	3	1	1
Number 9.	3	1	3
Number 10.	3	3	3
Number 11.	1	3	3
Number 12.	1	1	1
Number 13.	3	3	3
Number 14.	1	3	3

Table 1: Summary of NCTM Number and Operations Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets eleven of the standards, but does not adequately meet three of them. The lower scores were given because there are minimal requirements in the curriculum for students to select, develop, and analyze methods and algorithms. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- No calculators are used in *(6A)* or *(6B)*.
- Negative numbers are not studied until *(SL1)*.

CMP:

The scores in the table above show that the curriculum fully meets ten of the standards, adequately meets one of them, and does not adequately meet three of them. The lower scores were given because the curriculum does not enable students to adequately work fluently with fractions, decimals, percents, and integers at the mathematical level expected for grades 6-8. Furthermore, the curriculum does not address the associative law. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- Negative numbers are not studied until the 7th grade.
- The associative, distributive, and commutative laws are not encountered until the 8th grade. Moreover, these concepts do not build up from early arithmetic, which would make the algebraic notions stronger.
- Proportionality and ratios are not studied until 7th grade.

MIC:

The scores in the table above show that the curriculum fully meets nine of the standards, adequately meets three of them, and does not adequately meet two of them. The lower scores were given because the curriculum does not enable students to adequately work fluently with fractions, decimals, percents, and integers at the mathematical level expected for grades 6-8. There is also no mention of finding the common factors of two numbers anywhere in the curriculum. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- Only at the *(8/9)* level in the *Reflections on Number* unit do students see an algorithm for multiplying integers with multiple digits and that unit is not included in Plan B. It is conceivable, therefore, that students can complete a middle years mathematics program without having seen such an algorithm and having had to *analyze* why it works, or having seen its utility when numbers are not simple. This unit is also the only one in the curriculum where the topic of factors, multiples, and prime factorization of whole numbers are addressed.
- The names of the three fundamental mathematical laws (associative, communicative, distributive) do not appear in the student books.

3.3 Algebra Standard

3.3.1 Algebra Standard Question 1.

Does the curriculum enable all students to represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible symbolic rules?

Singapore: Score: 3
Evidence:

- *(5B)*. On p. 51 and p. 53, students represent tabular data using piecewise linear graphs. On p. 53, linear relationships are used to represent the exchange rate between U.S. and Singapore currency.

- *(6A)*. In Ch. 2, students work with proportional patterns seen as ratio tables, fractions, and ratios. Students are required to use statements such as "proportional to," "directly proportional to," and "the ratio of 5 to 3."

- *(6B)*. On p. 18, the relationship of the temperature (T) of water as a function of time (t) is generalized using the formula $T = 15 + 16t$. Students find T after 10 minutes and after 1/2 a minute. No graph is drawn.

- *(6B)*. On p. 15, tables are used to generalize a relationship. For example, if we have m stamps and 8 are in a set, then we have $m/8$ sets. This would mean if we had n in a set, then we have m/n sets.

- *(6B)*. On p. 21, students go from words to algebraic expressions in terms of a variable. They plug in values of the variable to evaluate the expression.

- *(SL1)*. On p. 43, number sequence patterns are studied with the goal of stating the general rule.

- *(SL1)*. On p. 44, tables are used to help find the pattern.

- *(SL1)*. On p. 53, examples of tabulation are given to help conclude the general rule for k layers is $3k + 3k(k-1)/2$. Patterns are generalized using symbolic notation. Students do 24 such problems in the Workbook for Ch. 3.

- *(SL2)*. In Ch. 8, students draw graphs.

- *(SL2)*. In Ch. 12, students work with representations of data in statistics.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time*. Patterns are introduced for square numbers and rectangular numbers on p. 43.

- *(7) Variables and Patterns.* In IV1, students collect data, and make tables and graphs for the same relationship. In IV2, p. 25, they analyze what could be true for points between the data points in the graph (getting to the idea of interpolation). Students go between words, tables, graphs, and simple symbolic equations to describe patterns in the context of a bicycle tour business. They look for a pattern in a table, fill in a table from a graph, write words and a formula. On p. 37 two bike companies send out information and one is in tabular form and one in graphical form. The students must make a decision which company has the better deals. On p. 39, they extrapolate to predict future profit from a graph.

- *(7) Moving Straight Ahead.* In IV1, students predict how the pattern will continue. They determine the two variables that are involved and state their relationship.

- *(8) Growing, Growing, Growing.* On p. 24, students state whether the pattern in the table is linear or exponential or neither. They write the equation, and draw the graph.

- *(8) Frogs, Fleas, and Painted Cubes.* In IV3, patterns that are quadratic are investigated, such as triangular and square numbers.

- *(8) Say It With Symbols.* In IV5, students generalize a pattern to a formula for the surface area of n staggered rectangular prisms as a function of n.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Patterns and Symbols.* This unit is an introduction to patterns. Students substitute numbers for symbols. They replace a symbol by three other symbols, represented as a tree. They investigate odd and even.

- *(6/7) Expressions and Formulas.* Tables are used for linear relationships and their relationship to arrow language is explored.

- *(6/7) Tracking Graphs.* Students read graphs, make graphs from story problems, and informally analyze them (ups, downs). They look at patterns in tide tables and temperature charts.

- *(7/8) Building Formulas.* Patterns are represented with strings of letters. Students generalize patterns with an algebraic formula, both recursive and direct.

- *(7/8) Ups and Downs.* Students match stories (p. 30) to graphs and study increase and decrease. In Section B, they study periodic graphs and cycles in such graphs.

- *(8/9) Graphing Equations.* Linear relationships are written as $y = sx+i$. Students generalize the relationship of car rental prices into an algebraic equation of a line and graph the line to answer various questions.

- *(8/9) Patterns and Figures.* Students write the symbolic expression for the general pattern.

Discussion: This standard is fully met.

3.3.2 Algebra Standard Question 2.

Does the curriculum enable all students to relate and compare different forms of representation for a relationship?

Singapore: Score: 3
Evidence:

- *(6B).* On p. 18, both words and an algebraic equation are used to represent temperature as a function of time.
- *(6B).* In the Teacher's Guide, p. 21, pictorial representations are used to explain that $x + 4$ means adding x to 4 or 4 to x.
- *(6B).* On p. 12, students make up words to go with an algebraic expression, such as x^2y.
- *(6B).* In Ch. 5, bars are used to represent and solve word problems involving proportions without algebra. Fractions and percents are represented the same way.
- *(6A and 6B).* Complicated word problems are made simpler through picture representations. This is a building block of algebraic thinking.
- *(SL1).* On pp. 149–153, before and after pictures are used for "age word problems." Relationships are represented by systematic lists followed by algebraic formulas. In problems on pp. 151–153, students would probably go between systematic lists and algebraic symbols and between pictures and equations as illustrated in the text, even though they are not explicitly told to do so.
- *(SL2).* On p. 73, tables and graphs show the same linear relationship.
- *(SL2).* In the Workbork on p. 72, an equation and a corresponding table of incomplete entries are given. Students must complete the table and then graph the equation to answer questions from the graph, see problems 116–123. This requires students to work with three different representations of the same relationship simultaneously.
- *(SL2).* On p. 101, given an equation, students fill in the corresponding table and draw the corresponding graph in Exercise 7B.
- *(SL2).* On p. 117, in problems 1–9, students go between equations, tables, and graphs for the same relationship.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns.* On p. 35 in the Reflection, students evaluate the advantages and disadvantages of using words, formulas, tables, and graphs to show the same relationship.
- *(7) Moving Straight Ahead.* The equation $y = mx + b$ is graphed. Values from the table are found on the graph.
- *(8) Thinking With Mathematical Models.* Tables, equations, and graphs are made for straight lines and nonlinear models.
- *(8) Growing, Growing, Growing.* Tables, equations, and graphs are made to examine exponential growth and decay.
- *(8) Frogs, Fleas, and Painted Cubes.* On p. 17, tables, graphs, and equations are used to describe quadratic relationships. The Arch in St. Louis is given as an example of a parabola on p. 7.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Patterns and Symbols.* Students use arithmetic trees to substitute numbers for variables to see growth patterns.
- *(6/7) Expressions and Formulas.* Students convert between arrow language and formulas, and between arithmetic trees and formulas.
- *(6/7) Comparing Quantities.* Students use combination charts, notebook notation, and equations for a linear relationship between two variables.
- *(6/7) Tracking Graphs.* Graphs and tables are made for temperature and tide data as a function of time.
- *(7/8) Building Formulas.* A formula, a table, a graph, and an arithmetic tree are used to represent linear relationships.
- *(8/9) Graphing Equations.* The algebraic equation and its straight line graph are matched on p. 62.
- *(8/9) Patterns and Figures.* Dot patterns, area diagrams, and number strips are used to model sequences of square numbers.
- *(8/9) Get the Most Out of It.* Students use tables to make graphs for straight line data on p. 12.

Discussion: This standard is fully met.

3.3.3 Algebra Standard Question 3.

Does the curriculum enable all students to identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations?

Singapore: Score: 1
Evidence:

- *(Up through 6B).* Students plot data from tables. They read graphs, but have not yet differentiated between linear and nonlinear relationships.

- *(SL1).* In Ch. 8, students construct linear equations from simple story problems. They practice going from the words to the equations. They have not yet used tables or graphs or nonlinear equations.

- *(SL2).* On p. 173, tables of values of x, y, and $3x + 2y$ and the corresponding graph and equation are compared. Students investigate the effect of changing m and b in the formula of a line $y = mx + b$. They produce tables and graphs of equations of quadratics and study the behavior of these graphs including min and max. They investigate the effect of translation and inversion.

Discussion: This standard is not adequately met. Although students work with linear and quadratic functions as evidenced above, they don't have to determine whether a given relationship is linear or nonlinear (say from a table of values). They are told what it is and they work with it accordingly.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns.* On p. 60i in the Teacher's Guide, plots of inverse relationships such as $t = 40/r$ are discussed. These equations are said to be nonlinear as opposed to linear.

- *(7) Moving Straight Ahead.* On p. 34 in the Reflection, students are asked "How do you decide whether a relationship is linear?" Then they say how to compare rates for a linear relationship. They find the y-intercept from the graph and the tabular data, and the equation. On p. 89, problem 23, students have to tell which graphs are linear or nonlinear.

- *(8) Thinking With Mathematical Models.* In IV2, students graph nonlinear relationships. On p. 34, they study equations like $y = a+2/x$ with graphic calculators. They look at a large number of graphs of both linear and nonlinear phenonemon. On p. 45, four graphs showing biological information about a caribou population are interpreted. On p. 21, students say whether data in a table represents a linear relationship.

- *(8) Growing, Growing, Growing.* On p. 24, students explain whether data in a table are linear or exponential or neither.

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 18g, tables of linear, exponential, and quadratic functions are compared for the teacher.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(7/8) Building Formulas.* On p. 64 the teacher is told that an area of a circle will increase quadratically as the radius doubles.

- *(7/8) Ups and Downs.* On p. 70, linear growth is discussed as a straight line corresponding to a constant rate of increase or decrease. On p. 80, a formula for linear growth related to a story problem is written and viewed as repeated addition. On p. 96 exponential growth is seen as repeated multiplication, and in Section E half-life is viewed as exponential decay.

- *(8/9) Graphing Equations.* The graph of a line is introduced on p. 62.

- *(8/9) Getting the Most Out of It.* Section F shows a line vs. a hyperbola to model rectangles with constant perimeter vs. constant area, respectively.

Discussion: This standard is fully met.

3.3.4 Algebra Standard Question 4.

Does the curriculum enable all students to develop an initial conceptual understanding of different uses of variables?

Singapore: Score: 3
Evidence:

- *(6B).* In Ch. 2, variables are used to represent both independent and dependent quantities. Variables stand for values of amounts in word problems. In the Teacher's Guide, p. 47, variables are used to represent unknown angle sizes.

- *(SL1).* In Ch. 8, students go from word problems to algebraic expressions that represent the problem.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns.* In IV1, students begin to understand that variables represent amounts of a quantity that changes, and begin to identify two variables to make a graph. The variables are used on the axis of a graph or in symbolic form.

- *(7) Moving Straight Ahead.* Variables are related to coordinates.
- *(8) Frogs, Fleas, and Painted Cubes.* In IV3, variables represent the general case in patterns.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Comparing Quantities.* Students begin to use two variables to describe linear relationships of the form $ax + by = c$.
- *(7/8) Building Formulas.* Variables are used to represent amounts of a quantity. Letters are used to represent types of things in a pattern, not necessarily an amount. Variables are used to represent temperature, time, and percentages.
- *(7/8) Decision Making.* Students use H and T to represent the number of houses and townhouses repectively, and graph inequalities to determine feasible regions for the amount of each to build.
- *(8/9) Getting the Most Out of It.* Two variables are used to represent the amount of two competing choices in linear programming problems and linear inequalities are used to isolate feasible regions in Section E.

Discussion: This standard is fully met.

3.3.5 Algebra Standard Question 5.

Does the curriculum enable all students to explore relationships between symbolic expressions and graphs of lines, paying particular attention to meaning of slope and intercept?

Singapore: Score: 3
Evidence:

- *(SL2).* In Ch. 7, students graph lines and study effects of shifting them to see where they intersect the y-axis. No formal words of slope and intercept are used, but students are indeed using these concepts as seen in problems 7 and 9 on p. 109. (Formal definitions of these terms are given in *SL3*, pp. 46–51 (and Pan Pacific, Book 3, Ch. 4, see p. 9 and p. 24 of the Workbook), the Singapore books corresponding to 9th grade.)

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Moving Straight Ahead.* In IV5, slope and y-intercept are formally introduced. Each is seen from the equation, the graph, and the table of data.

- *(8) Thinking With Mathematical Models.* On p. 13, students find the equation of a line given the slope and one point on the line. On p. 18, they find the equation of the line given two points on the line.
- *(8) Looking for Pythagoras.* In IV6, students use the slopes of two lines to test whether they are parallel or perpendicular.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(7/8) Decision Making.* Students graph linear inequalities to find feasible regions. This leads to slope and intercept concepts.
- *(8/9) Graphing Equations.* On p. 62 the connection between an algebraic equation of a line and its graph is shown. On p. 70 slope, y-intercept, equation of a line, and tangent of the inclination angle the line makes with the x-axis are introduced.
- *(8/9) Getting the Most Out of It.* The relationship of slope and the idea of fair exchange is used to solve optimization problems in Section E starting from the symbolic equation.

Discussion: This standard is fully met.

3.3.6 Algebra Standard Question 6.

Does the curriculum enable all students to use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships?

Singapore: Score: 3
Evidence:

- *(6B).* Symbolic algebra is used to represent word problems, but students work mainly with expressions, not equations at this stage.
- *(SL1).* Ch. 8 is devoted to translating from a word problem to an algebraic linear equation, and symbolically solving for the variable.
- *(SL2).* In Ch. 5, students solve systems of two linear equations in two variables. In Ch. 6, they solve linear inequalities in one variable.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns.* Students write a relationship between two variables symbolically. No equations are solved yet.

- *(7) Moving Straight Ahead.* On p. 54, students write an equation from a story problem that has one linear equation and one variable and find its solution. On p. 63 in the Reflection, students are asked "How do you solve $y = mx + b$ for x if you know y?"

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 39, students match a quadratic equation with its graph. They solve for the x-intercepts by checking in a data table or looking at the graph.

- *(8) Say It With Symbols.* In IV4, p. 53, students solve two equations in two variables symbolically. In IV4, p. 57, equations of the complexity $x^2 + 5x = 0$ are solved for x by factoring.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Comparing Quantities.* Students solve two linear equations in two variables using symbolic algebra and notebook notation.

- *(7/8) Decision Making.* Linear inequalities are investigated. The intersection regions and feasible solutions are found by graphing formulas such as $H + T \leq 12,000$.

- *(8/9) Graphing Equations.* On p. 82 students solve $ax + b = cx + d$ equations for x with a, b, c and d known numbers. This is done in a very concrete way using jumping frogs. On p. 90, an abstract representation of the same equation above and the necessary algorithmic skills to solve it are given.

- *(8/9) Getting the Most Out of It.* In Section E, students solve story problems involving linear inequality constraints.

Discussion: This standard is fully met.

3.3.7 Algebra Standard Question 7.

Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations?

Singapore: Score: 3
Evidence:

- *(SL1).* In Ch. 8, p. 141, the algebraic manipulations that are used to reduce an equation to a simpler equivalent equation are given. Students do a large number of practice problems with these operations to solve a linear equation in one variable. The standard is met here for one equation.

- *(SL2).* On p. 74, two linear equations in two variables are solved by the elimination method.
- *(SL2).* On p. 77, two linear equations in two variables are solved by the substitution method.
- *(SL2).* On p. 79, word problems with two linear equations in two variables are solved with pictures.
- *(SL2).* On p. 109, two linear equations in two variables are solved by graphing.
- *(SL2).* In Ch. 3, quadratic equations are solved by factoring.
- *(SL2).* In Ch. 7, quadratic equations are solved by graphing.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Moving Straight Ahead.* In IV4, a single linear equation is solved by converting to a simpler equivalent equation by symbolic manipulation.
- *(8) Say It With Symbols.* On p. 53, students solve two linear equations in two variables by generating equivalent equations.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Comparing Quantities.* Students solve linear relationships in two variables using notebook notation with row operations to generate equivalent forms of the same equations. Also the form of the equation with variables is given to show the correspondence to the notebook notation.
- *(7/8) Building Formulas.* The distributive, associative, and commutative laws are used to see the equivalence of expressions such as $9(2L+S)$ and $18L+9S$, and $x+(x+1)+2x$ and $5+4(x-1)$.
- *(8/9) Graphing Equations.* In Section E, equations of the form $5+3x=8+2x$ are solved.
- *(8/9) Patterns and Figures.* p. 58 shows visually that $(n+1)^2=n^2+2n+1$.

Discussion: This standard is fully met.

3.3.8 Algebra Standard Question 8.

Does the curriculum enable all students to model and solve contextualized problems using various representations such as graphs, tables, and equations?

Singapore: Score: 3
Evidence:

- *(6B)*. Students do many proportional reasoning story problems using the unitary method and ratio tables.
- *(SL1)*. In Ch. 8, p. 149, students solve word problems using algebra or picture models. These are linear equations in one variable. No graphing of linear equations is done in *(SL1)*.
- *(SL2)*. In Ch. 8, financial contexts are used for applications with linear equations, pp. 125-126.
- *(SL2)*. In the Workbook, p. 123, students see examples of the amount of petrol that is used to travel various distances. Graphs, tables, and equations are used as different representations of this relationship.
- *(SL2)*. On p. 119, practical problems involving travel graphs are introduced on pp. 126–129. Students are provided a large number of these problems.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns*. Students are just starting to see what graphs, tables and symbolic formulas are. They are not yet solving equations.
- *(7) Moving Straight Ahead*. Simple linear equations are written for story problems such as installment paying and forensic science, and are solved using the representations above.
- *(8) Growing, Growing, Growing*. The Unit Project is to model the half-life of iodine-124 by using sampling techniques to determine the amount that decays at a given time. Students are told to mark one face of a number cube. They draw number cubes from a hat to simulate the fraction of the iodine that decays at each step. They must come up with the equation that models the situation, and make tables and graphs to show their results.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Tracking Graphs.* Students represent or model speed problems with graphs.
- *(6/7) Comparing Quantities.* Students do contextualized problems using "notebook notation" (spreadsheet format), combination tables and equations, exchanging and substituting. (They are not facile yet with equation manipulation.)
- *(7/8) Looking at Angle.* Students use graphs of glide angle and glide ratio to solve problems.
- *(7/8) Building Formulas.* Students use the Centigrade-Fahenheit relationship in table, graph, and equation form in problems 6a and 6b on p. 82.
- *(7/8) Decision Making.* Students use graphs of feasible regions to solve linear programming problems that arise in decision-making contexts.
- *(7/8) Ups and Downs.* The context of carbon dating is used in Section F to investigate exponential decay.
- *(8/9) Graphing Equations.* A firefighting example is used in Section F to find and investigate the use of intersecting lines to locate a fire.
- *(8/9) Getting the Most Out of It.* The algebraic manipulation of two linear equations in two variables is done to find the solution. The same problem is solved by graphing the two lines and visually finding the intersection point. The pros and cons of both approaches are discussed.

Discussion: This standard is fully met.

3.3.9 Algebra Standard Question 9.

Does the curriculum enable all students to use graphs to analyze the nature of changes in quantities in linear relationships?

Singapore: Score: 3
Evidence:

- *(5B).* On p. 51, piecewise linear graphs show table data. Questions are asked about increase and decrease.
- *(5B).* In the Workbook, p. 61, students answer questions about increases and decreases in piecewise linear graphs.
- *(5B).* In the Workbook, p. 65, students fill in the blanks in a table that connects with a linear graph on exchange rates.
- *(5B)*: In the Workbook, p. 66, volume is given as a function of time in a linear graph. Students answer questions about how long it takes to fill a tank and answer questions that require reading the graph between values of time shown on the x-axis.

- *(SL2)*. In Ch. 7, linear and quadratic graphs are studied by the students. They analyze the nature of changes (increases, decreases, min, max) for quadratic graphs. They study graphs that model practical situations. They understand that linear graphs can show the conversion between the metric and customary measurement systems.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Covering and Surrounding*. Students make graphs of the diameter versus the circumference of a circle on p. 71.

- *(7) Moving Straight Ahead*. In problem 2.2, p. 18, students are asked how the rate (slope) affects the graph. In problem 5c, p. 26, students compare what graphs would look like based on tabular values. In problem 18c, p. 30, given the graph, students write the equation and generate a table. On p. 34 in the Reflection, students write about how to compare rates for two linear relationships from their graphs.

- *(8) Thinking With Mathematical Models*. On p. 25 in the Reflection, if the slope m is negative, students are asked what happens to y as x increases. In IV4, students sketch graphs that fit written descriptions.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Tracking Graphs*. On p. 22, students read "line graphs" to draw conclusions. On p. 46, these graphs are constructed from tabular data. The term "line graph" is presented as any graph that shows how quantities change over time. Examples given were not necessarily graphs of linear functions. This unit was an introduction to graphing and examining the changes in quantities as a function of time.

- *(7/8) Building Formulas*. Problem 6c on p. 84 requires students to read a straight line graph of Fahrenheit and Centigrade temperatures.

- *(7/8) Decision Making*. Students graph linear relationships and analyze them for the effect of exchanging a unit of one variable for a given quantity of another. This is working toward the idea of slope.

- *(7/8) Ups and Downs*. On p. 70, a linear graph is interpreted as a constant rate of increase or decrease and as repeated addition.

- *(8/9) Graphing Equations*. The relationship of the slope to the direction of the line and the change in vertical and horizontal directions is explored.

- *(8/9) Getting the Most Out of It.* In Section E, a rolling line (changing c in the equation $ax+by=c$) is used to obtain the optimal solution to an objective function in a feasible region formed by inequality constraints.

Discussion: This standard is fully met.

3.3.10 Algebra Standard Summary

Question	Singapore	CMP	Math-in-Context
Algebra 1.	3	3	3
Algebra 2.	3	3	3
Algebra 3.	1	3	3
Algebra 4.	3	3	3
Algebra 5.	3	3	3
Algebra 6.	3	3	3
Algebra 7.	3	3	3
Algebra 8.	3	3	3
Algebra 9.	3	3	3

Table 2: Summary of NCTM Algebra Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets eight of the nine algebra standards and does not adequately meet one of them. The lower score was given because students do not identify functions as linear or nonlinear from tabular data. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- All the algebra in *(6A)* and *(6B)* is with expressions and not equations. The first instance of modeling a word problem with an equation appears in *(SL1)*.
- Analyzing linear graphs starts in *(5B)*. This topic is not continued again until *(SL2)*, where both linear and quadratic graphs are studied.
- The concepts of slope and intercept start in *(SL1)* and continue into *(SL3)* (the text corresponding to 9th grade).

CMP:

The scores in the table above show that the curriculum fully meets all nine algebra standards. The evidence points to one issue worth mentioning that the scoring did not reflect.

- CMP only includes very minimal algebraic material in its 6th grade curriculum. (Reported in questions 1 and 9.)

MIC:

The scores in the table above show that the curriculum fully meets all nine algebra standards.

3.4 Geometry Standard

3.4.1 Geometry Standard Question 1.

Does the curriculum enable all students to precisely describe, classify, and understand relationships among types of 2D and 3D objects (e.g. angles, triangles, quadrilaterals, cyclinder, cones) using their defining properties?

Singapore: Score: 3
Evidence:

- *(5B).* In Ch. 7, students investigate angles by cutting out angles from paper parallelograms.
- *(6B).* In Ch. 4, the defining properties of rhombus, parallelogram, equilateral triangle, trapezoid, and cube are given.
- *(SL1).* In Ch. 13, p. 265 and p. 267, supplementary, complementary, acute, and obtuse angles are classified. Students work with cones and cylinders.
- *(SL1).* In Ch. 14 on p. 288, the defining properties of quadrilaterals are explicitly given.
- *(SL1).* In Ch. 17 on pp. 327–331, students work with the symmetries of regular polygons. In the exercises, students find planes of symmetry and axis of rotational symmetry for cones, cylinders, tetrahedra, and other solids.
- *(SL2).* In Ch. 11 on pp. 231–235, students find the volume of similar figures. These figures are cones, spheres, cylinders and cubes. They also use the property that the ratio of weights of similar solids made from the same material is the same as the ratio of their volumes.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* Students begin by recognizing angles, triangles, quadrilaterals, and regular and irregular polygons. On p. 45, equilateral and isosceles triangles are defined and on page 51 rectangles are defined.
- *(7) Filling and Wrapping.* The definition of a general right prism is given on p. 26. In IV4 and IV5, cylinders, cones, and spheres are defined. The relationship of cones and spheres to cylinders is given. Students build these and fit them into a cylinder.
- *(8) Looking for Pythagoras.* In Summarize, p. 16f, the teacher puts 2D figures on the board and students classify them depending on properties such as perpendicular sides, parallel sides, etc.

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 28, the parabola is studied in terms of its properties (max or min, x-intercepts, and line of symmetry).

Discussion: This standard is fully met.

Mathematics in Context: Score: 2
Evidence:

- *(5/6) Figuring All the Angles.* Right angle (p. 100), acute angle (p. 98) and obtuse angle (p. 102) are defined.
- *(7/8) Triangles and Beyond.* The defining properties of parallelograms, and isosceles, equilateral, and scalene triangles are given.
- *(7/8) Packages and Polygons.* Students classify 2D and 3D objects. The defining properties of regular polygons and polyhedra are given.
- *(8/9) Triangles and Patchwork.* The defining properties of similar triangles and corresponding and alternating angles are given.

Discussion: This standard is adequately met. Students do minimal work with cylinders and cones.

3.4.2 Geometry Standard Question 2.

Does the curriculum enable all students to understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects?

Singapore: Score: 3
Evidence:

- *(SL1).* In Ch. 15 on pp. 307–308, the formal definition of similarity is given. Students find the unknown sides of similar figures.
- *(SL1).* In Ch. 16 on p. 314, the concept of area scale is used to find the ratio between areas of similar figures.
- *(SL1).* In Ch. 16, p. 318, the relationship between the volumes of similar objects is found in Challenge Problem 3.
- *(SL2).* In Ch. 11, p. 222–230, the relationships of areas, volumes, and weights (same materials) of similar objects are studied.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Covering and Surrounding.* Students investigate perimeter and area of rectangles and parallelograms. They change the perimeter while keeping the area constant and vice versa. They build their own knowledge of the area of triangles, parallelograms, and trapezoids by constructing them on centimeter grid paper. They also compute the circumference and area of circles.
- *(7) Stretching and Shrinking.* In IV1, students draw similar figures. In IV2, they use the coordinate system to draw figures called Wumps that are mostly similar. They measure them to see the length scale and area scale for the similar ones. In IV3, the refining of figures to produce smaller congruent figures (called rep-tiles) allows students to study how areas of similar figures scale. This refining process is extremely useful in solving advanced practical problems and is used in developing computer graphics applications. In IV4 and IV5, the missing sides of similar figures are found.
- *(7) Filling and Wrapping.* In IV6, boxes are scaled up and down to create similar rectangular prisms. The scale factor is calculated and the volumes compared.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* The relationships of perimeter to area and surface area to volume are studied. The observation that if the length dimension doubles in each direction, the area increases by four is investigated.
- *(7/8) Triangles and Beyond.* If two triangles have equal angles, students discover that their shapes are similar.
- *(8/9) Triangles and Patchwork.* The formal definitions of similar triangles and corresponding and alternating angles are given. Ratios of corresponding sides of similar triangles are used to find lengths of unknown sides.

Discussion: This standard is fully met.

3.4.3 Geometry Standard Question 3.

Does the curriculum enable all students to create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity and Pythagorean relationship?

Singapore: Score: 2
Evidence:

- *(6B).* In Ch. 4, p. 42–54, students use deductive reasoning to find the size of missing angles in geometric figures. They use the general properties of equilateral triangles and rhombuses, for example.

- *(SL1).* In Ch. 15, p. 303, congruence and similarity are introduced and their formal definitions are given.
- *(SL1).* In the Workbook on p. 111, true-false problems for reasoning and constructing arguments about similarity and congruence are posed. The student must give the reason or justify with a counterexample.
- *(SL1).* On pp. 272–289, students use deductive arguments to find the missing angle in geometric figures. They must use the general properties of lines and quadrilaterals to do this.
- *(SL2).* In the Workbook, Ch. 10, pp. 78-84, students must determine whether given triangles are congruent. If so, they must write down the statement of congruency and "state the case" of congruency and name the other three pairs of equal measurements. They prove triangles congruent in problems 23 and 24 and triangles similar in problems 35–43 and 57, 58, and 60. In problem 69, they explain why the triangle is isosceles.
- *(SL2).* In Ch. 10, p. 191, the idea of minimum requirements for congruence is discussed. Students discover the SSS, SAS, and AAS rules. These are then used to make arguments about similarity.
- *(SL2).* On p. 215, problem 3, students are asked to make proofs.
- *(SL2).* On p. 220, problem 5, students prove two triangles are similar.
- *(SL2).* On p. 304, students are guided to discover the Pythagorean Theorem. Several proofs of this theorem using geometry are given on pp. 305–306.

Discussion: This standard is adequately met. Students do deductive proofs of congruence and similarity. We could not find instances of inductive reasoning related to geometry as described in the standards. (Students do use inductive reasoning in Chapter 3, *(SL1)*, but not for problems in geometry.) For these reasons, a score of 2 was given.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Stretching and Shrinking.* On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?"
- *(7) Filling and Wrapping.* In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.

- *(8) Looking for Pythagoras.* In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove the Phythagorean Theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.

- *(8) Hubcaps and Kaleidoscopes.* On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles.

- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation? Test several cases, and make a conjecture about the result." This is an example of inductive reasoning.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* In problems 4-6 on p. 44, students conjecture if there is a rule for finding the area of a quadrilateral whose corners touch the sides of a rectangle. If so explain it. Students try to disprove another student's rule since they are told it may work only on some examples. If so, they must describe when the rule works.

- *(7/8) Packages and Polygons.* In problem 4b on p. 86, students try to find the mistake in the reasoning given in the book.

- *(7/8) Triangles and Beyond.* Congruent shapes can be flipped, rotated, reflected and translated to align. On p. 94, it is stated, "If a figure is translated, rotated, or reflected, the resulting figure is congruent to the original figure."

- *(8/9) Triangles and Patchwork.* Students prove triangles are similar by investigating their angles and finding that the ratios of corresponding sides are equal.

- *(8/9) Going the Distance.* (Not in Plan B). Students investigate the Pythagorean Theorem and use it to make connections to glide angles of airplanes. Geometric proofs of this theorem are investigated. The tangent of the angle of inclination is calculated, and the inverse tangent button on a calculator used to find the angle.

Discussion: This standard is fully met.

3.4.4 Geometry Standard Question 4.

Does the curriculum enable all students to use coordinate geometry to represent and examine the properties of geometric shapes?

Singapore: Score: 3
Evidence:

- *(SL2).* In Ch. 7, graphs of lines are plotted using coordinates. Four lines are plotted and properties of the resulting figures are examined in problems 7 and 8 on p. 109. These figures either have parallel or perpendicular lines.

Discussion: This standard is fully met. (We note that coordinate geometry is studied in more depth in *SL3*, Ch. 3 (Pan Pacific Book 3, Ch. 4) which corresponds to 9th grade.)

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* On p. 60, problems 12-16, students figure out what four coordinates to plot to make a rectangle, and a parallelogram that is not a rectangle. They also plot triangles.

- *(7) Stretching and Shrinking.* In IV2, coordinates of a Wump are plotted. These coordinates are scaled and a new Wump is plotted. The similarity is investigated. On p. 27d, the teacher is told the rule that (x, y) maps to (nx, ny) is what produced the figures.

- *(8) Looking for Pythagoras.* On p. 15, students investigate the intersection of diagonals of squares, rectangles, and rhombuses to see that they are perpendicular. The lengths of the diagonals are also calculated.

- *(8) Frogs, Fleas, and Painted Cubes.* In IV2, students plot graphs of parabolas.

- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In IV3, images under symmetric transformations such as reflections, translations, and rotations are examined by coordinate maps, such as (x, y) maps to $(x + 3, y)$ for a translation.

Discussion: This standard is fully met.

Mathematics in Context: Score: 1
Evidence:

- *(6/7) Operations.* A coordinate system is introduced in Section F. A polygon's vertices are plotted, then questions about what happens to the polygon as the ordered pairs of numbers representing the vertices are multiplied by a constant, or either the abscissa or ordinate multiplied, or a number added to each, etc. This leads to enlargment, shrinkage, flips, or translations of figures.

Discussion: This standard is not adequately met. Coordinate geometry is not used to examine properties of the shapes such as parallel sides, perpendicular sides, or equal angles. This unit studies the effect an operation on the coordinates has on the original shape. This initial unit needs to be followed up by more coordinate geometry on shapes after the unit *(8/9) Graphing Equations*. This standard is started in *(6/7) Operations* but is not taken deeply enough by the end of the 8th grade.

3.4.5 Geometry Standard Question 5.

Does the curriculum enable all students to use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides?

Singapore: Score: 3
Evidence:

- *(SL2)*. In Ch. 7, coordinate geometry is begun. Graphs of lines are plotted using coordinates. Four lines are plotted and properties of the resulting figures are examined in problems 7 and 8 on p. 109. These figures either have parallel or perpendicular lines.

Discussion: This standard is fully met. (We note that more coordinate geometry is included in *SL3*, Ch. 3 (Pan Pacific Book 3, Ch. 4) which corresponds to 9th grade.)

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* On p. 60, problems 12-16, students figure out what four coordinates to plot to make a rectangle, and a parallelogram that is not a rectangle. They also plot triangles.

- *(7) Stretching and Shrinking.* In IV6, Turtle Math is used to draw rectangles, equilateral triangles and right trapezoids. The figures are scaled with Scale Tool and observed on a coordinate grid. Slopes are not yet used to analyze parallel or perpendicular lines.

- *(7) Moving Straight Ahead.* On p. 50, two lines are given on a coordinate grid and students are to conclude whether they are perpendicular based on their slopes. No geometric shapes are examined here.

- *(8) Looking for Pythagoras.* On p. 68, students find slopes of sides of figures from dot paper and their coordinates. They see how the slopes are related if the lines are perpendicular. In problem 15a, they look at the slopes of lines in a triangle formed by the diameter and chords of a circle.

Discussion: This standard is fully met.

Mathematics in Context: Score: 0
Evidence:

- *(6/7) Operations.* A coordinate system is introduced in Section F. A polygon's vertices are plotted, then questions about what happens to the polygon as the ordered pairs of numbers representing the vertices are multiplied by a constant, or either the abscissa or ordinate multiplied, or a number added to each, etc. This leads to enlargement, shrinkage, flips, or translations of figures. This is the only evidence found in the curriculum of the use of coordinate geometry to examine shapes.

Discussion: This standard is not met. Coordinate geometry was not used to study properties of shapes, such as pairs of parallel or perpendicular lines as discussed in this standard. The *(8/9)* units mention when two lines are parallel, but when two lines are perpendicular is not studied.

3.4.6 Geometry Standard Question 6.

Does the curriculum enable all students to describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling?

Singapore: Score: 3
Evidence:

- *(4A).* Lines of symmetry are investigated.
- *(6A).* In the Teacher's Guide, p. 97, a discussion is included about whether a shape can tesselate – whether it can flip, rotate, and move around to be congruent to itself. The words flip, rotate, translate, and congruent are not used in the student book. Students make tesselations and extend them.
- *(SL1).* In Ch. 16, p. 314, the context of maps is used to study scaling. The term area scaling is introduced. Proportional reasoning, similarity, the ratio method, and algebra are used to solve scaling problems.
- *(SL1).* In Ch. 17, pp. 323-324, rotational symmetry is used to investigate congruence of shapes under turns. A sophisticated treatment of lines of symmetry is on p. 324.
- *(SL2).* In Ch. 13, students reflect, rotate, translate, and enlarge plane figures. They use mirrors and do actual constructions.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* On pp. 52-54, flips and turns are introduced. No translations or scaling are done yet.
- *(6) Covering and Surrounding.* On p. 67, problem 23b., students discover that flipping a triangle over can make a parallelogram when combined with the original triangle. They use this fact to deduce the area of a general triangle from the area of a parallelogram.
- *(8) Stretching and Shrinking.* In IV3, students build similar figures called Wumps using scaling. Their positions are plotted on a cartesian grid and the transformations between the similar figures are determined. In IV4, students build larger similar figures, such as triangles, from smaller similar ones. The small triangle in the larger one is referred to as a rep-tile.
- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* This entire book is on symmetry transformations of flips, turns, and slides that lead to congruent figures. The students use these transformations and select tools to do these operations to make tesselations.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Operations.* A coordinate system is introduced in Section F. A polygon's vertices are plotted, then questions about what happens to the polygon as the ordered pairs of numbers representing the vertices are multiplied by a constant, or either the abscissa or ordinate multiplied, or a number added to each, etc. This leads to enlargement, shrinkage, flips, or translations of figures. This is the only evidence found in the curriculum of the use of coordinate geometry to examine shapes.
- *(7/8) Packages and Polygons.* Turns and flips of regular polyhedra and polygons are investigated.
- *(7/8) Triangles and Beyond.* Flips, turns, and slides are investigated.
- *(8/9) Triangles and Patchwork.* A scaling factor is used in the context of similar triangles.

Discussion: This standard is fully met.

3.4.7 Geometry Standard Question 7.

Does the curriculum enable all students to examine the congruence, similarity, and line of rotational symmetry of objects using transformations?

Singapore: Score: 3
Evidence:

- *(SL1).* In Ch. 17, pp. 323-324, rotational symmetry is used to investigate congruence of shapes under turns. A sophisticated treatment of lines of symmetry is on p. 324.

- *(SL2).* In Ch. 13, students rotate plane figures.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* On p. 5, simple rotational symmetry is introduced.

- *(7) Stretching and Shrinking.* In IV2, coordinates of a Wump are plotted. These coordinates are scaled and a new Wump is plotted. The similarity is investigated. On p. 27d, the teacher is told the rule that (x, y) maps to (nx, ny) is what produced the figures.

- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* Congruency is analyzed under similarity transformations. Tesselations are produced. Students locate lines of rotational symmetry.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(7/8) Packages and Polygons.* A square is folded in halves and quarters.

- *(7/8) Triangles and Beyond.* Congruence and the line of rotational symmetry using transformations are examined.

- *(8/9) Triangles and Patchwork.* This unit is all about similarity of triangles in the context of quilts and real life situations like finding the height of a tall tree using shadows.

Discussion: This standard is fully met.

3.4.8 Geometry Standard Question 8.

Does the curriculum enable all students to draw geometric objects with specified properties, such as side lengths or angle measures?

Singapore: Score: 3
Evidence:

- *(4A).* In Ch. 6, students investigate how two perpendicular lines are drawn.

- *(4A).* In Ch. 6, p. 98, students make parallel lines by sliding a set-square along a ruler.
- *(5B).* In Ch. 7, pp. 72-75, students draw parallelograms and rhombuses to specifications (angle and side length).
- *(6A).* In the Teacher's Guide, p. 75, students draw circles three different ways.
- *(6A).* On p. 82, students work in groups to make composite figures.
- *(SL1).* In Ch. 13, p. 276, students construct perpendicular bisectors. They draw lines perpendicular to another line, draw parallel lines, and use a compass and a set-square.
- *(SL1).* In the Workbook, p. 93, problems 57–63, students draw triangles to specification to reinforce the material up through *(6B).*
- *(SL2).* In Ch. 13, students do mathematical constructions to produce mirror images of planar objects under reflections, to rotate figures about a point, and to enlarge figures by specified factors. They also produce images that are combinations of these operations.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 2
Evidence:

- *(6) How Likely Is It?* Students make spinners for representing unequal probabilities. This requires angle measurements.
- *(6) Covering and Surrounding.* On p. 49, students construct parallelograms with specified measures. In problem 5.2, students use centimeter grid paper to draw rectangles and parallelograms with given dimensions and areas.
- *(7) Stretching and Shrinking.* On pp. 5–7, students use a rubber-band stretcher to draw similar figures that scale to twice or three times the original size.
- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* In the Unit Project, described on p. 71, students use dot paper, an angle ruler, and a protractor to make symmetry transformations and tesselations.

Discussion: This standard is adequately met. Students draw some simple objects with specified lengths and areas in *Covering and Surrounding.*

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* On p. 44, students draw general quadrilaterals with specified areas and break rectangles into parallelograms with specified areas. On p. 94, students use a compass to create specific enlargements of hexagons and circles.
- *(7/8) Ways to Go.* On p. 20, students use a compass to construct a triangle to scale (length of sides) representing the driving distances between three cities.
- *(7/8) Triangles and Beyond.* On p. 30, students do a compass construction of a triangle with specified lengths.

Discussion: This standard is fully met.

3.4.9 Geometry Standard Question 9.

Does the curriculum enable all students to use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume?

Singapore: Score: 3
Evidence:

- *(6B).* Students use nets of solids to visualize surface area. The problems at this level just ask which net goes with which solid.
- *(SL1).* In Ch. 10, pp. 177-186, volume and surface areas of solids (rectangular prisms, cylinders, and cylindrical rings (hollow cylinders)) are seen pictorially as cross-section slices and nets, repectively. For example, the volume of a cylinder is seen pictorially as a stacks of coins.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Ruins of Montarek.* This unit is entirely about spatial visualization. Students read information from a drawing to reason about and observe 3D objects to make 2D drawings. Students build from plans. For cubical objects, they determine the number of small cubes (volume) needed from a 2D representation. They learn to figure out how much surface area a 3D object has when part of it is hidden from view.
- *(7) Filling and Wrapping.* In IV1, a net for a unit cube is given on p. 5, a net for a rectangular box on p. 7, and a net for a cylinder on p. 39. Students make these nets and calculate the surface area and the volume of the 3D figures produced.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* Students cut out 2D patterns to make 3D objects. They make a 2D net for a box.

- *(7/8) Packages and Polygons.* Students build a large number of nets for 3D objects.

Discussion: This standard is fully met.

3.4.10 Geometry Standard Question 10.

Does the curriculum enable all students to use visual tools such as networks to represent and solve problems?

Singapore: Score: 1
Evidence:

- *(SL1).* In Ch. 3, p. 52, students use graphs to solve tournament pairing problems. On p. 53, students work combinatorial problems using a network formed by equilateral triangles.

- *(SL2).* On p. 109, in "Just for Fun," the nodes of a finite graph are labelled with numbers 1 to 8 in such a way that consecutive numbers are not joined by an edge.

Discussion: This standard is not adequately met. With the exception of the graphs for tournaments, students are not asked to construct visual tools themselves.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* On p. 57, the shortest path through a grid is found. Students explain why it is the shortest.

- *(8) Clever Counting.* IV3, students use networks to analyze the number of paths between cities and work other counting problems. Students create networks that satisfy given constraints.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(7/8) Ways to Go.* Students use networks to study combinatorial ideas such as the number of edges, $n(n-1)/2$, in a fully connected graph having n points. (This was one of the few instances in Mathematics in Context where the symbolic language of mathematics is used to describe a general case.)

- *(7/8) Packages and Polygons.* Bar (straw) models of platonic solids are used. These are similar to finite graphs.

Discussion: This standard is fully met.

3.4.11 Geometry Standard Question 11.

Does the curriculum enable all students to use geometric models to represent and explain numerical and algebraic relationships?

Singapore: Score: 3
Evidence:

- *(5A).* In Ch. 4, p. 66, a rectangle is used to clarify the relationship between a triangle's area and that of its related rectangle. This visual proof works for general triangles.
- *(6A).* On p. 94, students develop a formula for the area of a circle by cutting out circle sections and arranging them as a "rectangle."
- *(SL1).* On p. 169, the same circle proof above is given again, but this time the notion of having to add more sections to approach the shape of the rectangle is addressed.
- *(SL2).* On p. 34, an area model is used to explain how the expansion $(a+b)^2 = (a+b)(c+d)$ is connected to the distributive property.
- *(SL2).* On pp. 304-305, geometrical proofs are illustrated for the Pythagorean Theorem.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Filling and Wrapping.* In IV4, students examine two-dimensional nets for cylinders. They answer questions about the surface area of a cylinder from the geometric models.
- *(8) Looking for Pythagoras.* On p. 40g, the teacher is told how to use a geometric model to see that $(a+b)^2 = a^2 + 2ab + b^2$ and to prove the Pythagorean Theorem.
- *(8) Frogs, Fleas, and Painted Cubes.* Rectangles are used to go from story problems to algebraic expressions for quadratic relationships.
- *(8) Say It With Symbols.* Areas of rectangles are used to explain the distributive property of multiplication over addition on p. 1c and p. 20.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* A circle's sections are unfolded to see that the area approaches πr^2 as the number of sections increase.

- *(7/8) Packages and Polygons.* Students use models of regular polyhedra to verify Euler's Formula.

- *(7/8) Triangles and Beyond.* A semicircle is used along with the cutouts of the angles of a triangle to see that the sum of the angles of a triangle is 180 degrees.

- *(7/8) Building Formulas.* On p. 31, geometric pictures show terms in algebraic formulas.

- *(8/9) Patterns and Figures.* On p. 58, area models are used to explain that $(n+1)^2 = n^2 + 2n + 1$.

Discussion: This standard is fully met.

3.4.12 Geometry Standard Question 12.

Does the curriculum enable all students to recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life?

Singapore: Score: 3
Evidence:

- *(6B).* The displacement of solids in water to explain volume is a connection to science.

- *(SL1).* Most geometry is out of context. The exception is Ch. 16's treatment on scale drawings and maps.

- *(SL2).* On p. 125, students work with graphs for simple interest in financial contexts.

- *(SL2).* On p. 132, a graph of the length of a stretched spring under weight is a connection to science.

- *(SL2).* On p. 181, the displacement of solids in water are again used.

- *(SL2).* In Ch. 8 of the Teacher's Guide, a conversion graph from metric to customary units called a "ready rekoner" is used in everyday life.

- *(SL2).* On p. 297, tessellation investigations are related to "symmetry around us."

- *(SL2).* On p.319ff, symmetry is observed in real world objects.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* In IV1, tesselations with hexagons that are related to honeycombs are shown. On p. 4, a picture of the Pentagon is shown.
- *(6) Covering and Surrounding.* The ideas of perimeter and area are used in the context of floor plans. The Unit Project is to design a park that meets various area constraints.
- *(6) Ruins of Montarek.* Nets are shown for surface views. Blueprints are shown.
- *(7) Stretching and Shrinking.* On p. 61, the mirror method is used with similar triangles to find the heights of buildings and the width of irregular shapes. On p. 71, similar triangles are used to calculate distances in astronomy.
- *(7) Filling and Wrapping.* In IV7, volumes of irregular 3D objects are found by placing them in water. This is a connection to physics.
- *(8) Looking for Phytagoras.* In IV4, the Pythagorean Theorem is used to compute side lengths in applications, such as finding out if a barn wall is really perpendicular to the ground and how far a catcher must throw the ball to the 2nd baseman.
- *(8) Kaleidoscopes, Hubcaps, and Mirrors.* Symmetry transformations are used to create interesting artistic tesselations.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Figuring All the Angles.* The contexts of firefighting and air traffic control are used for finding angles.
- *(6/7) Reallotment.* Sewing patterns, Escher art for tesselations, and maps are used to estimate area and distance.
- *(7/8) Packages and Polygons.* Students make a package for a toy. Honeycombs that bees make in nature are viewed as tesselated hexagons, p. 52.
- *(7/8) Triangles and Beyond.* Reflections are seen in art pictures.
- *(7/8) Looking at Angle.* Blind spots, shadows, and flight paths for glide angle are used as contexts to study angle.
- *(8/9) Triangles and Patchwork.* Shadow heights used in movies are calculated. Quilt tesselations are examined.

Discussion: This standard is fully met.

3.4.13 Geometry Standard Summary

Question	Singapore	CMP	Math-in-Context
Geometry 1.	3	3	2
Geometry 2.	3	3	3
Geometry 3.	2	3	3
Geometry 4.	3	3	1
Geometry 5.	3	3	0
Geometry 6.	3	3	3
Geometry 7.	3	3	3
Geometry 8.	3	2	3
Geometry 9.	3	3	3
Geometry 10.	1	3	3
Geometry 11.	3	3	3
Geometry 12.	3	3	3

Table 3: Summary of NCTM Geometry Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets ten of the twelve geometry standards, adequately meets one standard, and does not adequately meet one standard. The lower scores were given because students do not do inductive reasoning related to geometry, and they only work minimally with visual tools such as networks. The evidence points to another issue worth mentioning that the scoring did not reflect.

- The curriculum does not make explicit that students should critique the arguments of others.

CMP:

The scores in the table above show that the curriculum fully meets eleven of the twelve geometry standards and adequately meets one standard. The lower score was given because the construction of objects to specification was only done for very simple objects, such as parallelograms and rectangles. The evidence points to other issues worth mentioning that the scoring did not reflect.

- We did not find evidence that students create or critique inductive or deductive arguments in 6th grade.

- Geometric models are not used to represent and explain numerical and algebraic relationships until 8th grade.

MIC:

The scores in the table above show that the curriculum fully meets nine of the twelve geometry standards, adequately meets one standard, does not adequately meet one standard, and does not meet one standard. The lower scores were given because students work minimally with coordinate geometry and with cylinders and cones. The evidence points to another issue worth mentioning that the scoring did not reflect.

- The unit *(8/9) Going the Distance* is the only unit in MIC that addresses the Pythagorean relationship. This unit is not included in Plan B, the recommended books for a three year middle-grades program.

3.5 Measurement Standard

3.5.1 Measurement Standard Question 1.

Does the curriculum enable all students to understand both metric and customary systems?

Singapore: Score: 3
Evidence:

- *(4B).* The conversions, 1 liter = 1000 cm^3, and 1 ml = 1 cm^3 are used on p. 112.
- *(5A).* Metric units of weight, length, and volume are used on p. 46.
- *(5B).* Metric units of length are used on p. 10.
- *(6B).* On p. 40, students work problems that reinforce 1 liter = 1000 cm^3.
- *(SL1).* On p. 180, units of density, $g/(cm^3)$ are used. Customary units are rarely included.
- *(SL2).* In the Workbook, p. 53, students make conversion graphs for the linear relationships between metric and customary units, such as centimeters to inches and miles to kilometers, etc.
- *(SL2).* On p. 124, students make linear graphs giving conversions between liters and gallons.
- *(SL2).* On p. 183, students make linear graphs giving conversions between acres and hectares.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces I.* An enlarged version of a 1 cm unit is divided into 10 parts on p. 60.
- *(6) Covering and Surrounding.* An area is given in m^2 units.
- *(7) Data About Us.* On p. 15, conversions between the systems are given, e.g. 1 $inch = 2.54\ cm$. Students select a familar object whose measurements are known in the customary system and convert it to the metric system to get a "benchmark" idea of its size in metric units. Grams and kilograms are given as units of mass. Ounces and pounds are given as units of weight. In the Summarize section on p. 22c, the teacher attaches the proper units to length, area, weight or mass, temperature, and time.

- *(8) Looking for Pythagoras.* Students use a centimeter ruler to estimate the length of a side of a square.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Made to Measure.* Students use m, cm, dm on p. 48 and liter and milliliter on p. 81 and on p. 186 of *Number Tools, Vol. 1.*
- *(6/7) Reallotment.* Students use customary and metric units for length, area, and volume.
- *(7/8) Cereal Numbers.* Students use quart, liter, decimeter and bushel volume units.
- *(7/8) Powers of Ten.* Students use the metric system to make a viewer for studying magnification problems.

Discussion: This standard is fully met.

3.5.2 Measurement Standard Question 2.

Does the curriculum enable all students to understand relationships among units and convert from one unit to another within the same system?

Singapore: Score: 3
Evidence:

- *(5A).* On p. 19 and p. 46, students convert within the same system.
- *(SL1).* Problems converting within the same system are observed throughout. Typical examples are found on pp. 178–179.
- *(SL2).* In the Workbook, p. 53, students make conversion graphs for the linear relationships between metric and customary units, such as centimeters to inches and miles to kilometers.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 1
Evidence:

- *(7) Filling and Wrapping.* In IV7, the cm^3 and millileter relationship is given.
- *(8) Clever Counting.* On p. 12a, the conversion of time units is seen. No conversion of units of length within the same system is found.

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 154, 3 $ft.$ = 1 $yd.$ is found in an Additional Practice problem.

Discussion: This standard is not adequately met. Students convert between systems, but hardly at all within the same system.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Made to Measure.* Students convert meters to centimeters.
- *(6/7) Reallotment.* Students convert feet to yards.
- *(7/8) Cereal Numbers.* Students make decimeters to meters conversions.
- *(7/8) Powers of Ten.* Students convert lengths within the metric system.

Discussion: This standard is fully met.

3.5.3 Measurement Standard Question 3.

Does the curriculum enable all students to understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume?

Singapore: Score: 3
Evidence:

- *(5A).* On p. 84, students measure angles with a compass.
- *(6A).* On p. 93, students estimate the area of a circle on grid paper.
- *(6A).* In Ch. 3, volumes are estimated and computed using centimeter cubes.
- *(SL1).* On p. 156, area is calculated in square centimeters, square millimeters, square meters, hectare, and square kilometers.
- *(SL1).* On p. 176, volume is calculated in cubic centimeters, and millimeters.
- *(SL1).* On p. 180, density is calculated in kilograms per cubic meter.
- *(SL1).* On p. 183, surface area is calculated in square centimeters.
- *(SL1).* On p. 265, angles are calculated in degrees.
- *(SL2).* On p. 188, in a class activity, students make a cardboard tetrahedron and measure its height.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* Students measure angles in degrees with an angle ruler.
- *(6) Covering and Surrounding.* Throughout this unit, students use centimeter-square grid paper to measure or approximate the area of figures. These figures range from simple qualrilaterals, for which they find the exact areas, to complex two-dimensional shapes, such as hands and feet.
- *(7) Data Around Us.* In IV2, students select a unit of appropriate size to describe data. These units are for length, area, surface, and volume in the context of the Exxon Valdez oil spill. (They must select units appropriate for very large numbers.)
- *(8) Looking for Pythagoras.* In IV2, p. 19, students find areas of two-dimensional shapes that are drawn on dot paper. The unit of length is taken to be the distance between two dots.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Figuring All the Angles.* On p. 54, students use a compass card for measuring angles.
- *(6/7) Made to Measure.* Students use a meter stick.
- *(6/7) Reallotment.* Students measure perimeter, surface area and volume using centimeter cubes. Students select a strategy for the measurement.
- *(7/8) Looking at Angle.* Students build an angle measuring device to measure the angle between a shadow on the ground and the tip of the object.

Discussion: This standard is fully met.

3.5.4 Measurement Standard Question 4.

Does the curriculum enable all students to use common benchmarks to select appropriate methods for estimating measurements?

Singapore: Score: 0
Evidence: none

Discussion: This standard is not met. No evidence was found where students use common-sense estimates or benchmarks to measure objects. The measurements that students do are with the standard tools, such as meter sticks.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Shapes and Designs.* In IV3, students estimate angles using common angular turns, such as 45^o and 90^o, as benchmarks.
- *(7) Data Around Us.* In IV2, students invent their own benchmarks to rewrite the Exxon Valdez article in units more understandable to their classmates.
- *(8) Looking for Pythagoras.* In IV5, students locate irrational numbers close to a benchmark fraction.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Made to Measure.* On p. 78 students choose measurements appropriate for large distances.
- *(6/7) Reallotment.* Students estimate the size of Cuba from the scale on the map or by comparison with the size of Florida.
- *(7/8) Ways to Go.* Students use the scale on a road map as a benchmark to estimate "as the crow flies" distances on the same map.

Discussion: This standard is fully met.

3.5.5 Measurement Standard Question 5.

Does the curriculum enable all students to select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision?

Singapore: Score: 3
Evidence:

- *(5A).* On p. 46, students measure angles using a compass.
- *(6B).* In Ch. 3, students place solids in water to estimate their volumes by measuring the height (length) of the displacement and by computing the volume of the displacement. They then prove (to a level of precision) that 1 liter is indeed 1000 cubic centimeters by pouring water from a 1 liter jar to a box with dimensions $10cm \times 10cm \times 10cm$.

- *(6B).* In Ch. 3, students use centimeter-cubes to build large volumes.
- *(SL1).* Students use protractors, compasses, meter sticks, and set-squares.
- *(SL1).* On p. 276, students do perpendicular bisection.
- *(SL2).* In Ch. 9, p. 116, students find areas of complicated shaded regions accurate to 1 decimal place by taking $\pi = 3.142$.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Bits and Pieces I.* Students use paper strips for fractions to measure the thermometer for their fund-raising event. On p. 21, they use fraction strips to explore the equivalence of fractions. In IV3, students have to determine how to cut a pan of brownies into equal pieces. The teacher is told that it is hard for students to make reasonable representations for doing this – that some students will select rulers and others will struggle to find a way to do it.
- *(6) Data About Us.* On p. 52b, it is suggested that students use a measuring tape to measure heights for a statistics project.
- *(6) Shapes and Designs.* Students use an angle ruler to measure angles approximately.
- *(6) Covering and Surrounding.* On p. 67, rules of thumb are used to think about the sizes of measurements.
- *(7) Filling and Wrapping.* In IV7, students study the relationship between a cubic centimeter and a milliliter. Then they use the water displacement method to find the volume of irregular objects to a certain precision.
- *(8) Looking for Pythagoras.* In IV2, p. 19, students use a centimeter ruler to measure the side length of a square (area of two square units) that was drawn on centimeter grid paper. Then a calculator is used to find the square root of two and precision of the measurement discussed.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* Students estimate how many snow geese there are in a picture. This estimate is contrasted with the precise number of people that will fit in a square kilometer given that 10 people fit in a square meter.

- *(6/7) Rates and Ratios.* Students measure lengths to find scale factors.
- *(7/8) Powers of Ten.* Students use length measurements to construct a viewer.
- *(7/8) Looking at Angle.* Students make a tool for measuring angles of shadows with the ground.
- *(7/8) Building Formulas.* Students measure stairs to build a scale model.

Discussion: This standard is fully met.

3.5.6 Measurement Standard Question 6.

Does the curriculum enable all students to develop and use formulas to determine the circumference of circles and areas of triangles, parallelograms, trapezoids, circles, and develop strategies to find areas of more complex shapes?

Singapore: Score: 3
Evidence:

- *(6A).* The Teacher's Guide on p. 81 and p. 94 recommends that students develop the formula for the area of a circle by cutting out sections and organizing them into a "rectangle." The CDIS computer software is referenced for circles and composite figures.
- *(6A).* In Ch. 5, pp. 97-99, students make composite figures and find their areas.
- *(6A).* In Ch. 5, students discover the circumference of a circle from its radius.
- *(SL1).* In Ch. 9, the Teacher's Guide says to spend more time on parallograms and trapezoids since the other shapes are understood from elementary school (up through *6B*). Here they investigate Pick's Formula using a geoboard.
- *(SL2).* In Ch. 9, p. 159, students find areas of rings with internal and external diameters.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Covering and Surrounding.* In IV5, students develop formulas for finding areas of triangles, rectangles, parallelograms, and trapezoids. The area of a circle is found approximately.
- *(7) Variables and Patterns.* On p. 49, the circumference of a circle is written symbolically as $C = \pi d$.

- *(8) Looking for Pythagoras.* In IV2, students find areas of figures on dot paper. They also use the strategy of dividing the figure into commonly known figures or by finding a length that is the side of a square of known area.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* On p. 52, the general area of a triangle is given as 1/2 the base times the height and the area of a rectangle as the base times the height. Students develop strategies for finding areas of more complex shapes. Area and circumference of a circle are also given.

- *(7/8) Cereal Numbers.* Areas of rectangles are determined.

Discussion: This standard is fully met.

3.5.7 Measurement Standard Question 7.

Does the curriculum enable all students to develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders?

Singapore: Score: 3
Evidence:

- *(up through 6B).* Students only study volumes of cubes. No surface area is formally studied. The idea of nets for volumes is explored.

- *(SL1).* Ch. 10, p. 177 gives the formal definition of a right prism and p. 183 derives its surface area and volume. On p. 186, the same thing is done for a cylinder by using a stack of coins analogy. Hollow cylinders are also considered. Volumes of composite figures are introduced on p. 192. Students compute surface area and volume for many problems.

- *(SL2).* Ch. 9, p. 170 discusses how to find the volume of a pyramid and composite solids and p. 174 discusses the same for a right circular cone. On p. 181, a displacement strategy for finding the volume of a sphere is outlined. The sphere is put in a cylinder of water and its volume determined by the amount of water displaced. On p. 175, the surface area of a cone is computed with paper cutouts.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Filling and Wrapping.* In IV2, the surface area of a rectangular prism is investigated. In IV3, the volumes of boxes and prisms are studied. In IV4, the surface area and volumes of cylinders are studied. In IV5, the volume of a cone is studied. On p. 55, problem 12b, students must describe methods for finding the surface area and the volume of a pyramid.

Discussion: This standard is fully met.

Mathematics in Context: Score: 2
Evidence:

- *(6/7) Made to Measure.* On p. 100, students investigate the net of a cylinder. In problem 8, p. 23, students think through how to find the volume of a soda can. The teacher is told that they are not expected to know the area of a circle yet (to find the area of a base). On p. 24, they estimate volumes of right prisms.

- *(7/8) Cereal Numbers.* Volumes of rectangular solids and their surface areas are found.

- *(7/8) Powers of Ten.* Students find the volume of a room and how many rectangular solids will fit in it.

Discussion: This standard is adequately met. By the end of the *(8/9)* books, the curriculum does not include the formula for the volume of a cylinder in its exposition or in problems that students do.

3.5.8 Measurement Standard Question 8.

Does the curriculum enable all students to solve problems involving scale factors, using ratio and proportion?

Singapore: Score: 3
Evidence:

- *(6B).* Ch. 5 has a section on complicated problems involving ratio and proportion. Many extension problems are included for abler students.

- *(SL1).* In Ch. 16, p. 314, scales of maps and linear and area scales are introduced. Students solve many problems using these concepts. They go between two different scales when solving the same problem. This chapter uses algebra as well as ratio, proportion and scaling.

- *(SL2).* In Ch. 11, pp. 222–228, scale factors are used to find areas and volumes of similar figures and solids.

- *(SL2).* The Teacher's Resource Manual, Ch. 11, recommends that students bring in a scale model of a toy car that has the scale written on it to be used in class.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Covering and Surrounding.* Students use scale to plan the layout of a park.
- *(7) Stretching and Shrinking.* On p. 61, mirrors are used to find heights using similar triangles.
- *(7) Filling and Wrapping.* In IV6, a scale factor is calculated to make a similar rectangular prism with specified volume.
- *(7) Comparing and Scaling.* On p. 41, problem 4.2b, students use a ratio table to solve problems involving proportion. On p. 95 in the Question Bank, the scale of 1 *in.* = 1 *million mi.* is used for a solar system calculation.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Some of Parts, (5/6) Per Sense, (6/7) Measure for Measure.* The ratio table is used for fractions, decimals, and percents.
- *(6/7) Fraction Times.* Students use ratio tables to find equivalent fractions.
- *(6/7) More or Less.* Reduction of photos is determined using percent and decimals. Students use a ratio table for the calculations.
- *(6/7) Rates and Ratios.* Section E is about scale factors (see p. 74).
- *(7/8) Powers of Ten.* Students use scale factors to construct a scale model of the solar system and investigate changing scales to fit the model into the classroom.
- *(7/8) Ways to Go.* A scale model of the driving distances between three cities is made.
- *(8/9) Triangles and Patchwork.* Students calculate a multiple (or scale) for the relationship between sides of similar triangles.

Discussion: This standard is fully met.

3.5.9 Measurement Standard Question 9.

Does the curriculum enable all students to solve simple problems involving rates and derived measurements for such attributes as velocity and density?

Singapore: Score: 3
Evidence:

- *(5B).* In Ch. 4, students study rates. Units such as liters/min are used. They use a double number line with minutes on one side and liters on another.
- *(6A).* Ch. 4 and Ch. 5 contain a large number of word problems and the concept of average speed is included in these problems.
- *(6A).* In the Teacher's Guide, p. 63, the CDIS computer software is recommended for rate and speed problems.
- *(SL1).* On p. 180, density is formally defined. Students do a large number of word problems, see problem 111, p. 59 of the Workbook.
- *(SL1).* On p. 215, problems on average speed are worked out for the student (and teacher). Students do a large number of word problems on p. 76 of the Workbook.
- *(SL2).* On p. 183, mass is formally defined as volume times density.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Variables and Patterns.* In IV4, rate-time-distance relationships are studied using tables, graphs, and equations. Population density (number of people per area) is also studied.
- *(7) Comparing and Scaling.* On p. 42, problem 4.3, average speed is calculated for a problem involving speeding up and slowing down. It is made clear that a single linear equation cannot describe the situation, that it is piecewise linear.

Discussion: This standard is fully met. (We note that density in the sense of mass per unit volume is not included – only population density is studied.)

Mathematics in Context: Score: 2
Evidence:

- *(6/7) Rates and Ratios.* Students calculate mi/hr and mi/gal ratios.
- *(7/8) Ways to Go.* On p. 12, students find the speeds in mi/hr that a route map uses for its driving times.

Discussion: This standard is adequately met. No treatment of density calculations were found in this curriculum.

3.5.10 Measurement Standard Summary

Question	Singapore	CMP	Math-in-Context
Measurement 1.	3	3	3
Measurement 2.	3	1	3
Measurement 3.	3	3	3
Measurement 4.	0	3	3
Measurement 5.	3	3	3
Measurement 6.	3	3	3
Measurement 7.	3	3	2
Measurement 8.	3	3	3
Measurement 9.	3	3	2

Table 4: Summary of NCTM Measurement Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets eight of the nine measurement standards, and does not meet one standard. The lower score was given because students do not use common-sense benchmarks to estimate or measure objects.

CMP:

The scores in the table above show that the curriculum fully meets eight of the nine measurement standards, and does not adequately meet one standard. The lower score was given because students work minimally with measurement conversions within the same measurement system. The evidence points to another issue worth mentioning that the scoring did not reflect.

- In the problems students do, density always refers to population density (which has units of mass per unit area). No examples from physics were included where density is interpreted as mass per unit volume.

MIC:

The scores in the table above show that the curriculum fully meets seven of the nine measurement standards, and adequately meets two standards. The lower scores were given because students do not work with volumes of cylinders or do calculations involving density.

3.6 Data and Probability Standard

3.6.1 Data and Probability Standard Question 1.

Does the curriculum enable all students to formulate questions, design studies and collect data about a characteristic shared by two populations or different characteristics within one population?

Singapore: Score: 3
Evidence:

- *(SL2).* On p. 250, students design a traffic survey about different types of vehicles that pass by the school. They collect data and present the results.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* On p. 42 students study two characteristics (arm span and height) of a population. They use scatter plots to see how these characteristics are related. The Unit Project is to design a study and collect data about the question, "Is Anyone Typical?"
- *(8) Samples and Populations.* In IV2, students design and conduct surveys. In IV3, p. 40, they learn how to take random samples and how sample size affects accuracy of the results.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* In the assessment project, p. 124, students select a topic and collect data from a list of topics and characteristics involving two populations. They are told what to calculate but must write a summary to describe the patterns they see.
- *(8/9) Insights Into Data.* Students analyze the growth of bean sprouts under different conditions.
- *(8/9) Great Expectations.* Students examine populations of men and women that wear glasses. This requires taking samples and making samples larger. They look at multiple samples from the same and from different populations. They formulate questions.

Discussion: This standard is fully met.

3.6.2 Data and Probability Standard Question 2.

Does the curriculum enable all students to select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots?

Singapore: Score: 2
Evidence:

- *(4A).* Students read and interpret bar graphs, histograms, and tables. (No box or scatter plots are used.)
- *(5A).* Students make line graphs from tabular data.
- *(6B).* Students use pie charts that present data as numbers, fractions, and percents.
- *(SL2).* Ch. 12 is the first introduction to statistical ideas. Uses of tally sheets, pictograms, bar charts, tables, pie charts, piecewise linear graphs and frequency charts are found on pp. 242–243. Box plots are not used. Scatter plots that show a single approximating line are not used. Problems are created such that it makes sense to connect the scattered points with a piecewise linear graph and interpolate between data points linearly.

Discussion: This standard is adequately met. Box plots and scatter plots with an approximating line are not used. However, the curriculum makes rich use of other graphical representations.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* On p. 8, histograms and frequency plots are used. On p. 42, scatter plots are used. In the ACE problems on p. 49, students select and create these representations.
- *(8) Samples and Populations.* On p. 7 box and whisker plots are used, and on p. 13 scatter plots are used to present results.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* Histograms, box plots, and scatterplots are used. The student must select the most appropriate representation to present data.
- *(6/7) Tracking Graphs.* Line graphs are used to represent continuous data as a function of time.
- *(8/9) Insights Into Data.* Students analyze various representations of the same data and make decisions about when to use a given representation.

Discussion: This standard is fully met.

3.6.3 Data and Probability Standard Question 3.

Does the curriculum enable all students to find, use, and interpret measures of center and spread, including mean and interquartile range?

Singapore: Score: 2
Evidence:

- *(4B).* In Ch. 4, mean is used. The other measures are not used.
- *(6A).* In Ch. 4, students do word problems involving average speed.
- *(SL2).* On p. 263, the formal definition of mean is given, and on p. 265 another formal definition involving frequency of occurrence is given.
- *(SL2).* On pp. 267–268, median and mode are defined. Students do many problems involving median, mode and mean.

Discussion: This standard is adequately met. Interquartile range is not mentioned up through *(SL2)*.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* The median is introduced on p. 12 and the mode on p. 9. Throughout IV5, students get a comprehensive view of mean by comparing different looking graphs that all represent samples with the *same* mean. The range of a data set is also discussed.
- *(8) Samples and Populations.* On p. 9, the interquartile range is discussed in relation to box and whisker plots.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* Box graphs include the center, spread, mean and interquartile range.
- *(8/9) Insights Into Data.* On p. 14, students find the mean visually from a scatter plot. On p. 72, students find the value in histograms about which the data cluster. In Section D, students work with the concepts of mean, median, mode, and interquartile range.

Discussion: The standard is fully met.

3.6.4 Data and Probability Standard Question 4.

Does the curriculum enable all students to discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots?

Singapore: Score: 2
Evidence:

- *(SL2).* On p. 272, pie charts are used.
- *(SL2).* On p. 273, frequent usage of histograms is seen. For example, a histogram of the number of families that have different numbers of children is given.

Discussion: This standard is adequately met. No evidence is found of scatter plots (except for piecewise linear graphs), box plots, or stem and leaf plots.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* Histograms are on p. 8, stem-and-leaf plots on p. 32, and scatter plots on p. 42. In IV1, students reflect on how tables and line plots are alike and how they are different. In prob 8a, p. 39, students have to describe which type of representation is best to use with the data.
- *(8) Samples and Populations.* On p. 23, in the Reflection, students describe in what situations various representations are useful.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* Histograms, stem-and-leaf plots, box plots, and scatterplots are used, and questions that ask where a particular datum was in each representation are included.
- *(8/9) Insights Into Data.* Students choose appropriate representations and tell what would be missing if another representation were chosen.

Discussion: This standard is fully met.

3.6.5 Data and Probability Standard Question 5.

Does the curriculum enable all students to use observations about differences between two or more samples to make conjectures about the populations from which samples were taken?

Singapore: Score: 0

Evidence: none
Discussion: This standard is not met.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) Comparing and Scaling.* In IV5, students use the capture-tag-recapture technique to draw samples from a population to estimate the size of the population. They do this by tagging 100 beans from a large container of beans. Applications to the size of a deer population are discussed.

- *(8) Samples and Populations.* In IV2 and IV4, students use samples to make predictions about a population and study, on p. 41, how sample size affects results.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* On p. 18, two samples from the same population are shown. One is representative and the other is chosen not be be representative.

- *(8/9) Insights Into Data.* Biases in samples are discussed. On p. 32, characteristics of several populations are given and the students must infer from which population they were drawn. In Section C, students draw conclusions about populations from samples. Sample sizes are increased to illustrate that the features deviate less.

Discussion: This standard is fully met.

3.6.6 Data and Probability Standard Question 6.

Does the curriculum enable all students to make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit?

Singapore: Score: 3
Evidence:

- *(SL2)*: On p. 258, piecewise linear graphs of data points are studied. (This assumes the data can be approximated in an interpolatory fashion rather than a single approximating line.) However, the spirit of this standard is here since many questions are asked about the relationships shown by the graph. For instance, a person's temperature between times given by the data is estimated. Predictions are made about these relationships between the data points.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* In IV4, the relationship of arm span to height is studied from scatter plots. A line $y = x$ is drawn through the data and students make statements about data that lie above or below the line.

- *(8) Thinking with Mathematical Models.* On p. 7, lines of fit to the data are discussed. Students draw a line that looks as if it fits the data and then they find its equation.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Dealing With Data.* On p. 120, in the Assessment section, students observe or conjecture about price increases based on a scatter plot.

- *(8/9) Insights Into Data.* Two characteristics are correlated based on scatter plots. The slope of a line tells the change in one given a unit change in the other. Students look at a scatter plot and tell what type of relationship, if any, exists. They also look at lines of fit.

Discussion: This standard is fully met.

3.6.7 Data and Probability Standard Question 7.

Does the curriculum enable all students to use conjectures to formulate new questions and plan new studies to answer them?

Singapore: Score: 0
Evidence:

- *(SL2).* On p. 262 in a class activity, students take samples but do not go further to design new questions based on initial results.

- *(SL2).* On p. 254, students collect data, show results, but do not design further studies.

Discussion: This standard is not met.

Connected Mathematics Program: Score: 0
Evidence:

- *(6) Data About Us.* Students plan a unit long study. On p. 25, the question "Can the question be answered by the graph? If so, what is the answer. If not, what additional information would be needed?" is a start toward designing new studies, but it is not evident from the curriculum that they actually do the new design.

- *(6) How Likely Is It?* On p. 49, problem 6.1, students create and do a simulation to answer probability questions.

- *(8) Samples and Populations.* On p. 53, students explain what is wrong with the conjectures made by others. On p. 63, they design a study and carry it out as a Unit Project. They make conjectures to know what questions to include. It is not clear whether based on the results they obtain, that they plan a second study.

Discussion: This standard is not met. The curricular materials do not explicitly ask students to design further studies based on results found from an initial investigation.

Mathematics in Context: Score: 0
Evidence:

- *(8/9) Insights Into Data.* In the Assessment section, students make up a test, provide conjectures, questions, and answers.

- *(8/9) Great Expectations.* On p. 38, the question "Do you believe the claim?" forces students to draw conclusions. On p. 58, students design a study to evaluate whether or not garlic actually lowers high blood pressure. They formulate the survey questions, plan the study, draw conclusions from their results, but do not ask further questions and plan new studies.

Discussion: This standard is not met.

3.6.8 Data and Probability Standard Question 8.

Does the curriculum enable all students to understand and use appropriate terminology to describe complementary and mutually exclusive events?

Singapore: Score: 0
Evidence: none
Discussion: This standard is not met. Probability is in *SL4*. We note that the Pan Pacific books also cover probability in Book 4, which corresponds to 10th grade.

Connected Mathematics Program: Score: 2
Evidence:

- *(6) How Likely Is It?* Students certainly see complementary events, such as Red or Not Red. The word complementary is not used.

- *(7) What Do You Expect?* Mutually exclusive events are in this unit, but these words are not used.

Discussion: This standard is adequately met. The desired vocabulary is not used, nor is it in the glossary of any of the probability units. Students do however work with mutually exclusive and complementary events.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Take a Chance.* The terminology "sure to happen" and "sure not to happen" demonstrate appropriate probability language to correspond to 100 percent and 0 percent probability, respectively.
- *(7/8) Ways to Go.* The term *probability* is formally defined in Section D. Students work with probability trees that model traffic flow. The branches show the different directions cars travel. The formal terms *complementary* and *mutually exclusive* are not used.
- *(8/9) Great Expectations.* In Section G, students study complementary events, independent events, and events that are not related.

Discussion: This standard is fully met.

3.6.9 Data and Probability Standard Question 9.

Does the curriculum enable all students to use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations?

Singapore: Score: 0
Evidence: none
Discussion: This standard is not met. Probability is in *SL4*. We note that the Pan Pacific books also cover probability in Book 4, which corresponds to 10th grade.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) How Likely Is It?* Students do simulations and use probability.
- *(7) What Do You Expect?* In IV5, students use an example from basketball to make conjectures about expected values. They predict the number of baskets made in a one-for-one free throw situation with 100 trips to the free throw line. This requires the principle of proportionality. The Unit Project requires students to design a carnival game, calculate the expected payout (make the game so the school wins money!), calculate the probability of winning, simulate it, and convince the carnival committee (in writing) to accept it.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Take a Chance.* Students experiment to test conjectures about coin tosses, and sums on dice. Students use proportionality in the context of 44 out of 100, 44:100.

- *(7/8) Ways to Go.* On pp. 92–98, students use probability trees to determine the expected number of cars taking certain routes. This requires ratios and proportional reasoning. The problems are not cast as simulations or experiments.
- *(8/9) Great Expectations.* Students start with a question about uncertainty, design an experiment and use probability applied to a sample to answer questions. Expected value questions are also investigated.

Discussion: This standard is fully met.

3.6.10 Data and Probability Standard Question 10.

Does the curriculum enable all students to compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models?

Singapore: Score: 0
Evidence: none
Discussion: This standard is not met. Probability is in *SL4*. We note that the Pan Pacific books also cover probability in Book 4, which corresponds to 10th grade.

Connected Mathematics Program: Score: 3
Evidence:

- *(7) What Do You Expect?* In IV4, the area model, lists, and probability trees are used to solve two-stage games, and maze problems with expected values that involve dependent events.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Take a Chance.* Students use organized lists and tree models to study fair games and probability.
- *(7/8) Ways to Go.* Students use probability tree models to study conditional probability.
- *(8/9) Great Expectations.* Students use probability trees, tables, lists and area models (page 104).

Discussion: This standard is fully met.

3.6.11 Data and Probability Standard Summary

Question	Singapore	CMP	Math-in-Context
Data Anal. and Prob. 1.	3	3	3
Data Anal. and Prob. 2.	2	3	3
Data Anal. and Prob. 3.	2	3	3
Data Anal. and Prob. 4.	2	3	3
Data Anal. and Prob. 5.	0	3	3
Data Anal. and Prob. 6.	3	3	3
Data Anal. and Prob. 7.	0	0	0
Data Anal. and Prob. 8.	0	2	3
Data Anal. and Prob. 9.	0	3	3
Data Anal. and Prob. 10.	0	3	3

Table 5: Summary of NCTM Data Anal. and Prob. Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets two of the standards, adequately meets three of them, and does not meet five of them. The lowest scores were given because there is a lack of any probability in the curriculum for the middle grades. (Probability begins in *(SL4)*.) The reasons for the other low scores are the lack of conjecturing about relationships between two characteristics of a sample, the lack of making conjectures to formulate new questions and plan new studies, and the lack of work with box plots, interquartile range, or stem and leaf plots. The evidence points to other issues worth mentioning that the scoring did not reflect:

- The only statistics in the *(6A)* and *(6B)* books involves reading data from pie charts.
- There is no statistics in *(SL1)*.
- The curriculum does not make explicit that students should critique the arguments of others.

CMP:

The scores in the table above show that the curriculum fully meets eight of the ten standards, adequately meets one standard, and does not meet one standard. The lower scores were given because students do not design further studies after an initial study has been completed and do not use the vocabulary of complementary and mutually exclusive events. The evidence points to another issue worth mentioning that the scoring did not reflect:

- With the exception of the capture-tag-recapture technique in the 7th grade *Comparing and Scaling* unit, there is no statistics in 7th grade CMP.

MIC:

The scores in the table above show that the curriculum fully meets nine of the ten standards, and does not meet one standard. The lower score was given because students do not ask new questions and design new studies after an initial study has been completed.

3.7 Problem Solving Standard

3.7.1 Problem Solving Standard Question 1.

Does the curriculum enable all students to build new math knowledge through problem solving?

Singapore: Score: 3

- *(4A to 6B).* Students practice *many* problems on every small building block of a concept.
- *(SL1).* This book contains an enormous number of problems. It is not uncommon for each workbook problem set to contain over 100 problems. This is in direct preparation for the Level O exams. The book explicitly discusses problem solving strategies. It is for this reason we were told that many teachers in Singapore are now using this book.
- *(SL1).* On p. 167, exercise 9D requires problem solving strategies to find non-standard areas that were not covered in the text.
- *(SL1).* On p. 228, problem solving strategies are made explicit and six challenge problems are given on p. 232.
- *(SL1).* On pp. 44, 47, 71, 149, 169, 222, 225, 228, 238, 241, 256 problem solving strategies are given. Students are expected to apply these strategies to build knowledge. They are not expected to experiment and test out their own strategies.
- *(SL2).* Sections on problem solving strategies are included in Ch. 1, 2, 3, 4, 5, and 8. Students are expected to apply these strategies to build knowledge. They are not expected to experiment and test out their own strategies.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3

- *(6) Prime Time.* On p. 48, students build the Prime Factorization Theorem by finding the longest string of factors of a number.
- *(6) Bits and Pieces I.* Students build knowledge of the equivalence between fractions, decimals, and percents by working with a 100 grid representation. On p. 23, they build knowledge of equivalent fractions by constructing fraction strips and matching them up on a number line.
- *(6) Shapes and Designs.* On p. 16, students build knowledge of the triangle inequality by trying to make a triangle from any three given lengths (which is not always possible).

- *(6) Covering and Surrounding.* In problem 19a, p. 65, students build their own formula for the area of a trapezoid by construction and reasoning.

- *(6) Bits and Pieces II.* By working in groups, students come up with the algorithms for addition and subtraction of fractions on p. 48. They discover the standard algorithm for multiplication of fractions on p. 68.

- *(7) Filling and Wrapping.* In IV4, students build a cylinder from a two-dimensional net made on centimeter grid paper. They build up the formula for its surface area and volume.

- *(8) Thinking With Mathematical Models.* Students build new knowledge about the nature of differences in linear and nonlinear relationships by using paper models of bridges that demonstrate each case.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3

- *(5/6) Sum of Parts.* Students work with fraction strips to see that 1/3 is bigger than 1/4 and 2/4 is the same as $2 \times 1/4$.

- *(5/6) Per Sense.* Students use percent strips to solve number stories.

- *(6/7) Fraction Times.* Knowledge of common denominator is developed through story problems and building solutions using fraction strips.

- *(6/7) More or Less.* Students work with interest problems, photo reduction problems, tax and discount problems to learn about ratios and percents.

- *(6/7) Rates and Ratios.* Scale factors are calculated for enlarging and computing sizes of small algae and big whales.

- *(7/8) Packages and Polygons.* Students build three dimensional insight into geometry by making nets, bar models, and by visualizing views.

- *(7/8) Triangles and Beyond.* Inquiry-based activities are used to see the effect of translations and rotations.

- *(8/9) Reflections on Number.* (Not in Plan B.) Students play games to build ideas of primes and perfect squares.

Discussion: This standard is fully met.

3.7.2 Problem Solving Standard Question 2.

Does the curriculum enable all students to solve problems that arise in math and in other contexts?

Singapore: Score: 2
Evidence:

- *(6A).* In Ch. 3, students do word problems that are related to finance, including selling price and cost price.
- *(6B).* In Ch. 3, the displacement of a solid in water to study volume is related to science.
- *(6A,6B).* The word problems have little relation to a real context. The exception is finance.
- *(SL1).* Ch. 12 is entirely on household finance.
- *(SL1).* Ch. 16 uses the context of maps and scale models to study proportion and scaling.
- *(SL2).* On p. 324, trigonometry is applied to finding the angle of elevation and depression.
- *(SL2).* In Ch. 12, data are collected from social situations (say traffic flow) and conclusions are drawn using statistics.

Discussion: This standard is adequately met. Students frequently solve problems that arise in math. With the exception of the chapter on finance, the enormous amount of word problems are mostly not from real contexts.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 38, students solve problems involving cycles of locusts using common multiples.
- *(6) Data About Us.* On p. 84, social contexts of political campaigns are used to discuss interpretations of average.
- *(6) Ruins of Montarek.* Architectural plans and side views are used for spatial visualization.
- *(6) Bits and Pieces II.* Sales taxes, tips, discounts, and ordering from a catalog are used as contexts to study percents.
- *(7) Stretching and Shrinking.* On p. 71-72, astronomy problems of planetary distances are used to motivate study with similar triangles.

- *(7) Comparing and Scaling.* Advertising and supermarket contexts are used to study rates and ratios.
- *(7) Accentuate the Negative.* Temperatures below zero are used to relate to negative numbers.
- *(8) Growing, Growing, Growing.* The half life of Iodine-124 is simulated to study exponential decay.
- *(8) Clever Counting.* In IV2 counting all the possibilities is motivated by finding how many lock combinations a thief would have to try if there are n numbers on the lock and r numbers make one combination.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Students solve cooking problems using fractions.
- *(6/7) Made to Measure.* Students measure sports races, and read tank gauges.
- *(6/7) Reallotment.* The contexts of sewing and interpreting area and distance on maps are used to study scale factors.
- *(6/7) Fraction Times.* Students use surveys and pie charts to make comparisons.
- *(6/7) More or Less.* Students solve interest rate, photo reduction, tax, and discount problems.
- *(6/7) Dealing With Data.* Students answer questions arising from history and animal population data.
- *(7/8) Cereal Numbers.* Students design boxes and find the amount of material needed to build them.
- *(7/8) Powers of Ten.* Students make a view-finder to solve scaling problems such as finding sizes of something on the ground from the air.
- *(7/8) Ways to Go.* Students work with graph-theoretic problems like paths through a network.
- *(7/8) Looking at Angle.* Students find the area that represents blind spots for boats and shadows.
- *(7/8) Decision Making.* Students construct feasible regions formed by constraint inequalities to make decisions facing cities.

- *(8/9) Triangles and Patchwork.* Students find the height of a tree using similar triangles and shadows.

- *(8/9) Getting the Most Out of It.* In Section G, decision problems are cast as linear programming problems from a variety of contexts.

Discussion: This standard is fully met.

3.7.3 Problem Solving Standard Question 3.

Does the curriculum enable all students to apply and adapt a variety of appropriate strategies to solve problems?

Singapore: Score: 2
Evidence:

- *(4A to 6B).* Appropriate strategies are given for the students to apply. Students do not need to do much, if any, adaptation of these techniques.

- *(6B).* Ch. 5 contains extension problems which do require some adaptation of the given strategies. This section is for the abler students, and the problems read a little differently from the given template problems in the text.

- *(SL1).* Students apply given strategies to solve an enormous number of problems. These problems have little to do with real contexts. The exception is household finance in Ch. 12 and maps and scale models in Ch. 16.

- *(SL2).* On p. 11, the book discusses four strategies to solve a given problem. These include making a diagram, making a before and after picture, using a ratio, and writing an equation.

- *(SL2).* On p. 20, the question, "What is the last digit of 9^{1997}?" requires the use of the strategy "look for a pattern."

- *(SL2).* On p. 137, students are given two different strategies that are slight adaptations of earlier ones. They apply these adapted strategies to solve problems where two vehicles move toward each other at different rates.

Discussion: This standard is adequately met. Only a small number of the many problems that students perform require any adaptation from the given strategy of the book. The book gives the adaptation, if needed, to the student. Students do apply a large number of strategies to solve problems. Based on the strength of the strategies they apply, a score of 2 rather than 1 is given.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 16c the teacher is told to give the students a chance to suggest a strategy to solve the problem.
- *(6) Data About Us.* On pp. 15–21, students adapt the concept of a bar graph to make a double-bar graph and a stem-and-leaf plot to make a double one to better present results from two populations.
- *(7) Filling and Wrapping.* In IV1, students investigate volumes of objects by filling them with unit cubes. In IV7, they use the strategy of seeing how much water the object displaces to find an estimate of its volume.
- *(8) Looking for Pythagoras.* In IV2, areas of complicated 2D shapes are found by splitting them into shapes of known areas and applying known strategies.
- *(8) Samples and Populations.* On p. 28, five sampling strategies are given. Students must analyze the sampling plans of four other groups and tell the pros and cons.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Students use a variety of strategies to study fractions informally.
- *(5/6) Per Sense.* Three strategies are used to introduce percent.
- *(6/7) Made to Measure.* Students understand decimals in different ways. For example, .35=.25 + .10 or .35 = 3/10 + 5/100.
- *(6/7) Comparing Quantities.* Combination charts, notebook notation, and symbolic equations are used to solve two linear equations in two variables.
- *(7/8) Triangles and Beyond.* Students cut apart angles and put them together in another way to see that they add to 180 degrees. Students use the "change the question strategy" to examine angles rather than distances between trees to see which trees are closer since closer sides (trees) will be opposite the bigger angle in a triangle.
- *(8/9) Reflections on Number.* (Not in Plan B.) Students use different strategies for multiplying and dividing whole numbers and explain why they work.
- *(8/9) Great Expectations.* Students use both area and tree models of probability. They design an experiment using a control group.

Discussion: This standard is fully met.

3.7.4 Problem Solving Standard Question 4.

Does the curriculum enable all students to monitor and reflect on the process of mathematical problem solving?

Singapore: Score: 0
Evidence: none

Discussion: This standard is not met. After not finding this standard in the books above, we looked in the Pan Pacific Book 1 and Book 2 (the other popular set of textbooks in Singapore corresponding to 7th and 8th grade). We found these books to have some evidence of this standard. For example, in Pan Pacific Book 1, p. 133 the question "Do you think ..." followed by the language "Investigate" was found. Also on p. 135 of this same book, the question "Can you use the same method to find primes > 200?" followed by "Investigate" was found.

Connected Mathematics Program: Score: 3
Evidence:

- *(6 to 8) All Units.* CMP makes it explicit that students should monitor and reflect on the process of mathematical problem solving. At the end of the investigation (usually 4 or 5) in each unit, students answer reflection questions. These questions require writing, talking to other students, and talking to the teacher.
- *(6) Bits and Pieces II.* In the IV4 Reflection, students write about how to add and subtract fractions with unlike and like denominators and how to multiply fractions. On p. 19, it states "One of the powerful things about math is that you can often find a way to solve one problem that will also work for solving similar problems." Five problems are then given and students have to find one strategy that works for all of them and compare with each other's strategy.
- *(7) Comparing and Scaling.* In IV2, students write about when it is good to use percent in rate comparisons.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Per Sense.* On p. 92 in the Assessment, students write an easy and a hard percent problem and explain why they are easy and hard.
- *(6/7) More or Less.* Students explain why two methods are both right in relation to two students' thinking about 20 percent less vs. 25 percent more.
- *(6/7) Dealing With Data.* Summary questions on pp. 20-21 ask students to reflect on math ideas.

- *(6/7) Operations.* On p. 114, students are asked, "Can you make a figure smaller by multiplying?"
- *(8/9) Reflections on Number.* (Not in Plan B.) Students reflect on the number structure of integers and whole numbers. For example, do two whole numbers divide to give a whole number?
- *(8/9) Insights Into Data.* Students reflect on the entire unit by making an exam for peers.
- *(8/9) Great Expectations.* In the Challenge Question on p. 118, students rewrite a problem so the same strategy won't work.

Discussion: This standard is fully met.

3.7.5 Problem Solving Standard Summary

Question	Singapore	CMP	Math-in-Context
Problem Solving 1.	3	3	3
Problem Solving 2.	2	3	3
Problem Solving 3.	2	3	3
Problem Solving 4.	0	3	3

Table 6: Summary of NCTM Problem Solving Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets one of the standards, meets two adequately, and does not meet the fourth. The lower scores were given because the curriculum does not frequently include contextual problems, does not provide problems where students must adapt the strategies given in the book, and does not explicitly ask students to reflect on the process of mathematical problem solving.

CMP:

The scores in the table above show that the curriculum fully meets all four of the standards.

MIC:

The scores in the table above show that the curriculum fully meets all four of the standards.

3.8 Reasoning and Proof Standard

3.8.1 Reasoning Standard Question 1.

Does the curriculum enable all students to recognize reasoning and proof as fundamental aspects of math?

Singapore: Score: 1
Evidence:

- *(6A).* In Ch. 2, p. 42, problem 29, changing ratio problems are being solved by pictures using reasoning and not algebra.
- *(6A).* In the Teacher's Guide, p. 77, the word *verify* is used, but little proof is done up through 6th grade. The notion of being precise is conveyed.
- *(SL1).* In the Workbook, pp. 6–7, students reason about the general term in a sequence. In problem 21 they "verify."
- *(SL1).* In "It's a Fact," Ch. 2, p. 26, students are asked to show all palindromic numbers with an even number of digits are divisible by 11. This gives the notion that things have to be proven.
- *(SL1).* Ch. 3 encourages pattern recognition in number sequences. Students generalize patterns in exercise 3B, problems 1-4.
- *(SL1).* In an investigation of π on p. 168, the book tells the students the conclusion right away.
- *(SL2).* On p. 304, the wording of the investigation of the Pythagorean Theorem gives away the answer, by asking if the relationship $c^2 = a^2 + b^2$ is observed.
- *(SL2).* In Ch. 10, p. 217, students develop a mathematical argument showing that two triangles are either similar or congruent. Students also come to the standard conditions for congruency of triangles by experimentation and formulation of these principles.
- *(SL2).* In the Workbook, pp. 79–84, students use deductive reasoning to prove triangles congruent or triangles similar.

Discussion: This standard is not adequately met. Students use inductive reasoning to generalize patterns and deductive reasoning to prove geometric properties. Our concern here is that the curriculum does not require students on a regular basis to explain their reasoning in the problems or in the assessments they do.

Connected Mathematics Program: Score: 3
Evidence:

- *All units.* Thoughout CMP students are asked to explain their reasoning. They answer why and why not and explain type questions in ACE problems and orally during the Investigations.

- *(6) Prime Time.* On p. 13, students are asked, "What is the best first move on a 49-board? Why?"

- *(6) How Likely Is It?* In problem 5.1, p. 43, students reason what a good strategy would be to win a probability game like Roller Derby.

- *(6) Covering and Surrounding.* On p. 33, the statement, "Find all possible pentominos and say how you know you have them all," illustrates the notion of proof. In IV4, students add tiles to pentominos and reason about how the area and perimeter changes with each addition.

- *(7) Bits and Pieces II.* On pp. 53c–53d, the curriculum promotes reasoning in the classroom by giving the teacher a dialogue used by students. The dialogue shows how students reason about the changing distribution of land based on clues they are given in one of the investigations.

- *(8) Looking for Pythagoras.* On p. 29, the definition of a theorem is given.

- *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person was based on clues given.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Per Sense.* Students solve percent problems by reasoning what 1 percent or 10 percent would be, and then use a percent bar rather than converting to a decimal and multiplying.

- *(6/7) More or Less.* Students rely on reasoning to multiply fractions and decimals since algorithms are not given yet.

- *(6/7) Expressions and Formulas.* In problem 16b, p. 14, students must show how they got their answers.

- *(7/8) Cereal Numbers.* Students reason without using a formal algorithm on problems such as, "If 80 percent is 2, then 20 percent is 1/2." This is possible since the numbers are rather simple.

- *(7/8) Packages and Polygons.* On p. 64 the question, "How can you be sure?" is posed. This is one of the first instances in this curriculum of a *convince me* or *prove it* request. *Explain your thinking* requests have been made, but the notion that proof is necessary has not been made explicit.

- *(7/8) Triangles and Beyond.* Students devise a way to reason that lines are really parallel. This illustrates a need to convince others or to prove the conjecture, but this language is not used.

- *(7/8) Decision Making.* On p. 34, an exchange between two students in the materials uses the "prove it" language.

- *(8/9) Triangles and Patchwork.* On p. 58 the "About the Mathematics" tells the teacher ways to prove angles congruent. On page p. 64, students are told "If corresponding sides have the same ratio then the triangles are similar and the corresponding angles are equal."

Discussion: This standard is fully met.

3.8.2 Reasoning Standard Question 2.

Does the curriculum enable all students to make and investigate math conjectures?

Singapore: Score: 1
Evidence:

- *(up through 6B).* Little or no conjecturing is done. Students practice problems as told.

- *(SL1).* On p. 228, "Making a supposition" is given as one of the problem solving strategies. Examples of doing this are given on p. 228. Students use this strategy to solve word problems.

- *(SL1).* In the Teacher's Guide, p. 95, problems 4, 5, and 6 (multiple choice section), students must say which statements are true. Then they have to investigate the statement.

- *(SL1).* In the Workbook, p. 111, students determine whether the statements are true or false. They must give reasons or provide counterexamples.

- *(SL1).* On p. 332, students examine logic statements such as "A parallelogram is a trapezoid" and determine whether it is true or false and then produce the supporting argument.

- *(SL2).* On pp. 15–16, students work problems using the problem solving strategies outlined on pp. 12–14. One strategy called "Making a Supposition" outlines the process of making an assumption and carrying through the related logic to arrive at a solution. The problem was such that the solution could then be verified as right or wrong.

- *(SL2).* In Ch. 10, pp. 191–197, groups of students build triangles with sides the same length and see if they are congruent. This leads to the the SSS property for congruent triangles. Then they are asked what happens if two sides and one angle are equal. They build triangles and compare. The conjecture is *given* to them in the book in leading questions such as "Are the triangles congruent?" "If yes, can we say that any two triangles that have three pairs of equal sides are congruent?"

Discussion: This standard is not adequately met. Students do evaluate conjectures. They don't frequently formulate their own conjectures. Isolated cases were found in the "Making a Supposition" problem-solving sections listed above.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* The word conjecture is defined on p. 28. Students make a conjecture and build a model to justify it on p. 29. For example, "The sum of two even numbers is an odd number."
- *(6) Shapes and Designs.* Students conjecture about the sum of interior angles of regular polygons on p. 44.
- *(7) Stretching and Shrinking.* In problems 2–5 on p. 23, students predict how changes in the rule affect changes in a figure. In problems 2b and 2c on p. 8, they predict what would happen as the anchor point moves up and down. They test this prediction and choose a new anchor point to see if their prediction is still true.
- *(8) Looking for Pythagoras.* In IV2, after experimentation, students make a conjecture about the relationship of the sides of a right triangle. This leads students to prove the Pythagorean Theorem geometrically.
- *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person is based on clues given.
- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are told to test several cases, and make a conjecture about the result.

Discussion: The standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Take a Chance.* Students do experiments with dice to test conjectures about likely outcomes.
- *(6/7) Reallotment.* Students are asked the question, "Do you think there is a rule for finding the area of a quadrilateral whose corners touch the sides of a rectangle?" The student must conjecture and then follow through.

- *(6/7) Dealing With Data.* Students collect data on heights of mothers and daughters to make statements about the data on p. 14. Further conjectures about heights are made.

- *(6/7) Tracking Graphs.* The materials ask if a particular graph is upside down based on what it is to supposed to convey.

- *(6/7) Operations.* On p. 60, students evaluate logic statements such as "A positive integer added to a positive integer gives a positive integer." In problem 4, p. 94, students are asked to write true and false statements about calculating with positive and negative numbers.

- *(6/7) Dealing with Data.* In problem 13, p. 92, students conjecture about whether they expect to detect a difference, and if so, what type of difference.

- *(7/8) Triangles and Beyond.* On p. 92, problem 3c, students figure out how to move the model to keep the shadow the same height to satisfy a movie director.

- *(8/9) Insights Into Data.* Students plan a simulation to test a conjecture.

- *(8/9) Great Expectations.* Students make conjectures about a population.

- *(8/9) Triangles and Patchwork.* On p. 64, students form the converse of a statement and investigate whether it is true. They first test it using sample triangles.

Discussion: The standard is fully met.

3.8.3 Reasoning Standard Question 3.

Does the curriculum enable all students to develop and evaluate math arguments and proofs?

Singapore: Score: 1
Evidence:

- *(up through 6B).* The curriculum does not explicitly give instructions for students to evaluate mathematical arguments and proofs.

- *(SL1).* In the Teacher's Guide, p. 49, teachers are encouraged to ask students to verify Pick's Formula in a geometry lesson.

- *(SL1).* In Ch. 4, p. 66, students follow the argument that the book develops that says the area of a triangle is 1/2 the area of its related rectangle.

- *(SL1).* On p. 102, the book develops and presents an argument why one can not divide by 0, but the student is not asked to explain this another way or to analyze the argument.

- *(SL1).* In Ch. 3, Sec. 3B, problems 1–4, students develop inductive arguments to find generalizations in patterns.

- *(SL2).* In Ch. 10 of the Workbook, students develop deductive proofs for geometry problems. On pp. 78-84, students must determine whether given triangles are congruent. If so, they must write down the statement of congruency and "state the case" of congruency and name the other three pairs of equal measurements. They prove triangles congruent in problems 23 and 24 and triangles similar in problems 35–43 and 57, 58, and 60. In problem 69, they explain why the triangle is isosceles.

- *(SL2).* In Ch. 10, p. 191, the idea of minimum requirements for congruence is discussed. Students discover the SSS, SAS, and AAS rules. These are then used to make arguments about similarity.

- *(SL2).* On p. 215, problem 3, students are asked to "prove."

- *(SL2).* On p. 220, problem 5, students prove two triangles are similar.

- *(SL2).* On p. 304, students are guided to discover the Pythagorean Theorem. Several proofs of this theorem using geometry are given on pp. 305–306.

Discussion: This standard is not adequately met. Students do make both deductive and inductive arguments. The book does present mathematical arguments. The students do not evaluate arguments made by others. (After examining the standard in detail, we interpret it to mean that students should evaluate the mathematical arguments of others.)

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 28, students use models to justify or prove their math conjectures.

- *(6) How Likely Is It?* Students are asked does the statement, "Nine out of ten dentists surveyed recommend sugarless gum for their patients that chew gum," mean that 90% of dentists think patients should chew sugarless gum.

- *(7) Comparing and Scaling.* In problem 1.2 on p. 7, students evaluate ratio and proportion statements for correctness.

- *(7) Stretching and Shrinking.* On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?" On p. 58, students develop an argument for whether two rectangles are similar or not similar.

- *(7) Filling and Wrapping.* In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.
- *(8) Clever Counting.* This entire unit is about using combinatorial arguments to make math conjectures about who the mystery person is based on clues given.
- *(8) Samples and Populations.* On p. 53, students figure out what is wrong with another student's logic.
- *(8) Say It With Symbols.* On p. 40, students develop a math argument for why three different strategies would work for finding the area of a trapezoid.
- *(8) Looking for Pythagoras.* In IV2, students figure out how some given puzzle pieces (that illustrate the Pythagorean Theorem) fit together. In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove this theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.
- *(8) Hubcaps and Kaleidoscopes.* On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles. In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combiniation of the two rotations equivalent to a single transformation?" Test several cases, and make a conjecture about the result. This is an example of inductive reasoning.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(6/7) Reallotment.* Students begin with a visual proof that two areas are equal by cutting and pasting and reallocation to see the proof. On page 52, students retrace another student's argument.
- *(6/7) Tracking Graphs.* In problem 16c on p. 36, students explain how a logic argument in the newspaper could have been correct.
- *(6/7) More or Less.* In problems 18a and 18b on p. 62, students are asked with which arguments they agree and to defend their position.
- *(6/7) Comparing Quantities.* In problem 16, p. 46, students explain Joe's reasoning.

- *(6/7) Operations.* The idea of preciseness is conveyed in problem 11a, p. 110. Students are shown cartoons of student dialogue and asked "Who is correct? Why?"

- *(6/7) Dealing with Data.* Students develop a mathematical argument to support their conclusions in problem 10, p. 28.

- *(8/9) Great Expectations.* Students evaluate the math arguments of others on p. 112.

- *(8/9) Patterns and Figures.* Students reason about the general pattern from specific examples. In Section G, they solve the choreography problem of rearrangement of people in the fewest moves.

- *(8/9) Going the Distance.* On p. 38 students do a puzzle proof of the Pythagorean Theorem.

Discussion: This standard is fully met.

3.8.4 Reasoning Standard Question 4.

Does the curriculum enable all students to select and use various types of reasoning and methods of proof?

Singapore: Score: 3
Evidence:

- *(up through 6B).* Students use the book's strategies with the aid of concrete representations. A lot of thinking goes into the word problems (especially those on proportions). The word problems reinforce the reasoning already made explicit in the text. Actual proofs are not done yet.

- *(SL1).* On p. 159, the problem solving in the book's margin requires spatial reasoning.

- *(SL1).* In exercise 17D, p. 331, students use logical reasoning to sketch a parallelogram that has all sides equal but not all angles equal.

- *(SL1).* In Ch. 16, students show triangles are similar by calculating scale factors between corresponding sides and show they are equal.

- *(SL1).* In Ch. 3, Sec. 3B, problems 1–4, students make inductive arguments. Also, the book gives strategies to help the students with inductive reasoning. These include tabulation, simplification, acting it out, drawing a diagram, and changing one's point of view. Students then practice these strategies and must *select* the proper one in exercises on pp. 50–52.

- *(SL2).* On p. 217, the notion that there is a minimum number of requirements that must be met for two triangles to be congruent is given. Students then conclude by SSS, SAS, AAS arguments about congruency. Students investigate the similarity of triangles as well.

- *(SL2).* In Ch. 13, p. 291, the notion of proof by construction is seen when students make constructions that enlarge a figure by a certain scale factor.

- *(SL2).* In Ch. 10 of the Workbook, students do deductive proofs in geometry. On pp. 78-84, students must determine whether given triangles are congruent. If so, they must write down the statement of congruency and "state the case" of congruency and name the other three pairs of equal measurements. They prove triangles congruent in problems 23 and 24 and triangles similar in problems 35–43 and 57, 58, and 60. In problem 69, they explain why the triangle is isosceles.

- *(SL2).* In Ch. 10, p. 191, the idea of minimum requirements for congruence is discussed. Students discover the SSS, SAS, and AAS rules. These are then used to make arguments about similarity.

- *(SL2).* On p. 215, problem 3, students are asked to "prove."

- *(SL2).* On p. 220, problem 5, students prove two triangles are similar.

- *(SL2).* On p. 304, students are guided to discover the Pythagorean Theorem. Several geometric proofs of this theorem are given on pp. 305–306.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 12, students are asked "How would you check to see if one number is a factor of another?" On p. 28, students use models to justify or prove their math conjectures.

- *(6) Shapes and Designs.* Discovery and reasoning are coupled to show that not every three side lengths necessarily make a triangle.

- *(7) Stretching and Shrinking.* In problem 4.2, p. 43, students must select a way to scale a picture to a given size or say why it can't be done in a such a way that the new picture is similar to the original. On p. 54, students determine whether triangles are similar and if so give a scale factor based on the lengths of the sides. In the Reflection, on p. 40, students are asked, "How can you decide whether two figures are similar?"

- *(7) Comparing and Scaling.* On p. 11, students choose a way to report results of an experiment that uses a spinner. The ways suggested include ratios, percents, or differences. On p. 13, students explain which reasoning methods should be used in particular circumstances.

- *(7) Filling and Wrapping.* In the Reflection on p. 72, students are asked to prove the relationship between cubic centimeters and milliliters. This leads to the water displacement method. In IV3, students are discovering an inductive argument for the volume of any right prism. In problem 18, p. 35, students explain how to find the volume of a right prism with an irregularly shaped base.

- *(8) Thinking with Math Models.* Students use a graph to reason about a linear relationship. As x increases, what happens to y. Does this change when the slope is negative?

- *(8) Looking for Pythagoras.* Students have to select a way to draw a line through dot paper that does not intersect other dots. In IV3, p. 27, the Pythagorean Theorem is introduced. Students make a table with the lengths of the sides and their squares, and use this to conjecture about their relationship based on the pattern seen. On p. 29, a definition of *theorem* is given. Students use geometric ideas to prove this theorem. On p. 40g, an algebraic proof based on a geometrical proof is given to the teacher.

- *(8) Clever Counting.* On p. 46d, reasoning is given to the teacher that leads to formulas for permutations and combinations. On p. 18, problem 2.3, students use inductive reasoning to find an equation for the number of combinations, c, in terms of the number of marks, m, on a lock.

- *(8) Frogs, Fleas, and Painted Cubes.* On p. 51c, the teacher pages explain that one can often first prove what something is not. For example, prove that the tables of values can not possibly represent a linear relationship.

- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are asked to test several cases, and make a conjecture about the result. This is an example of inductive reasoning. On p. 49, problem 2b, students are asked to find a transformation that moves one circle to another to check for congruence. On p. 51, problem 5 asks the same question for congruence of triangles. Students prove congruence by the similarity transformations of reflection, rotation, and translation.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Per Sense.* Students do multiplication and estimation problems using reasoning coupled with the ratio table and percent bars.

- *(6/7) Made to Measure.* Students reason which set of data is depicted in a picture on p. 52, problem 15.

- *(6/7) More or Less.* Various reasoning strategies are used to multiply decimals and find 35 percent of something. No formal algorithm has been discovered or given in the materials yet for multiplying decimals.
- *(7/8) Powers of Ten.* Students reason that a viewer that has its eye height the same as the length of its square can be used to solve problems of how big an area is as viewed from a given height. A model is built and tested.
- *(7/8) Triangles and Beyond.* Students devise a method to find out if a set of lines are parallel or just appear to be.
- *(8/9) Great Expectations.* Students reason about a population based on a sample.
- *(8/9) Patterns and Figures.* Students reason about the general pattern from the concrete. In Section G, they solve the choreography problem of rearrangement of people in the fewest moves.
- *(8/9) Getting the Most Out of It.* Students reason about slope of a line through the principle of fair exchange.
- *(8/9) Triangles and Patchwork.* The unit presents a more formal study of proof involving similar triangles.

Discussion: This standard is fully met.

3.8.5 Reasoning and Proof Standard Summary

Question	Singapore	CMP	Math-in-Context
Reason and Proof 1.	1	3	3
Reason and Proof 2.	1	3	3
Reason and Proof 3.	1	3	3
Reason and Proof 4.	3	3	3

Table 7: Summary of NCTM Reason and Proof Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets one standard and does not adequately meet three of them. The lower scores were given because the curriculum does not explicitly encourage students on a regular basis to explain their reasoning in the problems or assessments they do, does not ask students to frequently make and investigate their own mathematical conjectures, and does not explicitly ask students to evaluate the mathematical arguments of others. The evidence points to another issue worth mentioning that the scoring did not reflect.

- Students do both inductive and deductive reasoning and select between strategies for inductive reasoning. We note that they do not encounter problems that require them to select between inductive and deductive reasoning.

CMP:

The scores in the table above show that the curriculum fully meets all four of the standards.

MIC:

The scores in the table above show that the curriculum fully meets all four of the standards.

3.9 Communication Standard

3.9.1 Communication Standard Question 1.

Does the curriculum enable all students to organize and consolidate their math thinking through both written and oral communication?

Singapore: Score: 1
Evidence:

- *(6A to 6B)*. The Teacher Guides suggest that students work in groups for some activities. This gives oral communication opportunities. No evidence is found that students write about their mathematical thinking.
- *(6B)*. In the Teacher's Guide, p. 23, the teacher displays a card showing an algebraic expression. Students make up a word problem for that card.
- *(SL1)*. The Teacher's Resource Manual often suggests students work in groups. There is little evidence from this manual that written communication is required.
- *(SL1)*. On p. 34, the student is asked "What do you observe?" "Explain how you would proceed to determine whether or not a number is a perfect square?" This is an isolated example of communication in *(SL1)*.
- *(SL2)*. On p. 170, the Class Activity lets students discover the volume of a pyramid through a sand-pouring experiment. This group activity fosters oral communication.
- *(SL2)*. On p. 250, the Class Activity asks students to conduct a traffic survey, to display the information in a chart with a short title, then to interpret the information and draw a conclusion. This is one of the isolated examples of explanatory writing in this curriculum.

Discussion: This standard is not adequately met. The curriculum suggests some group work, but few opportunities are made explicit in the curriculum for written communication. The score of 1 reflects that the written communication part of this standard was not met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6th to 8th) All Units*. After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking. All units have group work. Most units have a Unit Project which requires both written and oral communication.

- *(6) Bits and Pieces I.* Students are asked to explain if two classes raised the same amount of money if they both reached 3/5 of their respective goals. (The classes had different goals.)
- *(6) Ruins of Montarek.* During the Unit Project, students must communicate in order to make a model building. They have to submit a written report as well.
- *(6) Bits and Pieces II.* On p. 63 in the Reflection, students are to think about what pattern there is when multiplying one fraction by another. This could lead to the standard algorithm of multiplying numerators and denominators. This will be shared with the class during the Summarize part of the lesson.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Questions in which students must explain their answers and methods are included.
- *(5/6) Per Sense.* In problem 22, p. 76, students write a percent problem where approximation is appropriate and another where an exact answer is needed.
- *(6/7) Made to Measure.* Students explain other people's thinking.
- *(6/7) Dealing With Data.* Students explain and describe questions about characteristics of data, data representation, and the statistic chosen.
- *(7/8) Cereal Numbers.* Students write about a cereal label by organizing their thinking about fractions and percents.
- *(7/8) Packages and Polygons.* Students pull together ideas to design a solid and its net, apply Euler's Formula, write a report, and make a class presentation.
- *(8/9) Insights Into Data.* In the Assessment, p. 132, students make up a test for other students on the math learned in the unit.
- *(8/9) Great Expectations.* Students write a letter to the principal saying why a certain survey should be disregarded. Given survey data, the student must present both sides of an argument.

Discussion: This standard is fully met.

3.9.2 Communication Standard Question 2.

Does the curriculum enable all students to communicate math thinking coherently and clearly to peers, teachers, and others? (both oral and written)

Singapore: Score: 1
Evidence:

- *(6A to 6B)*. The Teacher Guides suggest that students work in groups for some activities. This gives oral communication opportunities. No evidence is found that students write about their mathematical thinking.
- *(6B)*. In the Teacher's Guide, p. 23, the teacher displays a card showing an algebraic expression. Students make up a word problem for that card.
- *(SL1)*. The Teacher's Resource Manual often suggests students work in groups, so this fosters oral communication. There is little evidence from this manual that written communication is required.
- *(SL1)*. On p. 34, the student is asked "What do you observe?" "Explain how you would proceed to determine whether or not a number is a perfect square?" This is an isolated example of communication in *(SL1)*.
- *(SL1)*. Students communicate to the teacher in the many summative assessments by working problems. The summative questions do not require students to explain their thinking.
- *(SL2)*. On p. 170, the Class Activity lets students discover the volume of a pyramid through a sand-pouring experiment. This fosters oral communication.
- *(SL2)*. On p. 250, the Class Activity asks students to conduct a traffic survey, to display the information in a chart with a short title, then to interpret the information and draw a conclusion. This is one of the isolated examples of explanatory writing in this curriculum.
- *(SL2)*. On p. 122, the Class Activity asks students to investigate with others the linear graph that was produced in the book for cost versus the number of shirts to explain why it costs 50 dollars to buy 0 shirts. This provides for oral communication.
- *(SL2)*. On p. 294, students discuss observations of enlargements of figures.

Discussion: This standard is not adequately met. The curriculum suggested some group work, but few opportunities were made explicit in the curriculum for written communication. The assessments ask the students to work problems and rarely if at all require the students to explain their thinking. The score of 1 reflects the deficiency in the written communication part of this standard.

Connected Mathematics Program: Score: 3
Evidence:

- *(6th to 8th) All Units.* After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking with other students and the teacher. All units have group work and most have a Unit Project.
- *(6) Prime Time.* Students make up word problems that involve common factors and common multiples.
- *(6) Bits and Pieces I.* In the Reflection, students are asked to explain to others and to write in their journals how to tell which is bigger .57 or .559.
- *(7) What Do You Expect?.* In the Unit Project, students design, build, simulate, and analyze a new carnival game. They write a persuasive report to the carnival committee to accept the game at the next carnival.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Students are asked to communicate their mathematical thinking in the Assessment on p. 92.
- *(6/7) Rates and Ratios.* Assessments have a communication part. Students work together to study shadows and perform investigative experiments.
- *(6/7) Operations.* Students explain how a coordinate system works.
- *(7/8) Powers of Ten.* Students work together to build a scale model of the solar system.
- *(7/8) Looking at Angle.* Students write about the four contexts they used to study angle.
- *(8/9) Reflections on Number* (not in Plan B) A student communicates to a 4th grader why his method works and presents another strategy for solving the problem.
- *(8/9) Great Expectations.* Students write a letter to the principal saying why a certain survey should be disregarded. Given survey data, the student must present both sides of an argument.

Discussion: This standard is fully met.

3.9.3 Communication Standard Question 3.

Does the curriculum enable all students to analyze and evaluate math thinking and strategies of others?

Singapore: Score: 0
Evidence:

- *(4A to 6B).* The Teacher's Guides specify which activities students are to work on cooperatively. The strategies are given to the students. There is no evidence in the curriculum that students are to analyze the thinking or strategies of others.
- *(SL1,SL2).* The Teacher's Guides for these books recommends some group work. Opportunities in the student books exist in the form of Class Activities. For example, on p. 321 of *SL1* one of these activities is to make symmetrical masks. The curriculum does not encourage students to look at the thinking of others in a critical way.

Discussion: This standard is not met. Group work is suggested, but the expectation that students analyze or evaluate each other's thinking is not made explicit.

Connected Mathematics Program: Score: 3
Evidence:

- *(6th to 8th) All Units.* After each investigation in each unit, the Reflection section asks students to write about and discuss their mathematical thinking with other students and the teacher. All units have group work and most have a Unit Project.
- *(6) Data About Us.* Students analyze statements made by political candidates and decide which are correct.
- *(6) Bits and Pieces I.* On p. 52b, the Teacher's Guide contains a dialogue where two students communicate to analyze each other's thinking.
- *(6) Bits and Pieces II.* In problem 3c, p. 8, students evaluate whether the tip was added before or after the tax was added to the bill.
- *(7) Stretching and Shrinking.* In problem 11, p. 51, students are asked to provide the rule that Samantha used to make the new triangle.
- *(8) Looking for Pythagoras.* Students are given pieces that make up a puzzle that can be used to prove the Pythagorean Theorem. They have to decide how the theorem can be proven using these pieces.
- *(8) Say It With Symobls.* On p. 40, three strategies for finding the area of a trapezoid are given and students must explain why each works.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Assessment question, p. 115 and problem 6, p. 44 give the student a solution and ask how it was obtained.
- *(6/7) More or Less.* Students answer questions like "How did Mary solve it?" and "Explain what Jack did."
- *(6/7) Dealing With Data.* Students analyze others' reasoning on p. 12.
- *(6/7) Expressions and Formulas.* On p. 50, students explain why another person's formula does not match the arrow language.
- *(7/8) Powers of Ten.* Students analyze how the Power of 10 machine would work backwards and what that would mean.
- *(8/9) Great Expectations.* Students explain others' thinking on p. 112.
- *(8/9) Getting the Most Out of It.* The summary question on p. 102 requires evaluating the thinking of others and formulating and giving advice.
- *(8/9) Reflections on Number.* Students evaluate another person's algorithm for dividing whole numbers.

Discussion: This standard is fully met.

3.9.4 Communication Standard Question 4.

Does the curriculum enable all students to use the language of math to express math ideas precisely?

Singapore: Score: 2
Evidence:

- *(4A to 6B).* These books use the Concrete to Pictorial to Abstract approach. The books present the idea that mathematics is a *language* and it is used in the early books to communicate ideas precisely, even for the general case.
- *(6A).* The Teacher's Guide, p. 77, suggests that students be asked to *verify*.
- *(4A to 6B).* The idea that estimation is not precise is used throughout.
- *(SL1).* On p. 140, vocabulary words are in bold face and definitions are provided by the book both directly and indirectly through the context of the surrounding prose.

- *(SL1).* On p. 230, occassional summaries of vocabulary words are given in tables. This book contains many examples that mathematics is a *language*. On p. 25, it states "Mathematicans believed there were an *infinite* number of primes." On p. 143, "$x = x$ is an identity." On p. 183, the idea of *deriving* a formula is given. Throughout, terms such as multiple, factor, prime, and similar, are *defined* precisely. Through this vocabulary, students are given a glimpse of the preciseness of the subject.

- *(SL2).* On p. 174, one gets a right circular cone by "increasing to infinity" the number of sides of a polygonal base.

- *(SL2).* On p. 277, the word "invariant" is used in the context of rotational symmetry.

- *(SL2).* On p. 111, identical linear graphs of $x + y = 1$ and $3x + 3y = 3$ mean these equations have an "infinite number of solutions."

Discussion: This standard is adequately met. There are many rich instances in the curriculum that demonstrate that mathematics is a language with which we can communicate precisely with others. Students do frequently use mathematical symbols in the problems they do. Students do not frequently write about the mathematics which would give them opportunities to use proper mathematical terminology.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 15, "Is there a largest perfect number?" gives a glimpse of questions a mathematician might ask.

- *(6) Shapes and Designs.* On p. 21c, the teacher is told how a mathematician would write and discuss the triangle inequality that could be shared with the students.

- *(7) Comparing and Scaling.* On p. 51e, the teacher is told that the changing speed problems are related to *piecewise linear graphs*. In problem 4.4, p. 43, students write an equation that relates the cost C and the number of beads x for each type of bead.

- *(8) Looking for Pythagoras.* On p. 29, the definition of a theorem is given. On p. 1g, a proof by contradiction is given to the teacher that the square root of 2 is irrational. On p. 50, the notion of generalization is expressed by asking whether the Pythagorean idea works for more general 2D areas attached to the sides of a right triangle.

- *(8) Hubcaps and Kaleidoscopes.* In problem 27, p. 57, students are asked, "Investigate what happens when you rotate a figure 180 degrees about a point and then rotate the image 180 degrees about a different point. Is the combination of the two rotations equivalent to a single transformation?" Students are asked to test several cases, and make a conjecture about the result. This is an example of using correct mathematical terminology in a student problem. On p. 59, the term symmetry is precisely defined: "A geometric figure is symmetric if a reflection or rotation of the figure produces an image that matches the original figure exactly." On p. 63, the inverse of a symmetry transformation is defined. On p. 58j (to the teacher), equality is compared to congruence. The teacher is told if the students are ready, such a discussion could be done informally with them.

Discussion: This standard is fully met.

Mathematics in Context: Score: 2
Evidence:

- *(6/7) Reallotment.* Students use the formula $A = \pi r^2$ for the area of a circle.
- *(6/7) Expressions and Formulas.* Students begin to use formulas to express relationships.
- *(7/8) Ways to Go.* Students reason from specific to general to find there are $n(n-1)/2$ edges in a graph with n points that are maximally connected.
- *(7/8) Looking at Angle.* Students use the notation $\tan \alpha = h/d$ for the slope of a line.
- *(7/8) Building Formulas.* On p. 12 and p. 18, students describe patterns using recursive and direct formulas, respectively. On p. 66, in problems 5–7, students are asked to "unsquare." On p. 68, the book tells the students that mathematicians call this unsquaring process "taking the square root." In Section D, students use inductive reasoning to build a formula for temperature conversions.
- *(7/8) Triangles and Beyond.* Congruent shapes can be flipped, rotated, reflected and translated to align. On p. 94, it is stated, "If a figure is translated, rotated, or reflected, the resulting figure is congruent to the original figure." On p. 92 the "line of symmetry" is discussed.
- *(8/9) Great Expectations.* In notes to the teacher, n^n is the number of ways to order n numbers if the same number can be used in each of n slots while $n!$ is the number of ways to order n objects if each object is used once.
- *(8/9) Getting the Most Out of It.* In Section E, algebraic inequalities are used to represent feasible solution regions.
- *(8/9) Triangles and Patchwork.* On p. 58 the "About the Mathematics" tells the teacher ways to prove angles congruent. On page p. 64, students are told "If corresponding sides have the same ratio then the triangles are similar and the corresponding angles are equal."

Discussion: This standard is adequately met. The language of mathematics is often avoided in favor of an informal and concrete presentation that allows students to develop and hence understand strategies. The general cases listed above in *Great Expectations* are introduced for the teacher but not for the students. Based on the lack of mathematical language in the student books and the word *precisely* in the standard, a rating of 2 instead of 3 is given.

3.9.5 Communication Standard Summary

Question	Singapore	CMP	Math-in-Context
Communication 1.	1	3	3
Communication 2.	1	3	3
Communication 3.	0	3	3
Communication 4.	2	3	2

Table 8: Summary of NCTM Communication Standard Results

Singapore:

The scores in the table above show that the curriculum adequately meets one standard, does not adequately meet two standards, and does not meet one standard. The lower scores were given because the curriculum does not adequately provide opportunities for written communication, does not explicitly encourage students to look at each other's thinking in a critical way, and does not encourage students to write about mathematics in order to use mathematical terminology.

CMP:

The scores in the table above show that the curriculum fully meets all four of the standards.

MIC:

The scores in the table above show that the curriculum fully meets three of the four standards and adequately meets one of them. The lower score was given because mathematical terminology is not prevalent in the student books.

3.10 Connection Standard

3.10.1 Connection Standard Question 1.

Does the curriculum enable all students to recognize and use connections among math ideas?

Singapore: Score: 1
Evidence:

- *(6B)*. In the Teacher's Guide, p. 40, the unitary method that was used earlier in *(6A)* to study ratio and percentage problems is now applied to rate problems involving volumes.

- *(SL1)*. The strategies build on previously covered concepts and the Revision Tests are done frequently to test on past material. The unitary method is again used in scaling type problems.

- *(SL2)*. On p. 34, the connection between the area of a rectangle and the product of two numbers is used to explain the expansion of $(a + b)(c + d)$ geometrically. This is a connection to the distributive property. Likewise $3 \times 5 = 15$ is used to explain 3 and 5 are factors of 15 and hence a and $(x + y)$ are factors of $a(x + y)$.

- *(SL2)*. In Ch. 4, $a/b = (a \times c)/(b \times c)$ that was used earlier with real numbers is shown to also work for algebraic fractions.

Discussion: This standard is not adequately met. The texts present mathematics in a sequence of fairly self-contained pieces. Students rarely do activities that involve integrated concepts or different strands of math at the same time; therefore, do not have true opportunities to see connections.

Connected Mathematics Program: Score: 3
Evidence:

- *(6 to 8) All Units*. After each investigation, students do "C" or Connection problems that require connections to mathematics previously studied.

- *(6) Prime Time*. Students use areas of rectangles and sides of rectangles to find all the factors of whole numbers.

- *(6) Shapes and Designs*. On p. 41i for the teacher, a suggestion is made to get the students to make a line plot of all the angle measurements students found when measuring a a particular angle to see the spread and mean. This is a connection to statistics. Angles are also connected to the positioning of the hands of a clock.

- *(6) How Likely Is It?* Students read bar graphs to see if events are equally likely. On p. 45, the event that the sum showing on two dice will be prime is a connection to *Prime Time*.

- *(8) Frogs, Fleas, and Painted Cubes.* Students recognize that length is linear in its units and area is quadratic.

Discussion: This standard is fully met.

Mathematics in Context: Score: 1
Evidence:

- *(5/6) Per Sense.* Students connect percents and fractions but not percents and decimals.
- *(5/6) Take a Chance.* Students view probability as a fraction, proportion, and a percent.
- *(6/7) Made to Measure.* Students connect decimals and fractions.
- *(6/7) Rates and Ratios.* Students connect ratios and fractions.
- *(6/7) Comparing Quantities.* Students begin to see the connection between variables and unknowns in linear relationships.
- *(7/8) Triangles and Beyond.* Students see that two reflections may make a rotation.
- *(8/9) Great Expectations.* Students connect probability to ratios, percents and fractions.
- *(8/9) Triangles and Patchwork.* Students see the connection of similar triangles to shadow problems.
- *(8/9) Graphing Equations.* Students see that negative numbers represent negative slopes in linear graphs.
- *(8/9) Patterns and Figures.* Students see connections between tesselations and square numbers on page 76.

Discussion: This standard is not adequately met. As evidenced above, students do use some connections between math ideas to solve problems. However, these problems usually do not contain different strands of mathematics. The teacher's books do display prominently the connections Math in Context is trying to make, but none of this is evident in the student books through the problems they do.

3.10.2 Connection Standard Question 2.

Does the curriculum enable all students to understand how math ideas interconnect and build to create a coherent whole?

Singapore: Score: 1
Evidence:

- *(4A to 6B).* A small building block of a concept is introduced to the student who practices it many times. The next part is given and it is practiced many times. Students are then expected to comprehend the whole, or at least do all the subparts related to the whole. The flow of the mathematical concepts is logical and coherent and goes from a concrete representation to an abstract level. The Teacher's Guides for these grades do an excellent job of showing how the conceptual story builds.

- *(SL1, SL2).* The math does build logically from small pieces. It is not clear that this logical progress is pointed out to the students. The curriculum does not ask them very often (if at all) to stop and reflect. The curriculum presents the view that students will get the big picture by doing a lot of problems on the building blocks that make the whole.

Discussion: This standard is not adequately met. The *(6A)* and *(6B)* books point out to the teacher how the small ideas interconnect to form the whole. The scope and sequence chart in the *(6B)* Teacher's Guide is evidence of this. The students, though, practice only one part of one strand of math at a given time. The main concern here is the lack of this standard in the *(SL1)* and *(SL2)* books.

Connected Mathematics Program: Score: 3
Evidence:

- *(6th-8th grade units).* CMP makes it explicit to the teacher and the student how math ideas interconnect and build to create a coherent whole. In the beginning of each Teacher Guide, the big ideas of the present unit and the connections to the past and future units are given in a table. Students see the connections directly by doing the "C" (Connection) problems after each Investigation.

- *(6) Data About Us.* On p. 49, students make a graph of whole numbers vs. the number of factors. This is connecting to concepts studied previously in *Prime Time*.

- *(7) Moving Straight Ahead.* On p. 79h the teacher is shown how similar triangles are related to the slope of a line.

- *(8) Looking for Pythagoras.* On p. 68 and p. 16a, connections between similar triangles, proportionality and the Pythagorean Theorem are made.

Discussion: This standard is fully met.

Mathematics in Context: Score: 1
Evidence:

- *(6/7) Made to Measure.* Students see how place value and digits connect to build a number system. They see how decimals and fractions are related.
- *(6/7) Reallotment.* The idea of using a scale on a map coupled with measuring a map length to find actual area or length of an object is presented.
- *(6/7) More or Less.* Students see how percent, fractions, and decimals connect.
- *(7/8) Looking at Angle.* On p. 103, students write about the connection of the four contexts of angles that were presented. Problem 10 on p. 88 is a connection of equivalent fractions within a problem of glide angles in geometry.
- *(7/8) Building Formulas.* Students work with a model of stairs using risers and treads. They measure stairs, make graphs, and write a formula. This brings parts together as a whole in a real world carpentry application.
- *(8/9) Graphing Equations.* On p. 48, students see how the idea of fair exchange is related to slope in a linear relationship.
- *(8/9) Getting the Most Out of It.* Ideas of graph, symbolic linear equation, inequality, fair exchange, and rolling line come together to solve linear programming problems in Section E.

Discussion: This standard is not adequately met. As evidenced above, students do study mathematics in contextual situations. However, the problems usually don't include more than the current topic, and they do not contain different strands of mathematics. The teacher's books do display prominently the connections Math in Context is trying to make, but none of this is evident in the student books through the problems they do.

3.10.3 Connection Standard Question 3.

Does the curriculum enable all students to recognize and apply math in contexts outside math?

Singapore: Score: 1
Evidence:

- *(6A).* In Ch. 3, the context of finance is used heavily to study percentages. The problems students work are short examples of financial problems.
- *(6B).* In Ch. 5, the displacement of a solid in a liquid is a science context for studying volume.

- *(SL1).* Ch. 12 is entirely is about household finance.
- *(SL1).* On p. 88, the context of maps and geography is used to study scales.
- *(SL2).* On p. 27, the usefulness of scientific notation is mentioned in the context of physics (size of an electron) and in the context of biology (the number of times a heart beats in a lifetime).

Discussion: This standard is not adequately met. Our interpretation of this standard is that students should *regularly* work with contextual problems. As evidenced above, the Singapore curriculum includes the context of finance in the problems students do. Overall, however, from *6A* through *SL2*, this curriculum is basically context-free. Furthermore, the large number of word problems on a given topic are all very similar.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* On p. 11, connections to finding prime numbers and the Cray Computer are given. On p. 38, cycles of locusts are connected to common multiples. On p. 60 in Extension problem 17, students have to discuss why a minute is divided into 60 units rather than 61 or 59.
- *(6) Data about Us.* On p. 54, census data are used to make a frequency graph. On p. 84, political campaigns are examples of the convenient interpretation of the word average. Students analyze which candidate is telling the truth.
- *(6) Bits and Pieces I.* The locations of cars in drag racing on a linear race track are related to positions on a number line. Likewise, gauges in water containers are used in measurement studies. In IV6, surveys use percentages. In IV5, probems 42 and 43 use the Dewey Decimal system to study decimals.
- *(6) Shapes and Designs.* On p. 11, polygons are connected to street signs. In IV2, triangles are seen to produce stable structures, such as bridges. On p. 32, measurement error is related to Amelia Earhart's last flight. On p. 8, honeycombs are related to hexagonal tesselations.
- *(6) How Likely Is It?* Probability is used to forcast rain or snow. In IV7, probability is used to study events in genetics, such as having a blue-eyed child.
- *(7) Data Around Us.* In problem 17a, p. 68, students find the average number of miles per day travelled by the Galileo spacecraft.
- *(7) Moving Straight Ahead.* In IV1, the stretching of a spring under weight is a connection to physics.
- *(8) Thinking with Mathematical Models.* Compound interest is an example of a nonlinear relationship.

- *(8) Clever Counting.* Students do combinatorical problems including zip codes, phone numbers, and lock combinations.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Sum of Parts.* Students apply math to recipes, races, sharing in social situations (fractions), and populations of people.
- *(6/7) Made to Measure.* Students apply math to sports and measurement.
- *(6/7) Reallotment.* Students study 2D and 3D applications to sewing and find the volume of snow outside.
- *(6/7) Expressions and Formulas.* Students use counting on in supermarket problems to make change, and they find an equation for the total grocery bill.
- *(6/7) More or Less.* Applications to discounts, interest rates, tax, and photo shrinking are included in the problems students do.
- *(6/7) Rates and Ratios.* Students use scale factors to find sizes of things in nature, sizes from pictures that have been enlarged, and the design of new pictures.
- *(6/7) Dealing with Data.* Applications to historical data and animal characteristics are included in the problems students do.
- *(7/8) Cereal Numbers.* Students use the Consumer Price Index.
- *(7/8) Powers of Ten.* Students design a scale model of the solar system.
- *(7/8) Packages and Polygons.* The book shows shapes of regular polygons in honeybee combs, soccer balls, and the Pentagon.
- *(8/9) Great Expectations.* In advertising, students find the probability of creating more customers. They study the effects of particular drug remedies on populations.
- *(8/9) Triangles and Patchwork.* Students use similar triangles to analyze the takeoff paths of airplanes.

Discussion: This standard is fully met.

3.10.4 Connection Standard Summary

Question	Singapore	CMP	Math-in-Context
Connections 1.	1	3	1
Connections 2.	1	3	1
Connections 3.	1	3	3

Table 9: Summary of NCTM Connections Standard Results

Singapore:

The scores in the table above show that the curriculum did not adequately meet any of these standards. The lower scores were given because the problems rarely integrate different strands of math at the same time, that these problems are very similar on a given topic, and are mainly context-free. The evidence points to several other issues worth mentioning that the scoring did not reflect.

- The *(6A)* and *(6B)* books are much better than the *(SL1)* and *(SL2)* books in pointing out to the teacher how the mathematical ideas connect.

CMP:

The scores in the table above show that the curriculum fully meets all three of the standards.

MIC:

The scores in the table above show that the curriculum fully meets one of the three standards and does not adequately meet two of them. The lower scores were given because the connections between mathematical strands, though very explicit in the teacher books, are not reflected in student books nor in the nature of the problems students do.

3.11 Representation Standard

3.11.1 Representation Standard Question 1.

Does the curriculum enable all students to create and use representations to organize, record, and communicate mathematical ideas?

Singapore: Score: 3
Evidence:

- *(6A)*. In the Teacher's Guide, Ch. 3, teachers are encouraged to guide pupils to draw models to solve problems.
- *(6A)*. In Ch. 1, students model fractions with bar strips, fraction disks, folding paper, and pie charts.
- *(6A)*. In Ch. 2, ratios and proportions are modeled with ratio tables and the unitary method, and double number lines. Changing ratio problems and before and after problems are solved with pictures instead of algebra.
- *(6B)*. In the Teacher' Guide, p. 13, percentage circle charts are used.
- *(6B)*. In the Teacher's Guide, p. 21, pictures are used to represent algebraic expressions.
- *(6B)*. In Ch. 3, students use centimeter cubes to build larger cubes.
- *(SL1)*. In Ch. 2, p. 29, factor trees are used to understand prime factorization.
- *(SL1)*. In Ch. 3, p. 52, problem 4.7, students must determine which representation to use to solve word problems.
- *(SL1)*. In Ch.8, p. 141, diagrams showing pan balances are used as an introduction to algebra.
- *(SL1)*. In Ch. 10, p. 186, a cylinder is represented as a stack of coins.
- *(SL1)*. On p. 225, problems 1–9, students make a representation and use it to solve proportion problems.
- *(SL2)*. On p. 73, pictures are used to solve two linear equations in two unknown quantities to communicate the idea before symbols are used. This is immediately followed by a more systematic way to view equations and graphs.
- *(SL2)*. On p. 89, pictures on a number line are used to solve inequality problems.
- *(SL2)*. On p. 122, the connection between graphs and equations is made clear. Students are told, "Graphs are pictures of equations."

Discussion: This standard is fully met. (Here we do not interpret the standard as meaning students have to invent their own representation, just that they have to develop the habits of mind to use representations – standard or nonstandard).

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Prime Time.* This unit uses systematic lists, Venn diagrams, square tiles, areas to represent factors, and factor trees for prime factorization.
- *(6) Data About Us.* This unit uses line plots, bar graphs, frequency graphs, stem-and-leaf plots, pie charts, area models, double stem-and-leaf plots, scatter plots, and coordinate graphs.
- *(6) Bits and Pieces I.* On p. 5, students make thermometers to measure the percentage of a fund-raising goal that is met. Fraction strips, 100 grids, and area models are used to convey fractions and percent ideas.
- *(7) Data Around Us.* This unit uses base-10 units, rods, and flats to represent large integers and their relationships.
- *(7) What Do You Expect?.* This unit uses probability trees, spinners, colored balls, dice, coins, playing cards, thumb tacks (lands point up or down), area models, and number cubes.
- *(7) Accentuate the Negative.* In problem 3.1, p. 37, colored chips and number lines are used to develop meaning for addition, subtraction, and multiplication with negative numbers.
- *(8) Clever Counting.* Counting trees and networks are used to illustrate combinatorial problems.
- *(8) Samples and Populations.* Box and whisker plots, stem-and-leaf plots, scatter plots, histograms, spinners, dice, and random number generators are used to study statistics and probability.
- *(8) Growing, Growing, Growing.* Tree diagrams are used to illustrate exponential growth or decay.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Comparing Quantities*: Students use combination charts (and also state their limitations), notebook notation and symbolic equations.
- *(5/6) Sum of Parts*: Students use fraction strips, ratio tables, and the double line.

- *(5/6) Per Sense*: Students use percent bars with fractions.
- *(5/6) Take a Chance*: Students use probability trees and a probability ladder.
- *(6/7) Made to Measure*: Students use a number line, a circle for measurement, and a double number line.
- *(6/7) Fraction Times*: On p. 14, students use a bar chart, a pie chart, and a pie chart meter for fraction and percent problems.
- *(6/7) More or Less*: Rubber bands are used by students to create a way to model similar figures with given scale increases or decreases.
- *(6/7) Dealing With Data*: Students use histograms, scatter plots, box plots, tally charts, frequency diagrams, and stem and leaf plots.
- *(7/8) Cereal Numbers*: Students use the area model for multiplying fractions. Students make a triangular graph for comparing three quantities in a table.
- *(7/8) Powers of Ten*: Students use arrow language to successively multiply by 10. The book introduces a 10-machine to understand negative exponents with a base of 10.
- *(7/8) Ways to Go*: Students use graphs to model paths that airlines take. They also represent this information as incidence matrices.
- *(8/9) Reflections on Number.* (Not in Plan B.) Students use factor trees.

Discussion: This standard is fully met.

3.11.2 Representation Standard Question 2.

Does the curriculum enable all students to select, apply, and translate among mathematical representations to solve problems?

Singapore: Score: 3
Evidence:

- *(6A)*. In Ch. 2, students translate between a percent number line and a bar chart to solve story problems.
- *(6A)*. In Ch.2, students translate between ratio notation such as 3:5, fractions, bar charts, and the unitary method to solve proportion problems.
- *(6B)*. In the Teacher's Guide, p. 56, the teacher is encouraged to have pupils solve a problem different ways whenever possible. Different ways include: drawing a before and after picture, using the method of supposition, or using tabulation. Even so, the student book *(6B)* on pp. 66–72 only illustrates the before and after strategy.

- *(SL1).* In Ch. 3, pp. 47–52, a list of representations and problem solving strategies are outlined. This includes making diagrams, lists, and using equations. Then pp. 47-50 give concrete examples of each. Problems 4–7 on p. 52 require students to select a representation to solve problems.
- *(SL2).* On pp. 11–12, the same problem is solved four different ways. These ways include diagrams, before and after pictures, equations, and ratios.
- *(SL2).* In the Workbook, p. 104, problem 40, students make both a frequency table and a histogram for the same data. The same type of problem is also encountered in Test 7B, p. 78 of the *(SL2)* Teacher's Resource Manual.
- *(SL2).* On p. 242, tally sheets, pictograms, bar charts, histograms, tables, pie charts, piecewise linear graphs and frequency tables are used in statistics problems. Discussions about why students would benefit from using one versus the other are given.

Discussion: This standard is fully met.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* Both stem-and-leaf and scatter plots are used for the same data.
- *(6) Bits and Pieces I.* On p. 18e, the teacher is encouraged to help students go back and forth between words and fraction strips to reinforce the meaning of fractions. Teachers are encouraged not to rush to symbols until the meaning is clear.
- *(6) Ruins of Montarek.* Students translate between different views of a solid in order to build the solid. Alternately, they view the solid and fill in the missing parts in 2D views. This is a comprehensive spatial visualization unit.
- *(7) Accentuate the Negative.* On pp. 46–49, students translate between chip boards and number lines to add and subtract positive and negative integers.
- *(7) Stretching and Shrinking.* Students build a device called a rubber-band stretcher to enable them to produce similar figures.
- *(7) Variables and Patterns.* Students go between tables, graphs, equations, and words to discuss, represent, and analyze relationships.
- *(8) Looking for Pythagoras.* In IV2, students translate between dot paper and coordinate grid representations to find areas of plane figures.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Take a Chance.* Students select an appropriate representation between lists or probability trees.

- *(6/7) More or Less.* Students use ratio tables, 1 percent and 10 percent rules to solve problems. Some of these representations are lengthy and cumbersome, but no algorithms have been formally given.

- *(6/7) Dealing With Data.* Students represent data many ways, including histograms, scatter plots, box plots, tally charts, frequency diagrams, and stem and leaf plots.

- *(6/7) Expressions and Formulas.* Students switch between arrow language, equations, and arithmetic trees.

- *(6/7) Tracking Graphs.* Students discuss advantages and disadvantages of graphs and tables, especially as graphs must be compressed in a longer time frame.

- *(7/8) Cereal Numbers.* Ratio tables and area models are both used for fractions.

- *(7/8) Looking at Angle.* Students look at both side and top views for shadow problems.

- *(8/9) Insights Into Data.* Students select good representations for data from scatter plots, box plots, and histograms.

- *(8/9) Great Expectations.* Students use both area and tree models to solve conditional probability problems.

- *(8/9) Patterns and Figures.* On p. 75, students use dot patterns, area diagrams and number strips to solve problems with sequences.

Discussion: This standard is fully met.

3.11.3 Representation Standard Question 3.

Does the curriculum enable all students to use representations to model and interpret physical, social, and mathematical phenomena?

Singapore: Score: 2
Evidence:

- *(5B).* In the Workbook, p. 65, graphs of lines for money exchange in financial problems are given.

- *(6B).* In Ch. 5, a speed, distance, and time model is used to help students reconstruct the relationship between the three.

- *(6B).* In Ch. 3, a liter of water and a box whose volume is 1 liter (dimensions are $10cm \times 10cm \times 10cm$) is used in an investigation to show that 1 liter $= 1000cm^3$.
- *(SL1).* In Ch. 16, p. 312, a scale drawing is used to show floor plans for houses.
- *(SL1).* Ch. 12 is entirely about household finance. This includes profit, loss, and money exchange. Tables, diagrams, and before-after comparisons are used.
- *(SL2).* The Teacher's Resource Manual, p. 73, suggests that teachers assign a mini-project for groups of students since statistics can be a dry topic if students are taught by a direct approach. Several project topics on social phenomena were suggested.

Discussion: This standard is adequately met. The statistics mini-project above is one example where representations are used to solve a relatively large-scale, motivating, and significant applied problem alluded to in the standards. However, the curriculum predominently relies on shorter context-free problems. Students do use representations frequently to model mathematics, and based on this, a score of 2 rather than 1 is given.

Connected Mathematics Program: Score: 3
Evidence:

- *(6) Data About Us.* On p. 7 students make frequency graphs of the number of letters in people's names. On p. 54 census data are displayed using frequency graphs.
- *(6) Covering and Surrounding.* In the Unit Project, students build a plan for a park using a proper scale.
- *(6) Ruins of Montarek.* Isometric dot paper and architectural views are used to represent a solid figure.
- *(7) Variables and Patterns.* Students use graphs to extrapolate the relationship outside the data set and interpolation to find a reasonable relationship between the data points.
- *(7) Data Around Us.* In IV1, students use correct measurement units to better interpret large numbers in social situations, such as disaster results such as the Exxon Valdez oil spill.
- *(8) Frogs, Fleas, and Painted Cubes.* Rectangle models are used to come up with algebraic equations for quadratic relationships.
- *(8) Thinking with Mathematical Models.* This entire book is about using graphs, tables, and physical models to describe linear and nonlinear relationships and patterns that represent (as best as one can at this level) real-world situations. Students use these representations to model these situations.

Discussion: This standard is fully met.

Mathematics in Context: Score: 3
Evidence:

- *(5/6) Per Sense.* Students use graphs to show the percentage of people remaining in stadiums as a function of time.
- *(5/6) Take a Chance.* Students use trees to list combinations of clothes to wear.
- *(6/7) Rates and Ratios.* Students use scale factors to find the size of algae and whales.
- *(6/7) Dealing With Data.* Historical data and animal characteristics are represented differently. Students must then compare the representations. In problem 11b, p. 30, students are asked to say how the line helps them make conclusions about the data.
- *(6/7) Tracking Graphs.* Students use graphs to model the tides as a function of time.
- *(7/8) Cereal Numbers.* Students use triangular graphs to compare three ingredients in cereal.
- *(7/8) Ways to Go.* Students use probability trees to model traffic flow. They also use trees to illustrate tournaments and graphs to show car routes.
- *(7/8) Looking at Angle.* Students calculate blind spots in transportation problems (cars, boats). On p. 30, they use toy boats and centimeter grid paper to do this. On p. 10, they model the Grand Canyon by two student desks spread apart and talk about when they can see a boat in the water below. This progresses to a paper canyon to investigate angles. Then on p. 14, the idea is abstracted further to a 2D view of the blind spot problem.
- *(8/9) Triangles and Patchwork.* Students make side views using similar triangles.
- *(8/9) Insights Into Data.* Students find correlations between characteristics of bird eggs and other populations.
- *(8/9) Great Expectations.* Students calculate the percentages of different populations that wear glasses and answer questions about which population is more likely to need glasses. They use a mosquito population context to compare the effect of a new repellent.

Discussion: This standard is fully met.

3.11.4 Representation Standard Summary

Question	Singapore	CMP	Math-in-Context
Representation 1.	3	3	3
Representation 2.	3	3	3
Representation 3.	2	3	3

Table 10: Summary of NCTM Representation Standard Results

Singapore:

The scores in the table above show that the curriculum fully meets two of these standards, and adequately meets the third. The lower score was given because the curriculum does not frequently include problems that require students to model and interpret physical and social phenomena.

CMP:

The scores in the table above show that the curriculum fully meets all three of the standards.

MIC:

The scores in the table above show that the curriculum fully meets all three of the standards.

4 Comparisons with the 2000 NCTM Principles

4.1 The Equity Principle

Excellence in mathematics education requires equity-high expectations and strong support for all students. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

4.1.1 Equity Question 1.

Does the curriculum provide materials and suggestions to the teacher for individualizing instruction?

0	1	2	3
All students do the same tasks	Low	Medium	High number of materials and tips for individualizing

Singapore: Score: 1
Discussion: This principle is not adequately met. In the *(6A)*, *(6B)*, and *(SL1)* and *(SL2)* books, a large number of problems are given for students at different levels. In the *(SL1)* and *(SL2)* books, three sets of exams are given, based on the level of the students. The *(6A)* and *(6B)* Teacher's Guides occassionally give hints of what to exclude for weaker students. The individualizing is through summative assessment. Embedded assessment that would help the teacher to adapt instruction to meet the needs of individual children in the class is not evident in the curriculum.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. The curriculum gives Extension problems that can be used for abler students. Extra challenges are given to the teacher for those that finish early. The teacher is told how to alter the assessments for inclusion students to individualize instruction for them. Teachers are encouraged to find out by group work who understands and who is confused. The Embedded Assessments focus on what students know and feeds back into the planning of lessons.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. In each book, the Overview section explicitly states that students should be allowed to work at different levels of sophistication. "Some students may need concrete materials, while others can work at a more abstract level." Also, the Overview suggests that students should share their ideas since they will have different strategies to solve problems. The Teacher Guide for each lesson has a "Hints and Comments" column where Comments About the Problems are included. These Comments frequently tell what to do if students are having difficulties or how to challenge students.

4.1.2 Equity Question 2.

Are the curriculum materials likely to be interesting, engaging, and effective for girls and boys?

0	1	2	3
No sensitivity to gender issues	Low	Medium	High sensitivity to gender issues

Singapore: Score: 1
Discussion: This principle is not adequately met. The pictures (cartoons) in the *(6A)* and *(6B)* books include girls and boys alike. No gender discrimination was observed in these materials. The situation, however, is different for *(SL1)* and *(SL2)*. In these secondary books, most of the story problems involve males, or females in traditional roles. For example, in *(SL1)*, p. 211, the typist was female, while the renter, worker, shopkeeper, and cook were male. In the Chapter 1 "Challenge Yourself" problems, only one out of five involves a female, and she is a housewife. In *(SL2)*, one positive example of a female engineer was found on p. 246. We find the overall curriculum to be below standard on this topic, hence the score of 1.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. Equal attention is paid to boys and girls in the curriculum. The materials should be equally beneficial to boys and girls.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. Equal attention is paid to boys and girls in the curriculum. The materials should be equally beneficial to boys and girls.

4.1.3 Equity Question 3.

Are the curriculum materials likely to be interesting, engaging, and effective for underrepresented and underserved students (e.g., ethnic, rural, with disabilities)?

0	1	2	3
No sensitivity to underrepresented and underserved students	Low	Medium	High sensitivity to underrepresented and underserved students

Singapore: Score: 1
Discussion: This principle is not adequately met. An isolated example is found in *(SL1)*, p. 253, Problem 6a, which is a tax liability calculation problem showing a man with a handicapped child. Overall, the Singapore materials would require more attention to ethnic and rural issues if used in American classrooms.

Connected Mathematics Program: Score: 3

Discussion: This principle is fully met. For United States students, this curriculum does a thorough job of including problems with stories about different parts of the country, both urban and rural and includes the groups outlined above.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. The materials include pictures and problems that are inclusive of these groups. The materials would appeal to all groups equally.

4.1.4 Equity Question 4.

Are the curriculum materials likely to be interesting, engaging, and effective for mathematically capable students?

0	1	2	3
No Level too low even with the extension materials	Low	Medium	Highly interesting, engaging, and effective without any extension materials

Singapore: Score: 2
Discussion: This principle is adequately met. This curriculum includes a large number of problems that would be appropriate for highly capable students. What the curriculum does not do for these students is to ask them to be independent thinkers. That is, they are told what method, what tools, and what strategy to use to solve these difficult problems. They are not helped to get to the next level of selection, development, or analysis of methods. For these reasons a score of 2 rather than 3 is warranted.

Connected Mathematics Program: Score: 2
Discussion: This principle is adequately met. The materials, even though beneficial to these students, is not enough. More problems at different levels should be included to help the teacher individualize the instruction. In CMP classooms we visited, the teachers brought in more challenging problems to supplement the curriculum. The combination worked well. CMP helped them slow down to make sure the concept was totally understood for all and they supplemented for the most capable students.

Mathematics in Context: Score: 2
Discussion: This principle is adequately met. While mathematically capable students would benefit from these materials, they would be able to go beyond the level of these materials in some units. An example is the last geometry unit in grades *(8/9)* called *Going the Distance*. On page 96 of this unit, the students are still developing the formula for the area of a triangle, the area of a circle, and the circumference of a circle. These topics were introduced in *(6/7) Reallotment*. Primary curriculum, for example *Everyday Math*, have similar problems for 4th and 5th grade students. One would like to see many more problems included that the teacher could use to individualize instruction for the

high-end students. We note that some Extensions are included in the teacher book, but not the student books.

4.2 The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. The interconnections between the mathematics strands should be displayed prominently in the curriculum and in instructional materials and lessons. A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills. The curriculum should help teachers understand the mathematics that has been studied at previous levels and what is the focus at successive levels. A well-articulated curriculum gives teachers guidance regarding important ideas and major themes and depth of study warranted at particular times and when closure is expected for particular skills or concepts.

4.2.1 Curriculum Question 1.

Is the mathematics curriculum coherent?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 1
Discussion: This principle is not adequately met. The interconnection between the concepts of the *(6A)* and *(6B)* curriculum is outlined on pages *ix* to *xviii* of the Teacher's Guide for *(6B)*. The *(SL1)* and *(SL2)* books do not display any help to the teacher or student in seeing how the big ideas of the different mathematical strands interconnect. All the books present mathematics in a sequence of fairly self-contained pieces, but there are few if any problems that are clearly designed to involve different strands of mathematics at the same time. Students do review very frequently in the Revision exercises, but these exercises do not explicitly connect the current topics to those previously studied in an integrated way.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. The curriculum is very coherent for the teacher. In the Teacher's Guide for each unit, the "Connections to Other Units" gives a table with the three columns "Big Idea," "Prior Work," and "Future Work." This includes work in other mathematical strands. The curriculum is also explicitly coherent for the student. At the end of each Investigation, there are three sets of problems, called ACE. The "A" problems apply to the current topic, the "C" are the connection problems that

connect the current topic to other strands and to concepts previously learned in the current strand, and the "E" problems are extension problems.

Mathematics in Context: Score: 1
Discussion: This principle is not adequately met. In the *About the Mathematics* that appears in every lesson of the Teacher Guide, the teacher is alerted to how the math ideas build. In the Overview in the Teacher Guide for every unit, it is suggested that the class should discuss what was learned from the unit and what the activities have in common. This Overview gives the connections within the given strand and the connections to other strands. The curriculum is very coherent for the teacher. The prose in the student pages have none of this and do not connect the material for the student. The belief appears to be that if the teacher sees it, the students see it. It is not evident from the curriculum that the students are shown the coherence, except for what happens to come out in the summarizing discussions. The problems in the student book connect the mathematics they do to real world contexts, but very few connections are made to other strands and math topics. These problems in the student book would correspond to the "A" or "E" problems from CMP, but not the "C" type problems. For these reasons, the materials are considered below standard for the student and a score of 1 is given.

4.2.2 Curriculum Question 2.

Does the curriculum focus on important mathematics?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 3
Discussion: This principle is fully met. The topics the curriculum focusses on are very important. The lack of the probability strand in the middle school grades was seen as an unfortunate delay.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. All the standards are represented in this curriculum. The curriculum focusses on the understanding of the big ideas in these strands.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. All the standards are represented in this curriculum. The curriculum focusses on the understanding of the big ideas in these strands.

4.2.3 Curriculum Question 3.

Is the mathematics curriculum well-articulated across the grades (6-8)?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 1
Discussion: This principle is not adequately met. The *(6A)* and *(6B)* books were very well articulated and deserve a score of 3. The *(SL1)* and *(SL2)* books, on the other hand, make almost no attempt to help the teacher understand which math has been previously studied and which is to be studied in the future. The score of 1 is given, since the curriculum as a whole is not at the level 2 of being adequate.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. In the Teacher's Guide for each unit, articulation is given in a table called "Connections to Other Units." This table shows how the big ideas of the present unit were built from prior units, and how they are building to future units. More articulation is given to the teacher in the "Getting to Know Connected Mathematics" pamphlet. This gives the scope by strands, the recommended order by grade and a flowchart of the connections of the units in each grade.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. The articulation in Math-in-Context is very easy to follow. In each Teacher Guide, the Overview gives the Prior Knowledge expected for the unit. It gives the mathematical Goals for the unit, a graphical flowchart of the Pathways through the curriculum showing which units are pre-requisites, and a grade by grade brief description of the mathematics represented in the particular strand being studied.

4.3 The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks.

4.3.1 Teaching Question 1.

Does the curriculum help the teacher to deeply understand the mathematics he or she needs to teach the students?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 3
Discussion: This principle is fully met. The Teacher Guides for *(6A)* and *(6B)* work out selected problems for the teacher and give the teacher questions to ask the students about the math. The *(SL1)* and *(SL2)* books contain many worked out examples that the teacher as well as the students can see. Also, the Teacher's Resource Manuals work out a lot of the challenge problems for the teacher.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. In the first four or five pages of the Teacher's Guide for each unit, the "Mathematics in the Unit" explains the mathematics to the teacher. Helpful hints also appear in the "Teaching the Investigation." Answers to the problems often have worked out solutions in the Teacher's Guide.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. The teacher is given sufficient hints about the mathematics in the About the Mathematics column. Sample solutions and samples of student work also are included in the Teacher Guides, as are solutions to the student Try This! sections.

4.4 The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. A major goal of school mathematics is to create autonomous learners who when challenged with appropriate tasks are confident in their ability to tackle difficult problems, eager to figure things out on their own, are flexible in exploring mathematical ideas and trying alternative solution paths, and are willing to persevere.

4.4.1 Learning Question 1.

Does the curriculum promote learning with understanding?

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 0                1       2              3
No               Low     Medium      Very much so
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Singapore: Score: 2
Discussion: This principle is adequately met. Students practice the strategies given in the book. These strategies are valid ones, so students may develop understanding by doing many, many problems and see concepts for themselves, or they may become very sophisticated in carrying out the operations they are told to do. *(SL1)* and *(SL2)* contain an enormous number of problems on a given topic that are all very similar to each other. Furthermore, the curriculum's assessments rarely ask students to explain "why" in the problems they do. The emphasis in the assessments appears to be to

determine only what students do not understand. This, to us, does not adequately promote learning with understanding. On the other hand, the clear exposition in the student books gives many examples to explain the mathematics and how to apply the book's strategies. By carefully reading the book, we feel students would gain some understanding of the mathematics. (We note that we are not experts in how students actually gain understanding. For this reason, we have included our detailed thinking about how we arrived at the score above.)

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. The emphasis in this curriculum is that students should understand the conceptual development of the mathematics.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. The emphasis in this curriculum is that students should understand the conceptual development of the mathematics.

4.4.2 Learning Question 2.

Does the curriculum encourage students to be autonomous learners?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 0
Discussion: This principle is not met. Students are told the successful ways to think about mathematics. Perhaps they solve enough problems that they are confident in tackling new (and differently cast) problems, but the problems in the curriculum are very much the same on a given topic. They don't have to worry about figuring out what to use to solve a given problem–the strategy is given. They don't have to rely on themselves.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. Students are encouraged to take charge of their own learning. They choose appropriate strategies and tools. They explain, analyze, reflect on, and refine their mathematical work. They decide when using a calculator is appropriate. This mode of operation is intended to be carefully monitored by a thoughtful and well-qualified mathematics teacher.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. Students are encouraged to take charge of their own learning. They choose appropriate strategies and tools. They explain, analyze, reflect on, and refine their mathematical work. They decide when using a calculator is appropriate. This mode of operation is intended to be carefully monitored by a thoughtful and well-qualified mathematics teacher.

4.5 The Assessment Principle

Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions.

4.5.1 Assessment Question 1.

Does the curriculum include and encourage multiple kinds of assessments (e.g. performance, formative, summative, paper-pencil, observations, portfolios, journals, student interviews, projects)?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 0
Discussion: This principle is not met. Teacher Guides for *(6A)* and *(6B)* recommend students do work in cooperative groups, but the teacher is not given suggestions for what to look for during this work to be used in formative assessment. The only form of assessment is summative through the many end of the unit tests. *(SL1)* and *(SL2)* recommend the use of quick multiple choice tests to get a reading on what the students know. This is not true formative assessment, since the work that the students do (their thinking) is not analyzed to see what they *do* know as well as what they *do not* know. (The Singapore teachers may indeed do proper formative assessment in order to individualize instruction, but it is not evidenced by the curriculum.)

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. These kinds of assessments are made explicit in each unit of the curriculum. Checkups, Quizzes (A and B), Unit Tests, Question Banks, Self-Assessment Forms, Student Notebook Checklists, and Partner Quizzes are all included with the curriculum. Students keep Journals of their work.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. In the Overview for each unit, Ongoing Assessment Opportunities are explicitly given for each mathematical goal of the unit. Students also keep portfolios, do self-evaluation, and participate in summary discussions.

4.5.2 Assessment Question 2.

Does the curriculum provide well-aligned summative assessments to judge a student's attainment?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 3
Discussion: This principle is fully met. There are many Revision Tests and Unit Tests that do align with the work that students do as evidenced in the curriculum.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. Well-aligned summative assessments are given at the end of each unit for the teacher's use.

Mathematics in Context: Score: 3
Discussion: This principle is fully met. Well-aligned summative assessments appear in the Teacher's book for each unit.

4.6 The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving. Technology should not be used as a replacement for basic understandings and intuitions; rather it can and should be used to foster those understandings and intuitions.

4.6.1 Technology Question 1.

Does the curriculum use technology as a tool for learning and doing mathematics?

0	1	2	3
No	Low	Medium	Very much so

Singapore: Score: 2
Discussion: This principle is adequately met. There is no calculator usage up through *(6A)* and *(6B)*. The curriculum frequently calls for CDIS software to assist in some lessons in these grades. Students begin using the calculator in *(SL1)* with lots of calculator drills. CDIS software is referenced once in *(SL2)* to support instruction on solving quadratic equations. The score of 2 reflects that students in Primary 6 do not learn when it is appropriate to use calculators versus pencil and paper calculation.

Connected Mathematics Program: Score: 3
Discussion: This principle is fully met. CMP teaches students *when* and *how* to use calculators and graphical calculators. Students also use a LOGO program called Turtle

Math and its Shape, Grid, and Scale Tools to study geometry. The use of graphical calculators starts in 7th grade.

Mathematics in Context: Score: 2
Discussion: This principle is adequately met. Students use calculators. In the Letter to the Teacher at the beginning of each unit, Teacher Preparation Videos are mentioned that would help the teacher prepare the unit. We tried to order these and were told they were not made. Instead, there are Videos produced by Mathsphere that are recommended on the mathematical topic of the unit. We decided that since these Videos were optional and not part of the curriculum, not every school district would purchase them, and we should not consider them an integral part of the curriculum for review. Our conclusion is that calculators are probably the main technology that is used. This adequately meets the standard, since throughout the grades 6-8, students learn to decide *when* and *how* to use calculators.

4.7 Summary of Alignment with NCTM Principles

Question	Singapore	CMP	Math-in-Context
Equity 1.	1	3	3
Equity 2.	1	3	3
Equity 3.	1	3	3
Equity 4.	2	2	2
Curriculum 1.	1	3	1
Curriculum 2.	3	3	3
Curriculum 3.	1	3	3
Teaching 1.	3	3	3
Learning 1.	2	3	3
Learning 2.	0	3	3
Assessment 1.	0	3	3
Assessment 2.	3	3	3
Technology 1.	2	3	2

Table 11: Summary of Alignment with the NCTM Principles

Singapore:

The scores in the table above show that the curriculum fully meets three of the thirteen principles, adequately meets three of them, does not adequately meet five of them, and does not meet two of them. The lower scores were given because of the lack of coherence, articulation, multiple forms of assessment, use of calculators in *(6A)* and *(6B)*, sensitivity to gender issues, and sensitivity to underrepresented and underserved students in the curricular materials. One of the lowest scores was due to the curriculum's lack of attention to encouraging students to learn with understanding and to be autonomous learners.

CMP:

The scores in the table above show that the curriculum fully meets twelve of the thirteen principles and adequately meets one of them. The lower score was given because the materials will have to be supplemented to keep the interest of the more capable students.

MIC:

The scores in the table above show that the curriculum fully meets ten of the thirteen principles, adequately meets two of them, and does not adequately meet one of them. The lower scores were given because the connections between the mathematical strands is not made evident in the problems that students do, the materials will have to be supplemented to keep the interest of the more capable students, and calculators are the only technology that is frequently used in the curriculum. (In the teacher books, videos are listed that correspond to a given topic, but they appear to be optional.)

5 Direct Comparisons of the Curricula

In Chapters 3 and 4, we compared each curriculum with the 2000 NCTM Principles and Standards. That comparison was not intended to be a direct comparison of the three curricula with each other. The purpose of this chapter is to perform a direct comparison of each curriculum against each of the others, drawing upon the information we have gained from the previous chapters in conjunction with our joint expertise as mathematicians and applied mathematicians.

In the sections that follow, we compare the three curricula with each other in the areas covered by the ten overarching standards and principles. However, our discussion is not confined to those standards and principles.

5.1 Number Comparison

Ratio and Proportion:

All three curricula include good strategies for finding ratios and proportions. Singapore's strategy is called the unitary method. CMP calls it the rate table. MIC calls it the ratio table. In CMP, this topic, however, is only studied in 7th grade. MIC treats this topic as early as the *(5/6)* units and continues with it through the *(8/9)* units. Singapore's students work much more difficult problems on this topic in grades *6A* and *6B*, *SL1* and *SL2* than students using the American texts. Algebra is ultimately used to solve these problems in the Singapore texts.

The Commutative, Associative, and Distributive Laws:

CMP only introduces these important topics in 8th grade. MIC's students work with these laws in all grades *(5/6)* through *(8/9)*, but the terminology is not used in the student books. Singapore's texts, on the other hand, offer students many opportunities to work with these laws by name throughout the middle grades. Moreover, they work fluently with them. We note that the Everyday Mathematics elementary program (K-6) requires students to work with these laws in fifth grade and by name in sixth grade. In this respect, CMP aims too low.

Algorithms:

In CMP and MIC, students invent and analyze their own algorithms. The 2000 Standards say that students should compare their invented algorithms to the standard ones. We note that these standard methods can be taught with understanding and can be just as conceptual as a rate table, for instance, and have far greater utility for more complicated situations. In CMP, these standard methods will probably surface during the Summarize part of the lesson. MIC introduces the standard algorithm for multiplication of two digit integers in the *(8/9)* unit *Reflections on Number* – we feel this is much too late. Furthermore, this unit, though included in our study, is not included in Plan B, the books recommended by the publisher for a three-year middle school program. If introduced earlier and analyzed, students would have yet another conceptual way to organize and solve problems and another algorithm with which to compare their invented ones, as recommended in the standards.

The Singapore texts have a different philosophy. They present what are considered the best algorithms for the type of problem considered. Students then practice these algorithms in solving complicated problems. The *SL1* and *SL2* textbooks appear to be just books of problems. The philosophy is to provide practice, rather than opportunities to select, develop, and analyze.

Exponent Usage and Multiple Bases:

In MIC, the general case $a^b a^c = a^{(b+c)}, a \neq 0$ is only studied when the base $a = 10$. Other bases for a are used when b and c are both positive. CMP introduces students to functions of the form $y = a^x$ and $y = a^{-x}$ in the 8th grade, and uses graphing calculators to study the exponential growth and decay. On the other hand, the Singapore texts use exponents in a very general way. The laws for exponents are given explicitly in the student texts. The bases are general, the exponents are both positive and negative, and students work many problems using these laws.

On a related issue, we note that the 2000 NCTM Number Standard for grades 6-8 does not mention the use of multiple base arithmetic as a way to build up the concept of place value using exponents. This might account for the fact that MIC does not deal with powers of numbers other than 10. For example, 10^5 is related to a certain place value in the base 10 number system (100000) and 2^5 is related to the base 2 number (100000). None of the curricula we examined makes this important connection.

We also note that the Everyday Mathematics elementary curriculum works extensively with place value in grades 4 and 5.

Calculators:

CMP and MIC make use of calculators across the middle grades. Singapore makes it clear that there is no calculator usage allowed in *6A* and *6B*. Calculator usage begins in earnest in *SL1*.

Fraction, Decimals, and Percents:

CMP and MIC students do not work fluently with this topic. The calculations are on the whole too simple for these grades, especially those done toward the end of 8th grade or in the *(8/9)* books. Students are not working with general fractions to compare them by finding a common denominator. By the end of 8th grade, we feel this is a skill students should have. Instead, they use a calculator, which converts the fractions to an approximate decimal form.

CMP and MIC were designed to adhere to the 1989 NCTM Standards, which had very low standards with regard to fluency and skills involving fractions. The 2000 NCTM Standards now require a higher level of fluency and skills, which are not met by

CMP and MIC. We think that the discussion in the 2000 NCTM Standards may also lead curriculum developers to aim too low in the area of computation. In particular, an algorithm for division by a fraction discussed in the 2000 NCTM Principles and Standards on p. 217 as repeated subtraction is flawed (e.g. 5 divided by 3/4 as $5-3/4-3/4-3/4-3/4-3/4-3/4 = 1/2$. How should one divide 1/2 by 3/4 using repeated subtraction? How can one use repeated substraction when the denominator is bigger than the numerator?), especially without the subsequent generalization to (and proof of) the more general "invert and multiply" algorithm the students will need in algebra.

The problems the Singapore students do with this topic are much more complicated than those in the American texts. In fact, the Singapore *SL1* text tells the teachers to go quickly through these topics because they were covered in depth in lower grades. We note that *SL2* does not teach these topics at all. This makes sense for the Singapore students using these texts, since students have been streamed already. Even though the topics are more complicated, Singapore students are not expected to select, develop, or analyze the algorithms they use. They simply practice them.

We note that CMP and MIC teach students "when" it is appropriate to use a decimal versus a fraction or a percent. Singapore's texts do not do this.

5.2 Algebra Comparison

MIC includes algebra throughout the middle grades. CMP has no algebra in its 6th grade curriculum. Singapore's *6A* and *6B* include algebraic expressions, but not equations. Singapore's treatment of linear graphs starts earlier in *5A* and *5B* and doesn't continue until the 8th grade book *SL2*. If Singapore's texts were used in middle schools in America, the pre-requisite work expected by *SL2* might be missing. Coordination with elementary curriculum is extremely crucial and should be kept in mind before selecting any of the three curricula.

The Algebra level in CMP and MIC appear to be almost two grade levels lower than in the Singapore materials. Division of one polynomial by another or multiplying two polynomials of order higher than one is not taught even by the 8th grade in these American curricula.

5.3 Geometry Comparison

Creating and Critiquing Arguments Related to Geometry:

This topic is missing from 6th grade CMP, but is done later in the curriculum. CMP and MIC both develop the habits of mind to critique the arguments of others, especially in the classroom setting. Singapore's curriculum, especially *SL1* and *SL2*, give the impression that students are doing problems in isolation. The curriculum does not explicitly call for this mode of operation. However, Singapore's curriculum contains a fair number of geometry proofs based on deductive reasoning that are at a much higher mathematical level than those in the American curricula. Inductive reasoning is not found in the Singapore texts in relation to geometry.

Coordinate Geometry:

CMP and Singapore both met the standards related to coordinate geometry. CMP has this topic across all grade levels. Singapore starts it in *SL1* and picks it up again for more proficiency in *SL3*. The evidence in Chapter 3 shows that MIC is very deficient in coordinate geometry.

Pythagorean Relationship:

All three curricula address this relationship in the 8th grade books. The unit *(8/9) Going the Distance* is the only unit in MIC that addresses the Pythagorean relationship. This unit is not included in Plan B, the recommended books for a three year middle-grades program. It is surprising to us that this topic is not treated in much earlier grades in all three curricula.

Geometric Models to Explain Algebra:

Singapore starts at the 5th grade and continues to use these models through 8th grade. MIC also makes use of these models from units *(6/7)* up through *(8/9)*. CMP, on the other hand, doesn't use them until 8th grade.

5.4 Measurement Comparison

Both CMP and MIC make use of common benchmarks to develop measurement skills. The Singapore curriculum does not.

The Singapore curriculum includes problems that require students to go between two different scales in the same problem. Linear and area scales are also used. CMP and MIC also include good sections on scaling. Singapore includes more work on problems involving density than CMP. MIC does not include problems involving density.

All three curricula provide problems that require conversions between the customary and metric system, but CMP does very little with conversions within the same system.

5.5 Data and Probability Comparison

The three curricula differ widely in the amount of coverage of probability and statistics in the middle grades. MIC includes these topics in all of the middle grades. CMP does not include statistics in its 7th grade books, but does have statistics in its 6th and 8th grade books. Singapore does not include statistics in its 6th or 7th grade books. With the exception of data representation, which is done in the 4th grade books, and reading pie charts in the 6th grade books, statistics begins at 8th grade in the *SL2* books. Singapore has a total lack of probability in all of Primary school (1-6) and the books *SL1*, *SL2*, and *SL3*. Probability begins at 10th grade in *SL4*. We felt this delay to be unfortunate.

This issue would have to be seriously addressed if this curriculum were to be used in American classrooms.

5.6 Problem Solving Comparison

CMP and MIC are equally good at including problem solving in the curriculum. These curricula take the point of view that it is the student's job to learn how to select, adapt, and reflect on problem solving strategies. The Singapore curriculum does not require students to monitor and reflect on the process of problem solving or to adapt strategies. The philosophy seems to be that good strategies are given the students and a student's job is to apply these strategies to solve complicated, but mostly non-contextual problems.

5.7 Reasoning and Proof Comparison

CMP and MIC are both very strong in explicitly requiring students to explain their reasoning. Singapore's curriculum does not do this, especially in the *SL1* and *SL2* books. Moreover, Singapore students don't have to select strategies or make their own conjectures as often as students from CMP and MIC. We feel that MIC could provide more of a distinction between explaining one's thinking which can be flawed and providing an argument which can not be refuted.

5.8 Communication Comparison

The Singapore texts use the language of mathematics in a very rich way, but this usage doesn't carry over to the expectations for the students. They are rarely asked to explain their reasoning in writing, or to analyze or evaluate the mathematical thinking of others. The problems they work on and the assessments they do require very little writing using mathematical terminology.

MIC is the opposite of Singapore in this regard. The MIC student books avoid the precise language of mathematics. (Mathematical terminology and discussion of the general case are seen in the teacher book.) MIC students are, however, asked frequently to use written and oral communication to explain their reasoning and analyze or evaluate the mathematical thinking of others.

CMP uses the language of mathematics in the student books as well as the teacher books. CMP students are also asked frequently to use written and oral communication to explain their reasoning and analyze or evaluate the mathematical thinking of others.

5.9 Connection Comparison

A well-connected mathematics program includes problems that span multiple subject areas that incorporate different mathematical strands. The curriculum should provide the teacher with the explicit strands that are involved in the lesson. The CMP curriculum does this better than either MIC or Singapore. In CMP, such connections are made explicit for the teacher and the student. The MIC curriculum is very strong on the use of contextual problems, but not as good as CMP on making the connections explicit to the student. MIC makes the connections explicit to the teacher, but the problems the students do don't draw in these connections as well as those in CMP. We sense a philosophical difference between CMP and MIC in this regard. MIC gives the impression that the goal is to make everything available to the teacher, on the theory that if the teacher sees it, the students see it. CMP provides the connections explicitly in the ACE Problems (A is Application, C is Connection and E is Extension). Students see through the C (Connection) problems how the current topic connects back to previous work. The problems in MIC corresond to CMP's "A" and "E" problems.

Singapore's *6A* and *6B* books contain quite a lot of guidance to the teacher on how the math ideas interconnect. The *SL1* and *SL2* books present none of this. All of the Singapore books, however, only require the student to practice one strand of mathematics at a given time. The work that students do does not require the integration of different mathematical strands. Even though a mathematician can clearly see the logical progression of the mathematical ideas in these books, the connections of these ideas are not nearly as evident in the work students do as those in the CMP books.

On the other hand, the connection issue can also be interpreted as vertical connections in the internal structure of mathematics. In this regard, the exposition in Singapore's books follows a more logical progression of mathematical ideas, uses more precise mathematical language, and provides more abstraction and generalization than the exposition in the American texts. To mathematicians this logical vertical connection is very familiar and easy to appreciate, although it could be made more apparent in the problems that students do.

5.10 Representation Comparison

All three curricula are very strong in the use of representations. Singapore's students do not use representations as often in physical or social contextual problems as do the American students. They do, however, use many representations in the mathematical problems they do.

5.11 Principles Comparison

The main deficiencies of the Singapore curriculum include its lack of coherence, articulation, multiple forms of assessment, use of calculators in *(6A)* and *(6B)*, sensitivity to gender issues, and sensitivity to underrepresented and underserved students. The most serious single deficiency, however, is its lack of attention to encouraging students to be autonomous learners. The curriculum is based on the philosophy that practicing many,

many problems will lead to mathematical understanding and will produce autonomous learners.

Both CMP and MIC are more in line with the 2000 NCTM Principles than the Singapore curriculum. Both are well-articulated, provide multiple forms of assessments, use calculators throughout, are sensitive to gender issues and to underrepresented and underserved students. CMP provides problems with more coherence (connections) across mathematical strands and makes more use of technology than MIC. These American curricula take the point of view that it is the student's job to learn how to select, adapt, and reflect upon strategies for solving problems, and that by doing so, the student will develop mathematical understanding and become an autonomous learner.

Both CMP and MIC lack the mathematical level necessary to challenge the brightest students. These curricula will have to be supplemented in the number, algebra, and geometry strands in particular to meet these needs. Singapore's curriculum, on the other hand, has the mathematical level and rigor required for these students, but does not require them to rise to the level of being able to select or adapt strategies or reflect about them in writing.

6 Conclusions

In Chapters 3 and 4 we compared two American curricula, Connected Mathematics Program (CMP) and Mathematics in Context (MIC), and the Singapore material, at the middle school levels of 6th, 7th and 8th grades with the 2000 NCTM Principles and Standards. In Chapter 5 we compared these three curricula with each other. The two American curricula are actually quite similar to each other in philosophy and execution. Both were developed during the nineties to satisfy the 1989 National Council of Teachers of Mathematics (NCTM) standards, namely the *Curriculum and Evaluation Standards for School Mathematics*, the *Professional Standards for Teaching Mathematics*, and the *Assessment Standards for School Mathematics.* They represent the new thrust in American mathematical education of inquiry-based, discovery-based and student-centered learning. The American curricula strive to produce independent thinkers who can analyze problems, select appropriate tools to solve them and achieve conceptual understanding of the mathematics behind the algorithms, usually through a real-world context. Singapore mathematics, on the other hand, is more traditional in orientation and emphasizes the acquisition of proficiency in mathematical skills and teacher-directed learning. Although there are some signs that the latest Singapore curriculum is attempting to include more discovery-type tasks for students, the implementation is, at this stage, nothing more than a simple retrofit. The two American curricula, on the other hand, were designed from the beginning to reflect this new approach to learning; MIC and CMP appear to us to be much better conceived along these lines and have much better teacher support for implementing the lessons than the Singapore curriculum, especially at the the secondary grades (7th and 8th). When compared with the 2000 NCTM Principles and Standards, the Singapore curriculum scores lower than the two American texts.

This result is not surprising. The Singapore texts were designed for Singapore students to prepare them for the General Certificate of Education (GCE) examinations. The students have already been divided into different streams when they come to middle school. Therefore the curriculum targets a very specific audience at very specific student levels. The Singapore teachers are educated more uniformly at their normal school, and their continuing professional development is supported by the government at a level not achieved in the United States. Consequently, the lack of teacher support in the textbooks themselves does not appear to pose a problem for the Singapore teachers. It is interesting to point out that at the primary grades, the Singapore texts provide excellent teacher support, probably because it is thought to be necessary at these lower levels where the teachers are not mathematics or science specialists. The Singapore mathematics curriculum appears to do what it was designed to do quite well for the audience intended. The mathematical problems are more advanced than those of the two American curricula, and use the language of mathematics in a rich way. Mathematicians may find it attractive because it lays a logical foundation for the college mathematics with which they are familiar. It may be useful as supplemental and enrichment material for teaching mathematics in American schools and at home, but for the reasons outlined above, and because the Singapore secondary curriculum does not mesh well with

American elementary grade material, is probably not suitable as the source of principal texts.

As mathematicians and applied mathematicians, we feel that a major shortcoming of the two American curricula, ironically, is that they adhered to the 1989 NCTM Curriculum Standards too literally at the expense of the level of the mathematics taught and the mathematical proficiency expected of the students. Responding to criticisms of its earlier Standards, the 2000 NCTM Principles and Standards now raises the bar with respect to arithmetic and computational skills. The Standards now state that sixth through eighth grade students should "develop and analyze algorithms for computing with fractions, decimals and integers, and develop fluency in their use." CMP and MIC do not meet these new standards in the number strand, one of the most fundamental subjects in mathematics. For example, division of fractions is not discussed at all even through 8th grade in CMP and only discussed at the level of "how many 1 1/4's are there in 8 1/2" in MIC. This shortcoming has been a source of criticism from both mathematicians and parents. The developers of the curricula are aware of it, and will probably remedy this deficiency in the future editions. We also expect that in the next edition some of the typographical errors, on which much has been written and which are usually interpreted by critics of these curricula as mathematically incorrect reasoning, will also be corrected. (e.g. Complaints have been raised about "Find the slope and y-intercept for the equation $10 = x - 2.5$", CMP 7th grade *Moving Straight Ahead*, where y was mistakenly printed as 10.)

The impression we gained from comparing the American curricula with the NCTM Principles and Standards is that the curriculum developers take the standards seriously and consequently perhaps adhere to them too literally. Therefore it is incumbent upon the community of mathematicians and mathematics educators to be explicit in providing guidance on the NCTM Principles and Standards. We noted some shortcomings in this respect in the 2000 NCTM Standards which may have implications for the new curriculum revisions from the publishers. To use an example discussed earlier, the 2000 NCTM Standards set a higher standard in the number strand as compared to the 1989 version by requiring "fluency" in computing with fractions. Yet in the guidance following that statement, it appears to suggest that division should be done by repeated subtraction (such as in the cutting ribbon example), which is a flawed algorithm in our opinion and not generalizable to all fractions. Although such a conceptual understanding could be taught as one of the many ways for understanding the meaning of division of fractions, there is the danger that the curriculum developers may interpret this guidance as the definition for "fluency" required and stop at that level. To us, doing division by repeatedly cutting off pieces of a ribbon does not remotely demonstrate "fluency."

The level of the mathematics in both CMP and MIC is not as advanced as that in the Singapore curriculum (with the exception of probability, which is delayed until the 10th grade in Singapore). Some of the mathematics in CMP and MIC has already been covered in Everyday Mathematics, an exemplary elementary school curriculum with which we are familiar. It is also our prediction that students wishing to take calculus before the end of their 12th grade year are likely not to be on track to do so after completing 8th grade CMP or MIC, but would be ready to do so after completing Singapore's *SL2*. We are not advocating that calculus in high school should be a goal for all students, but if this is the desired goal for certain students, the proper supplementation of CMP and MIC at an accelerated pace cannot be ignored. Moreover, we are skeptical about the possibility of maintaining the interest of high-end students while progressing at the pace necessitated by the discovery process, if care is not taken to individualize these discoveries for the students. In schools where student achievement is higher, it may be beneficial to use 8th grade texts for 7th grade students and to use 7th grade texts for the 6th grade students, as is already being done in many schools across the country.

CMP and MIC meet the 2000 NCTM algebra standard, although the mathematical level is much lower than that covered in the Singapore texts. Generalization and abstraction of concepts discovered and learned, which could easily have been included in the curricula, are mostly absent in the American texts. It appears that this may be done deliberately in the authors' attempt to offer easily visualizable problems and concrete examples. For example, CMP's 8th grade algebra text has an excellent introduction to the distributive law of multiplication using the example of calculating the area of a rectangle with two parts. It can be calculated either by first finding the area of each part and then adding them, or by adding the two lengths and two widths and then multiplying the sums to get the total area. By seeing "in context" that the two ways of calculating area are equivalent, a student can then discover the distributive law, without being told that it is true. However, multiplying polynomials that are higher than first order is not covered in the entire curriculum. This could be because it is difficult to come up with a context for multiplying an area by an area, or it could be the result of a decision that the topic is non-essential to a middle school student since it is not explicitly called for by the NCTM Standards. In either case, it is an omission which requires attention for students who wish to be on an accelerated track in high school. Similarly, the division of a polynomial by another polynomial of lower order is not covered, probably because it would have required conceptual understanding of long division at a level not covered by the curricula, which encourages the use of calculators for these problems.

It is not clear to us that a curriculum is the major component of mathematical success for students. The successful implementation of any curriculum depends on a mathematically proficient teacher. As we said in our Introduction, we have avoided the issue of implementation in our comparison studies in Chapters 3 and 4. Instead we have compared the curricula to the 2000 NCTM Principles and Standards under the assumption that ideal conditions exist for their implementation. Many of the criticisms of the new curricula we hear expressed by parents are probably related to poor implementation. It is our judgment that CMP and MIC are more difficult to implement well than a traditional curriculum such as that of Singapore. ("Let us not forget that training *by rote* is a less dangerous weapon in the hands of a teacher of limited mathematical preparation and understanding than are attempts to foster understanding in others that one does not have oneself." Herbert Clemens, 2000, *Notices of the American Mathematical Society*, vol. 74, p. 1074). CMP and MIC also present difficulties to parents who want to help their children, or even to keep track of where they are mathematically. Many familiar landmarks have changed in character or disappeared altogether. A mathematician looking at the materials may initially be frustrated by the fact that the unit titles in CMP, clearly chosen with the intention of making them attractive to middle schoolers, give absolutely no direct information about whether the content is measurement, algebra, geometry, trigonometry, statistics, probability, etc., though the subtitle for each unit is more informative. Similar frustration can result from the fact that in MIC information such as a glossary and index is hidden from the students and parents.

In both MIC and CMP, the lessons are rooted in discovery-based learning. However, it is a truism that teaching in the "Socratic mode" requires more knowledge on the part of the instructor than traditional teaching in the "lecture mode." Since these curricula stress discovery of mathematics by the students, requisite for the success of the curricula is mathematical proficiency on the part of the teachers. In particular, leading students toward mathematical knowledge, without just feeding it to them, requires the teacher to have a broad idea of where the books are leading the students, and of what mathematics precedes and succeeds the current lesson. While each of the teachers' manuals contains an overview that attempts to lay out this very progression, this material is likely only to be useful to teachers with a firm grasp of the underlying mathematics. Teachers will have to judge the mathematical arguments of their students, which will often be novel and unique; they will need to discern the value of algorithms students develop and whether they are sufficiently general to encourage the students to refine them further. This level of mathematical expertise will have to be gained by professional development or with the aid of a mathematical specialist at the school. Alternatively, as some countries have done, we may need to have specialists take over the teaching of mathematics, especially when attempting to implement learning in the "discovery mode." In addition, while a discovery-based format can allow children with different learning styles to engage the material in the manner most useful for them, the teacher must herself be comfortable with the resulting diversity of activity in her classroom and with aiding children who have different needs. Thus we feel that success with these new curricula will require American education schools to educate future teachers with an increased mathematical understanding.

A related comment is that discovery-based learning naturally takes more time than the traditional lecture-then-practice format. If students are to be provided with the chance to experiment, to create, and to test their own hypotheses (some of which will be dead-ends), and to explain their results and their reasoning, more time will have to be allotted to mathematics in the middle grades. This is something which any school district implementing CMP or MIC should take into account.

Finally, we return to the questions raised by the TIMSS study alluded to in our Introduction. Is the Singapore curriculum in mathematics responsible for its students' top ranking in the TIMSS tests? Is the abysmal performance of American middle and high school students in the TIMSS tests attributable to flaws in the American mathematics curricula? Would American students' performances move to the top if only we adopted the Singapore curriculum here? Of course, if the TIMSS tested rote memory, recall of facts, and manipulation skills, Singapore students would have an edge over American students; their curriculum emphasizes practice problems and makes sure that the students attain fluency and computational skills at a level which we judge to be one to two grades higher than their American counterparts. However, the TIMSS tests were designed to test understanding of concepts in addition to competency. The inferences that can be made thus become more murky. On one hand, we must acknowledge that Singapore's educational system – the curriculum, the teachers, the parental support, the social culture, and the strong government support of education – has succeeded in producing students who as a whole understand mathematics at a higher level, and perform with more competence and fluency, than the American students who took the tests. Simply adopting the middle-grades Singapore curriculum is not likely to help American students move to the top, not to mention the fact that the middle-grades Singapore curriculum cannot be adopted without also adopting the elementary Singapore curriculum, and that the teachers at the secondary grades have to receive more training in the subject matter. On the other hand, we doubt very much that any tests, no matter how well designed, can accurately test creativity and independent thinking, qualities which the new American curricula, such as CMP and MIC, strive to foster. Nevertheless, given the fact that the students who took the TIMSS tests probably did not have the benefit of the new curricula, their exceedingly low scores may simply mean that the American students were less well educated in mathematics and science at the time than their Singapore counterparts. The new curricula, and their expected revised versions, may change that. We have found much to like in the new American curricula, especially their emphasis on conceptual understanding and on educating independent thinkers, qualities we value in our society. Can future tests be devised to test gains in these areas? We will watch with great interest the efforts to answer this question.