

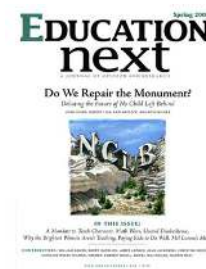
# An A-Maze-ing Approach To Math

*A mathematician with a child learns some politics*

By **BARRY GARELICK**

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I am not a mathematics teacher, but I have a degree in mathematics and an intense interest in how the subject is taught. When I retire, I would like to teach math, which is why I started tutoring high school students in my spare time three years ago. My first student was a 9th grader having difficulty with geometry. He stated his problem succinctly: “I don’t know how to do proofs.” Confronted with what I thought could be a common problem, I was still unaware that what I was really seeing was a national crisis in mathematics education.

Here’s some of what I would soon learn:

–Only 55 percent of 8th graders taking the National Assessment of Educational Progress (NAEP) exam in math correctly answered the question, “How many pieces of string will you have if you divide  $3/4$  yard of string into pieces each  $1/8$  yard long?”

–In an international math test taken by students worldwide in 1995 (the Third International Mathematics and Science Study, or TIMSS), U.S. student math proficiency for 8th graders fell below the international average (28th out of 41 countries). For 12th grade, U.S. math performance was among the lowest (18th of the 21 countries participating).

–In 1989 the National Council of Teachers of Mathematics (NCTM) published its *Curriculum and Evaluation Standards for School Mathematics*—an extensive set of mathematics standards for grades K-12 which de-emphasized memorization of number facts, the learning of proofs, and algebraic skills, but encouraged the use of calculators and “discovery learning.”

–The National Science Foundation (NSF) promoted the NCTM standards beginning in 1991 and awarded millions of dollars in grant money for the writing of math texts that embraced them and to state boards of education whose math standards aligned with them (see Figure 1).

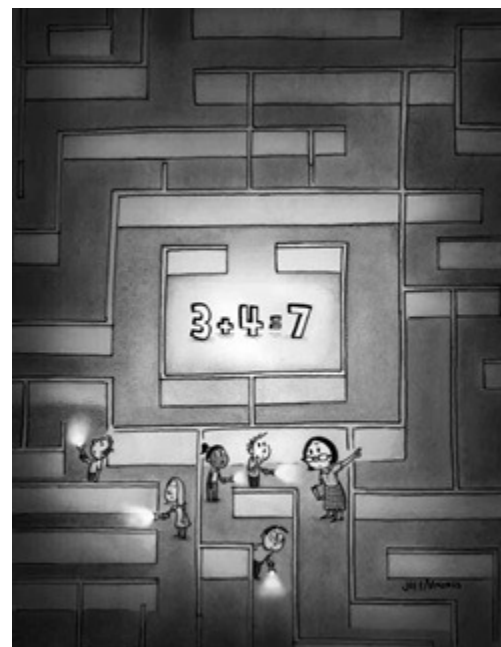


ILLUSTRATION BY JEFF MOORES

## Textbook Taxes (Figure 1)

Since 1990, the National Science Foundation has awarded more than \$83 million to programs that developed textbooks promoting approaches that are favored by the National Council of Teachers of Mathematics.

Organization	Program name	Grant amount (in millions of dollars)
University of Illinois at Chicago	Math Trailblazer	6.4

TERC, Inc.	Investigations in Number, Data and Space	11.4
University of Chicago	Everyday Mathematics	5.4
Education Development Center	MathScape	4.3
Michigan State University	Connected Mathematics Project	4.9
University of Wisconsin at Madison	Mathematics in Context	6.2
University of Montana	MATH Thematics	5.7
Institute for Research on Learning	Middle School Mathematics through Application Project	2.8
Connecticut Business and Industry Association Education Foundation	Math Connections (Secondary Mathematics Core Curriculum Initiative)	4.1
Western Michigan University	Contemporary Mathematics in Context (Core Plus)	13.9
San Francisco State University	Interactive Mathematics Program	11.6
Consortium for Mathematics and Its Applications, Inc.	Mathematics: Modeling Our World (ARISE)	6.1
Montana Council of Teachers of Mathematics	Systemic Initiative for Montana Mathematics and Science	1.0

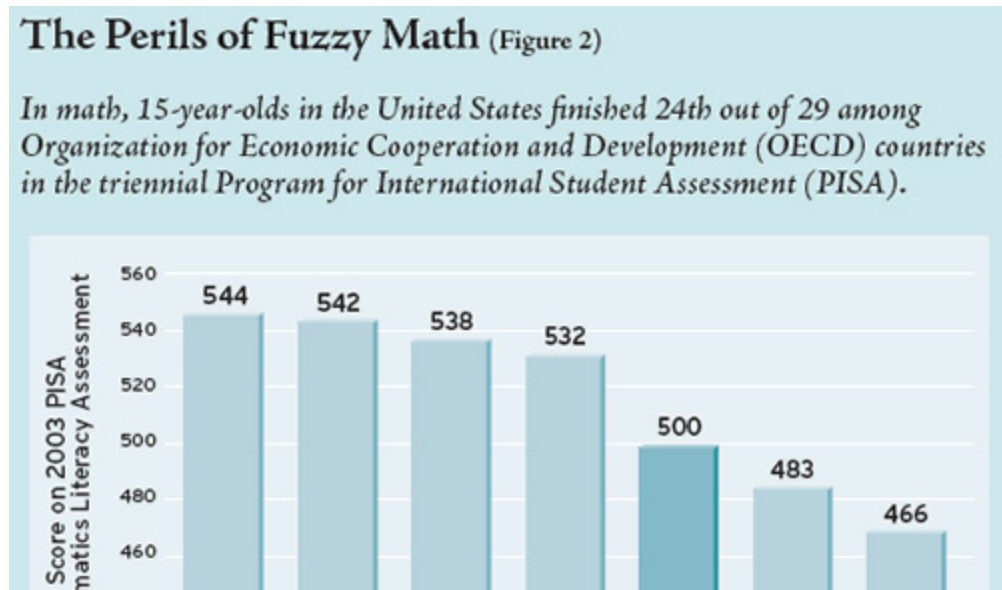
SOURCE: National Science Foundation

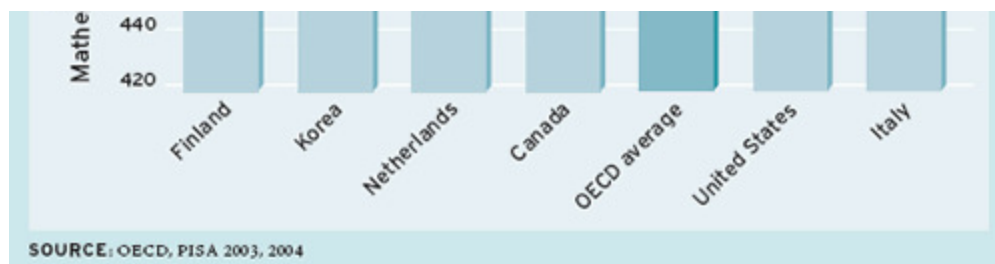
Not knowing about these significant events (all of them before last December's release of the Program for International Student Assessment report ranking American 15-year-olds 24th out of 29 in math among industrial countries (see Figure 2), I had no idea that our children were being deprived of a math education, thanks in no small part to a dubious education theory, watered-down standards, and a well-meaning but intellectually bankrupt federally subsidized program of math illiteracy.

What I did know was that my 9th grader didn't know how to do proofs.

I looked through his textbook, one of whose authors was a recent president of NCTM, and I was surprised to find very few proofs of anything. More troubling, most theorems in the book were stated as postulates—that is, propositions stated without proof—and students were told to memorize them. The problems at the end of the chapter required students to do only a few simple proofs.

Proofs in geometry class have been a mainstay of mathematics. In fact, proofs were always considered an essential part of high school geometry, not only because of their importance in higher math, but because learning the rules of logical argument and reasoning has applications in science, law, political science, and writing. To see proofs being shortchanged in a geometry textbook was shocking.





Algebra texts were in no better condition, in terms of presentation and content—or, rather, their lack of content. Even if you accept the argument that geometry in general, and proofs in particular, are unnecessary for students to learn, at least algebra should be taught properly, since algebra is the common language of, and gateway to, all of higher math. The absence of clear explanation and logical development left students I later tutored in algebra as lost as my geometry student. Their textbooks (and, probably, their teachers too) encouraged them to use a graphing calculator. Operations with algebraic fractions, like  $a/b + c/d$ , were given little attention, to say nothing of quadratic equations, once the pinnacle of any first-year algebra course. Instead, the quadratic formula is presented for the students to memorize and apply—if it is even mentioned at all.

At the time I started tutoring, my daughter was in 2nd grade. I was concerned that she was not learning her addition and subtraction facts. Other parents we knew were saying the same thing. Teachers told them not to worry because kids eventually “get it.”

One teacher told me her understanding of the new method. “It used to be that if you missed a concept or method in math, then you were lost for the rest of the year. But the way we do it now, kids have a lot of ways to do things, like adding and subtracting, so that math topics from day to day aren’t dependent on kids’ mastering a previous lesson.”

In a world where it doesn’t matter when you learn something, because you’ll get it eventually, there seem to be few if any critical junctures, no mastery of procedure, no building on what you’ve learned—no learning.

### **The Hell-Hath-No-Fury Postulate**

Coincidentally, I had the opportunity to find out how my experiences related to the politics of math education. I work for the federal government, which has a program that gives employees a chance to work on Capitol Hill to gain experience and knowledge of legislative and congressional procedures, which is valuable information when working in government. I applied for and received a six-month detail to work in a Democratic senator’s office. Senator X (so called, in keeping with mathematical convention to describe a class of variables, because, as I was also to learn, both the good intentions and the shortcomings of Congress are institutional) was interested in establishing a science project to nurture a “homegrown” breed of scientists and engineers who would then support that state’s burgeoning technology industry. Since I thought a likely place to start would be math education, the staffers working the education issue asked me to see what I could come up with.

I compiled a list of questions that I sent by e-mail to various mathematicians involved with the math education issue. The questions focused on the quality of textbooks and teaching, with emphasis on algebra and geometry. I also wanted to know whether K-6 texts taught arithmetic well enough to prepare students to learn algebra.

The nice thing about working on the Hill is that you almost always get responses to e-mails and phone calls. Fifteen minutes after I sent an e-mail to Harvard mathematics professor Wilfried Schmid, he called. I found out that his initiation into the world of K-12 math education was similar to mine—through his daughter. He explained how she was not being taught her multiplication tables. He was shocked at the math instruction she was receiving in the 3rd grade. Its substance was shallow, memorization was discouraged, students were kept dependent on mental crutches (her teacher made her work with blocks or count on her fingers), and the intellectual level was well below the capability of most of the kids in his daughter’s class.

Schmid’s reaction to the problems of math texts and teaching was similar to that of other mathematicians I talked to in the course of my Capitol Hill assignment, particularly those with children. Those dialogues led me to develop an ad hoc theory that I will postulate (this means I don’t have to prove it): Hell hath no fury like a mathematician

whose child has been scorned by an education system that refuses to know better. In Schmid's case, he talked to parents, school boards, and ultimately with the Massachusetts commissioner of education. Along with others, he succeeded in revamping Massachusetts's math standards, much to the dislike of the education establishment and textbook publishers.

## Framing the Debate

The controversy over K-12 math education has come to be known as the "math wars." Like Schmid, mathematicians have been active in this debate, as has the "mathematics community" at large, including not only mathematicians at the university level, but teachers and others involved in the education establishment. They believe that students must master basic skills (the number facts, standard algorithms for adding, subtracting, multiplying, and dividing) in tandem with larger concepts about mathematics.

On the other side of the debate are the followers of an education theory that promotes discovery learning, minimization of both teacher instruction and repetitive drills, and a disdain for standard procedures (algorithms). The math being protested—by the mathematics community—is called a variety of things: "reform math," "standards-based math," "new new math," and, most commonly, "fuzzy math."

Although the education theories on which much of fuzzy math is based are promoted in many education schools in the United States, it is not accurate to say that everyone in the education arena buys into these ideas. Therefore, for purposes of clarity, and to be consistent with a vocabulary used by others describing the math wars, I will use the term "educationist" to refer to those who promote the contested theory of math education known generally as discovery learning.

## Early Skirmishes

The math wars revolve around a four-part problem: A disputed theory of education that informs NCTM's standards; state boards of education that base their standards of learning for mathematics on the NCTM standards; textbooks written to incorporate these standards; and teachers and others in the education establishment who are indoctrinated in the disputed education theory and who may not possess enough knowledge of mathematics to overcome the first three factors.

The education theory at the heart of the dispute can be traced to John Dewey, an early proponent of learning through discovery. But for all practical purposes, the story begins on October 4, 1957, when the Soviet Union launched Sputnik. This event signaled the American malaise in science and mathematics education and the need to overcome it if we weren't to fall further behind the Soviet Union and lose the cold war. Thus the U.S. Congress was motivated to pass appropriations that triggered the development of the "new math," a national effort, spurred by the National Science Foundation, that eventually found its way into most of our schools.

The curriculum, designed primarily by mathematicians, had problems, but it introduced to algebra, geometry, and trigonometry a long-missing formalism, logic, and consistency, and it resulted in calculus being taught in high school. The problem, however, was that a similar formalism was introduced into K-6 texts and curricula, with the result that students (and elementary school teachers, who were caught totally off guard) were exposed to number bases, set theory, and axioms long before they were ready for them. And soon enough the new math was being blamed for not teaching basic arithmetic: it was often said that new-math students could tell you that  $5 + 3 = 3 + 5$ , but didn't know that it was equal to 8.

Mathematicians have agreed for years that emphasizing sets and number bases in math programs designed for the lower grades was a horrendous mistake. Notwithstanding these errors, however, the difference between the current slew of textbooks and those from the new-math days of the 1960s is definitely worth noting: Accomplished mathematicians wrote many of the texts used in that earlier era, and the math—though misguided and inappropriate for the lower grades and too formal for the high school grades—was at least mathematically correct. Some of the high school texts were absolutely first-rate, and new-math-era textbooks like Mary Dolciani's "Structure and Method" series for algebra and geometry continue to be used by math teachers who understand mathematics and how it is to be taught. (They usually use them on the sly, since most teachers are required to use

the books that the schools have adopted.)

During the new-math era, which spanned the period between Sputnik to the early 1970s, mathematicians dominated the design of math texts and curricula for the first, and almost last, time. Up to that point mathematicians had been kept out of the math education picture, and K-12 mathematics tended not to include any examination of the logical structure of mathematics itself, with the single exception of Euclidean geometry. Students in the first half of the 20th century had instruction in practical matters: consumer buying, insurance, taxation—everything but algebra, geometry, or trigonometry, which, when they were taught, were frequently lacking in depth.

Significantly, the new-math era was one of the only times that mathematicians were given an opportunity to make proper math education available to the masses. (Not until the past few years, working with several state education departments, would they be allowed back into math education decisions.) And some believe that had certain prominent mathematicians who had started working with the development of the new-math programs managed to maintain their influence on those programs, the math education that would have emerged from new math—both lower grades and high school—would have been on par with the best of the math programs overseas.

Eventually, however, the problems with K-6 formalism and the logic and formalism of the program in general doomed new math. The general public, the education community, and even mathematicians themselves judged the new-math programs a failure. Mathematicians were assigned the blame, and the education establishment took back the reins. That establishment received an inadvertent boost in 1983 with the publication of *A Nation at Risk*, the shockingly pessimistic assessment of the nation's schools by the National Commission on Excellence in Education. The report sounded another alarm about student math performance, and the NCTM, increasingly dominated by educationists, took advantage of this new education crisis to write revised math standards. *The Curriculum and Evaluation Standards for School Mathematics*, published in 1989, purported to put the country back on the math track. But because it was, in part, a reaction to the new math and those believed responsible for it, NCTM did not, as mathematicians point out, promote a lively public debate, as had the creators of the new math, but suppressed it.

### **Some Secrets about Discovery Learning**

The NCTM standards were a brew of progressivism—a nod to the 1920s when math was supposed to be practical—and constructivism, which was progressivism that adapted research from cognitive psychology to the task of teaching and called it discovery learning. The standards were based on theories of learning that assumed that children had an innate ability to understand math. The group's math curricula were thus structured to allow children to *discover* math concepts rather than to be *given* them, through direct instruction. The standards also expanded their reach to include, in addition to basic arithmetic, algebra, geometry, and trigonometry.

The NCTM's view was that traditional teaching techniques, known as “drill and kill,” numbed student minds, turned them off math, and taught them nothing. And so the new standards recommended that students learn “strategies” for learning number facts rather than memorize those facts. It emphasized the use of calculators in all grades. Most important, however, the standards recommended certain areas that should receive “decreased attention” in grades K-4, including “complex paper-and-pencil computations,” “long division,” “paper-and-pencil fraction computation,” “use of rounding to estimate,” “rote practice,” “rote memorization of rules,” and “teaching by telling.” This last item, teaching by telling, is a reference to direct instruction (telling students what they need to know), which NCTM believed should be replaced by “discovery.”

Discovery learning has always been a powerful teaching tool. But constructivists take it a step beyond mere tool, believing that only knowledge that one discovers for oneself is truly learned. There is little argument that learning is ultimately a discovery. Traditionalists also believe that information transfer via direct instruction is necessary, so constructivism taken to extremes can result in students' not knowing what they have discovered, not knowing how to apply it, or, in the worst case, discovering—and taking ownership of—the wrong answer. Additionally, by working in groups and talking with other students (which is promoted by the educationists), one student may indeed discover something, while the others come along for the ride.

Texts that are based on NCTM's standards focus on concepts and problem solving, but provide a minimum of exercises to build the skills necessary to understand concepts or solve the problems. Thus students are presented with real-life problems in the belief that they will learn what is needed to solve them. While adherents believe that such an approach teaches "mathematical thinking" rather than dull routine skills, some mathematicians have likened it to teaching someone to play water polo without first teaching him to swim.

The *Standards* were revised in 2000, due in large part to the complaints and criticisms expressed about them. Mathematicians felt that the revised standards, called *The Principles and Standards for School Mathematics (PSSM 2000)*, were an improvement over the 1989 version, but they had reservations. The revised standards still emphasize learning strategies over mathematical facts, for example, and discovery over drill and kill.

Concept still trumps memorization. Textbooks often make sure students understand what multiplication means rather than offering exercises for learning multiplication facts. Some texts ask students to write down the addition that a problem like  $4 \times 3$  represents. Most students do not have a difficult time understanding what multiplication means. But the necessity of memorizing the facts is still there. Rather than drill the facts, the texts have the students drill the concepts, and the student misses out on the basics of what she must ultimately know in order to do the problems. I've seen 4th and 5th graders, when stumped by a multiplication fact such as  $8 \times 7$ , actually sum up 8, 7 times. Constructivists would likely point to a student's going back to first principles as an indication that the student truly understood the concept. Mathematicians tend to see that as a waste of time.

Another case in point was illustrated in an article that appeared last fall in the *New York Times*. It described a 4th-grade class in Ossining, New York, that used a constructivist approach to teaching math and spent one entire class period circling the even numbers on a sheet containing the numbers 1 to 100. When a boy who had transferred from a Catholic school told the teacher that he knew his multiplication tables, she quizzed him by asking him what  $23 \times 16$  equaled. Using the old-fashioned method—one that is held in disdain because it uses rote memorization and is not discovered by the student—the boy delivered the correct answer. He knew how to multiply while the rest of the class was still discovering what multiples of 2 were.

### **Enter Stage Left: The National Science Foundation**

The NCTM standards received a boost of credibility in 1991 when the Education and Human Resources Division of the National Science Foundation funded two grant programs related to math education. The first was for state education departments that aligned their math standards with NCTM's and school districts that adopted constructivist math programs aligned with NCTM's standards; the second, for the development of commercial mathematics texts that also followed the NCTM party line, a spending program that now took NSF into the textbook business.

Eventually, NSF supplemented grants to school districts by funding "distribution centers" that promoted the very programs NSF had helped to create. This "Invasion of the Body Snatchers" technique has worked well in enhancing the NCTM constructivist ethic and the texts that NSF helped develop. Meanwhile, NSF continues to fund revisions to some of those texts.

The imprimatur of the National Science Foundation on both NCTM's standards and the various textbooks they funded and NSF's grant program caused states to revise their standards of learning for mathematics. Case in point: In 1992, the California State Board of Education adopted the California Mathematics Framework—a set of standards based prominently on NCTM's standards of 1989.

### **And Stage Right: the Right**

By this time, people were beginning to notice. NSF's embrace of NCTM's philosophy of education, among other problems with the teaching of mathematics, became grist for Lynne Cheney, then an active senior fellow at the American Enterprise Institute. With well-articulated essays in leading media, Cheney took out after the educationists and won the respect of mathematicians and scientists as she helped raise awareness among a wider audience across the United States. Isolated math revolts began to occur. One of the first was in Silicon Valley, where parents are engineers, scientists, programmers, and mathematicians—and they didn't like the way their children

were being taught mathematics. Two local mathematicians, Jim Milgram of Stanford and Hung-Hsi Wu of Berkeley, became key players in the rewriting of California's math standards and the elimination of NSF-funded books from the state's curriculum. Several years later, as I mentioned earlier, Wilfried Schmid from Harvard became active and helped rewrite Massachusetts's standards. Two other states, Minnesota and Michigan (Milgram and Wu went to the Wolverine State as well), also just recently revised their math standards. Minnesota's standards were changed due in large part to the efforts of Dr. Larry Gray, chair of the mathematics department at the University of Minnesota, and of concerned and outraged parents.

But the education bureaucracy did not roll over, and in the fall of 1999 the U.S. Department of Education released a list of ten recommended math programs, designated as "exemplary" or "promising," all of them aligned with the NCTM standards and based on texts funded by NSF.

The reaction was swift. More than two hundred university professors—including Milgram and Wu, Schmid, and several winners of the Fields Medal, the highest international award in mathematics—wrote an open letter to Secretary of Education Richard Riley, calling on the Department of Education to withdraw the recommendations. The open letter was also published as a full-page ad in the *Washington Post*, paid for by the Packard Humanities Institute, long a critic of constructivist education. The Department of Education did not withdraw the recommendations, but instead added two more NCTM-aligned books to the list.

The Education Department's 1999 list, as it represented a more visible—and seemingly overtly political—assertion of political will, was a critical point in the new-math wars. And when Lynne Cheney slammed the Clinton administration as she criticized math textbooks and the NSF in her editorials, she helped ensure that the math wars would become (as they remain) partisan. Republicans tended to be sympathetic to the issues, and some hearings were held. But nothing came of them, and no investigations into NSF and its funding practices were launched. No questions were asked about why the Department of Education didn't rely on mathematicians in the review of proposals for these programs, nor was anyone in the department ever questioned about the NCTM's education philosophy and the millions of tax dollars spent on texts that were the subject of fierce objections from 200 prominent mathematicians and scholars.

### **An Evidentiary Interlude**

In the various arguments about how best to teach math, educationists make the point that research shows that their approaches work best. I tend to be suspicious of that research, and apparently others are as well. Ironically, a recent study by the National Academy of Science's Mathematical Sciences Education Board, sponsored by the NSF itself, was also skeptical. "Evaluations of mathematics curricula provide important information for educators, parents, students and curriculum developers," concluded the NAS about 19 specific mathematics curricula, including all 13 NSF-sponsored curricula, that it analyzed. They all "fall short of the scientific standards necessary to gauge overall effectiveness."

While the proponents of these NSF-sponsored math programs may be able to claim that the research shows no evidence that the programs are "ineffective," the mathematics community, and parents who are protesting to the various school boards across the United States, can now claim that the research cannot be used to support claims of superior effectiveness—or any effectiveness at all.

Is there a common ground in this war? It seems that for now both sides agree that subject matter should be as important as pedagogy and that improving the math education of teachers is probably the most important weapon in the battle. What constitutes a proper math education for prospective teachers, however, is subject to debate. A mathematician I know who teaches at a small college told me that a graduate of his education school caused much embarrassment when, during the interview for a job in an elementary school, the job seeker was unable to add two fractions when asked to do so. The college's education department subsequently decided to put its math content courses under control of the math department.

### **Meanwhile, Back on the Hill**

Though academic debate about mathematics curricula will no doubt continue, the field of argument is increasingly muddied by politics. It was in this context that I began my investigation into math education in 2002. I recall

meeting with Senator X’s deputy chief of staff and two other staffers not long after completing my research on math curricula and the battles that had shaped—often, misshaped—them. “So what are your ideas on how math and science education can be enhanced?” they asked. My answer was something like, “You can enhance a car by painting it, but if the car has no engine, it’s not going to do much good.” This was not what they were expecting to hear. Nor were they expecting to hear that Lynne Cheney had also taken up the cause of anti-fuzzy math. At that point, the discussion took a decidedly troubling turn. These staffers—Democrats—now worried that they could not support policies that were also advocated by the wife of a powerful Republican.

I told them about the open letter from the two hundred mathematicians and urged them not to confuse the message with the messenger. “This is a real issue,” I said. “Kids aren’t learning the math they need to learn.”

I had discussions and sent e-mails in the hopes that I would at least get a chance to brief Senator X on the issue and, perhaps, persuade him to ask some tough questions of NSF when it came time to fund their programs. But I felt that at any moment everything was going to be whisked away.

And one day it was. The staffers in my office talked with other Democratic staffers on the Hill, who told them that it would be wise to stay away from the “fuzzy math/Lynne Cheney/Bush agenda” issue. Ultimately the staffers I was working with told me they couldn’t take a chance on having Senator X “come off like Lynne Cheney.”

This development was not surprising to any of the mathematicians with whom I had been working—most of them Democrats, like me. The senator was never briefed, and no investigation into NSF was launched. I was thanked for my hard work. I went back to my regular job and started tutoring middle school students in math at a school in D.C. while continuing to work with high school students in my neighborhood. That year a 9th-grade girl was having problems in geometry and came to me for help. “What seems to be the problem?” I asked. “I don’t know how to do proofs,” she said.

“I know,” I replied. “Don’t worry. It isn’t you.”

All politics is local, I decided.

*Barry Garelick is an analyst with a federal government agency in Washington, D.C. The opinions expressed in this article are those of the author.*

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