Hands-On Operations: Using Manipulatives for Understanding of ALL Four Operations

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Passionate about Singapore Math, Teacher Trainer/Coach, Treasure Hunter, Learner, Answer to the ultimate question? 42.

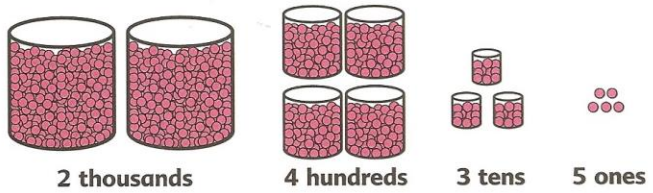
Slides will be posted:

https://goo.gl/j4aBY

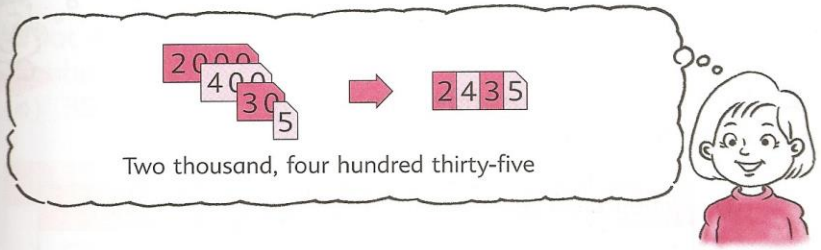
Big Ideas in Singapore Math:

1. Number Sense
2. Making Connections
3. Visualization
4. Communication
5. Variation

Concrete-Pictorial-Abstract approach

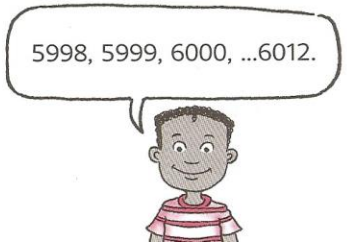


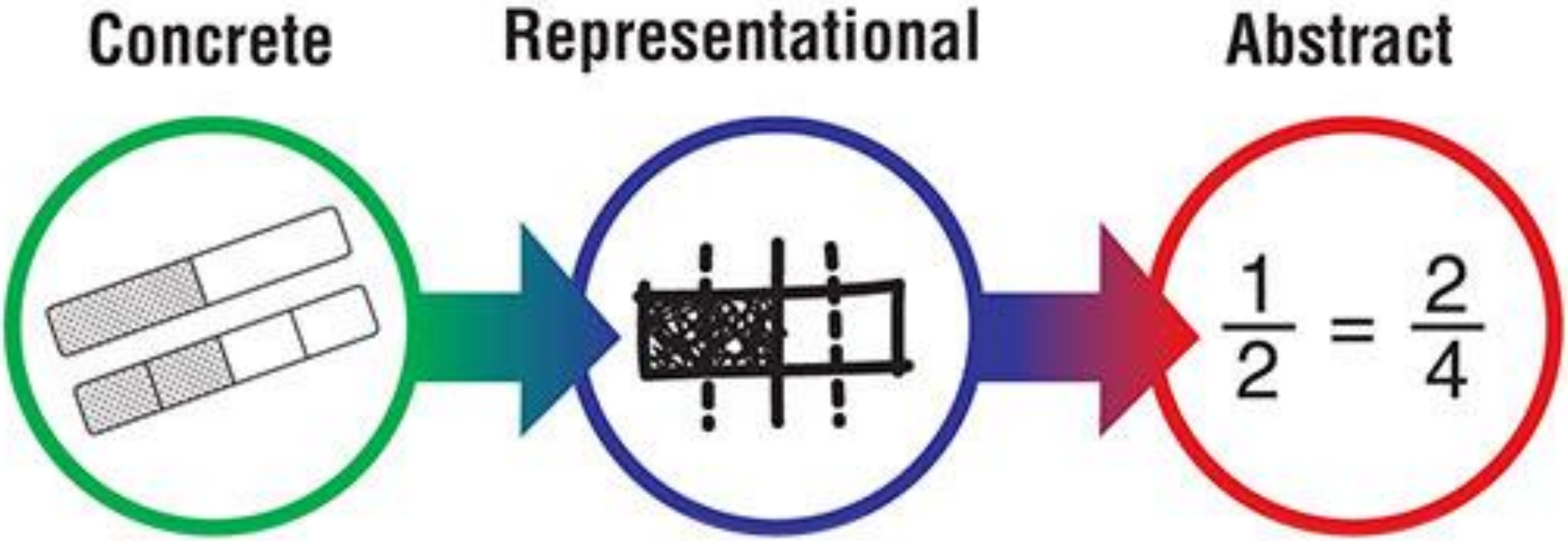
2000 + 400 + 30 + 5 = ■
How many marbles did she collect?



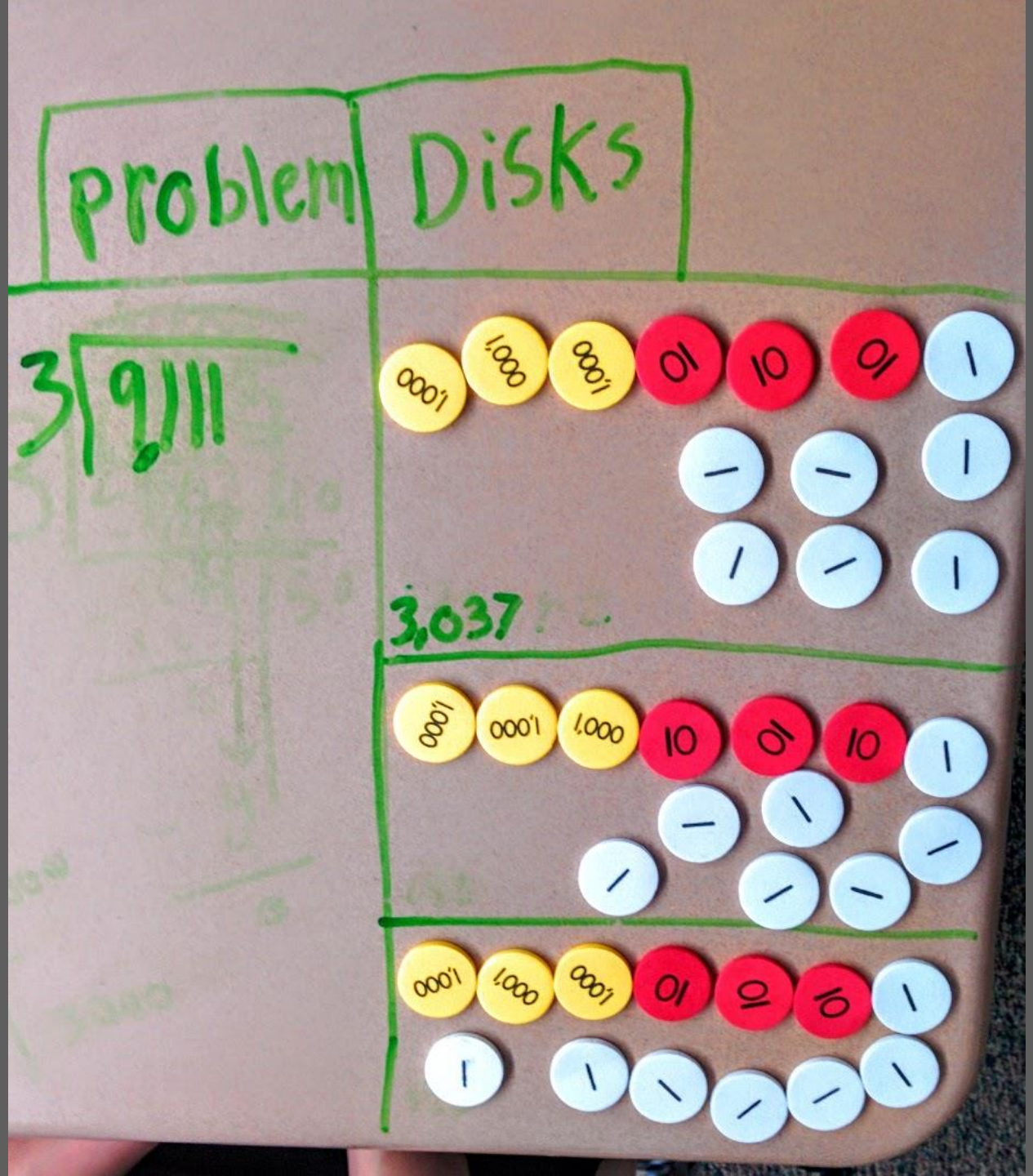
(c) Read the numbers 5998 and 6012.

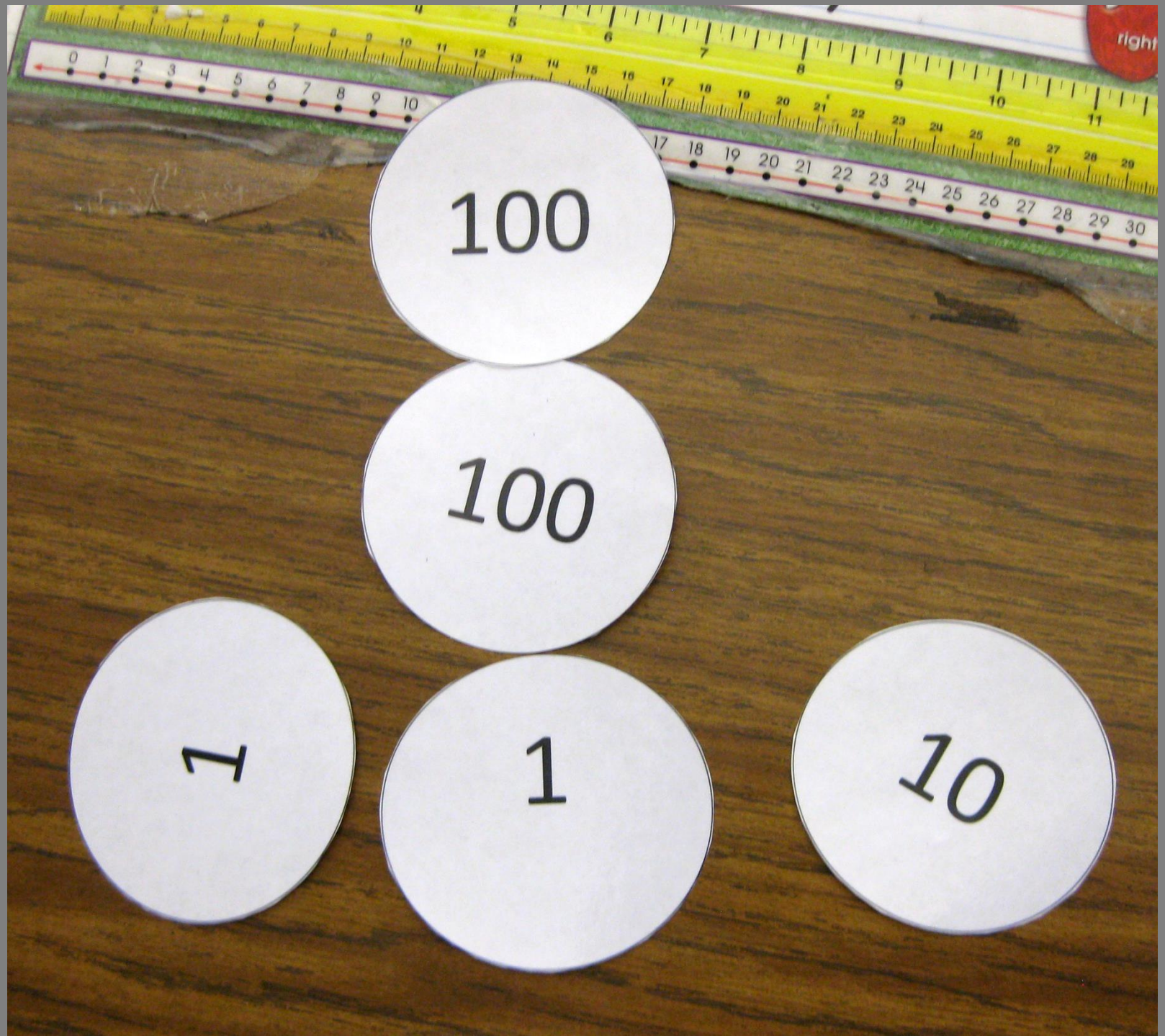
(d) Count from 5998 to 6012.

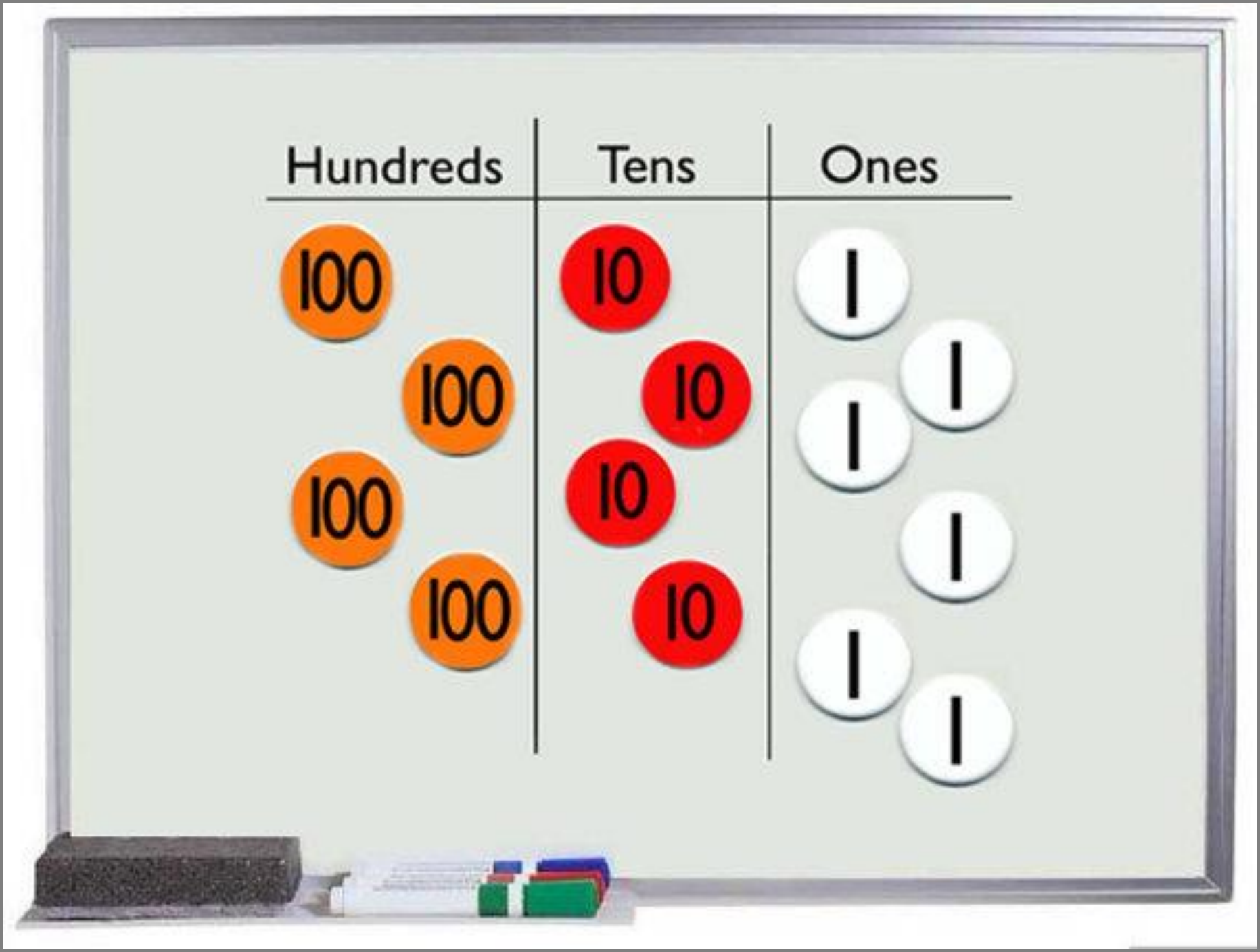



Concrete
Representational
Abstract
$\frac{1}{2} = \frac{2}{4}$

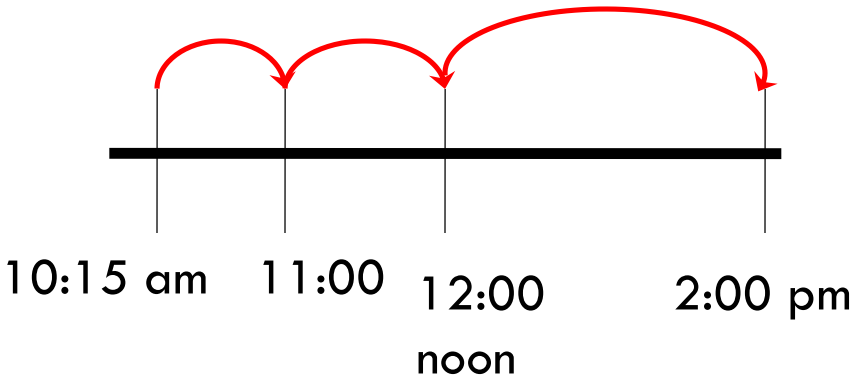
Ones (1)
Tens (10)
Hundreds (100)
Thousands (1,000)

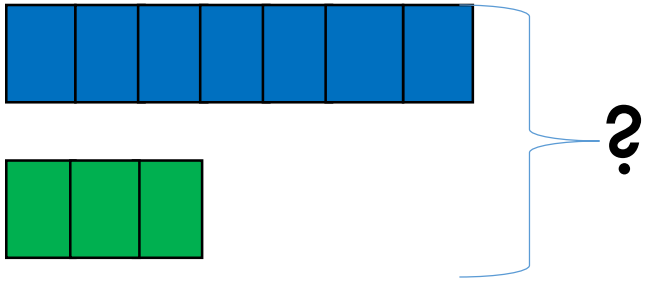

problem
Disks
3 9,111
3,037

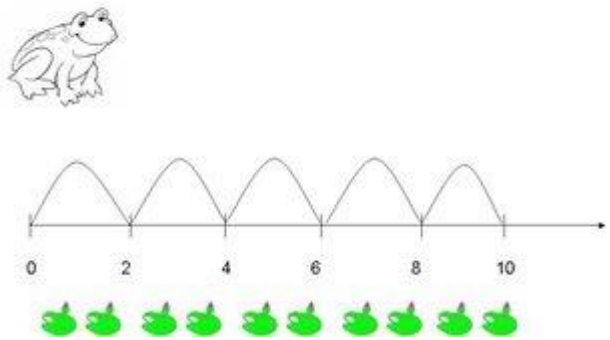

100
100
1
1
10

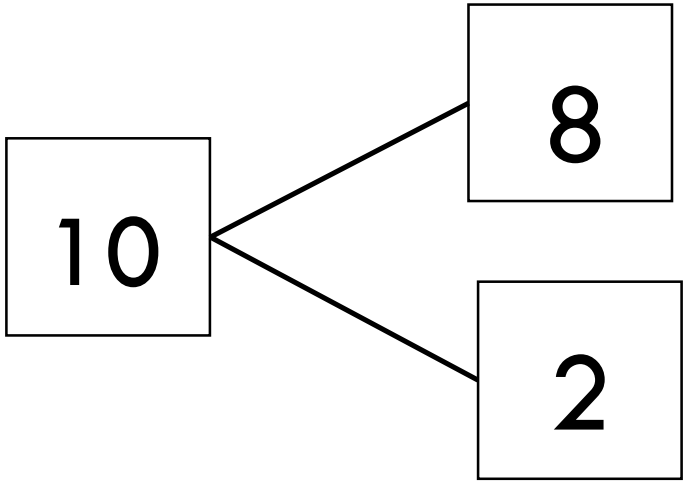

Hundreds
Tens
Ones
100
100
100
100
10
10
10
10
1
1
1
1
1
1

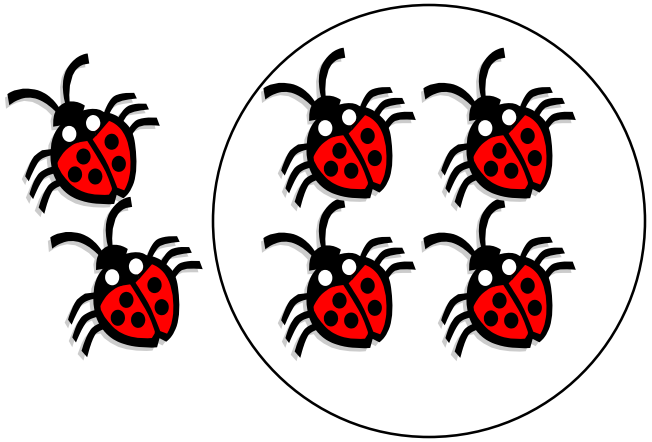
Pictorial







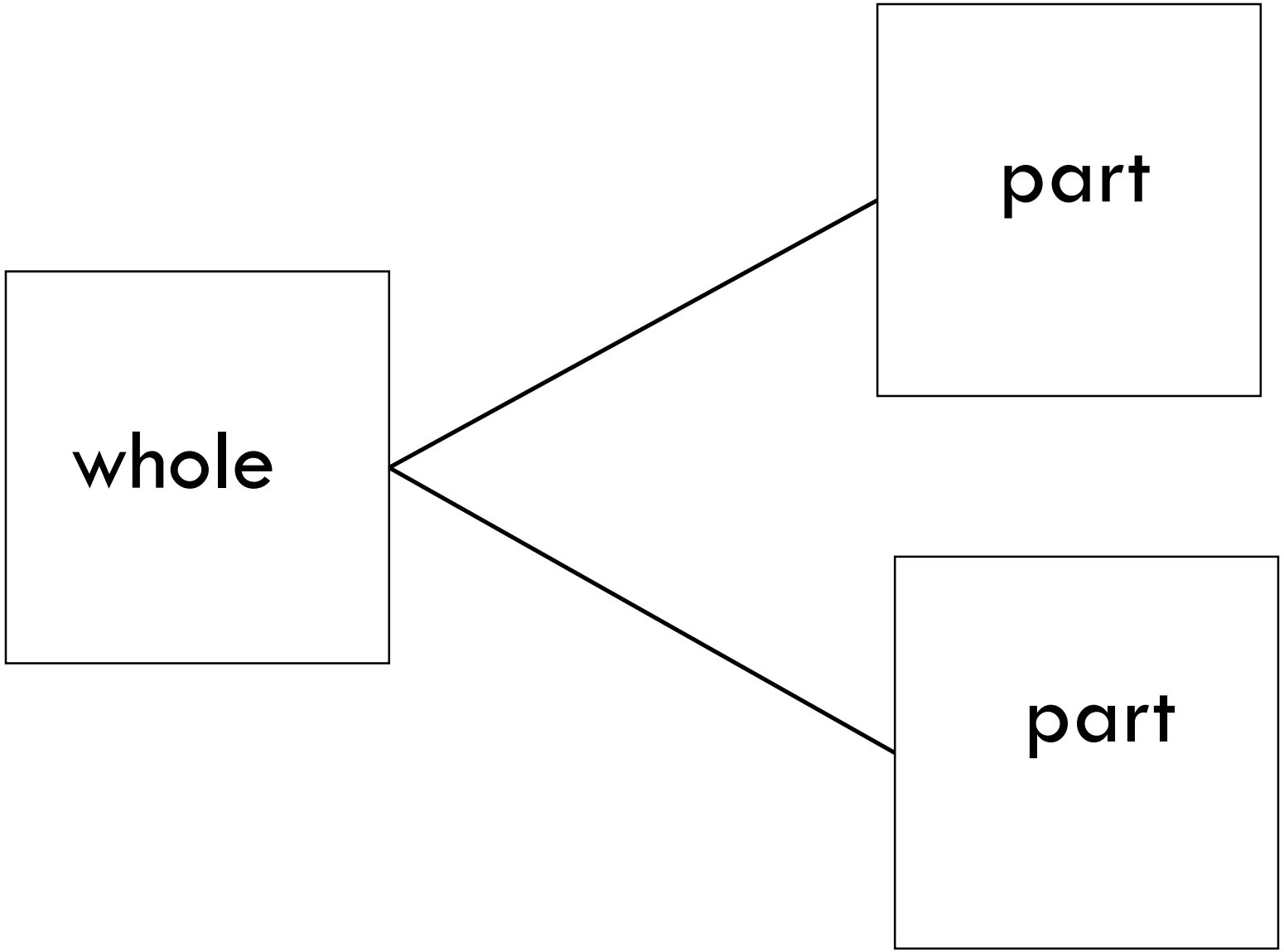


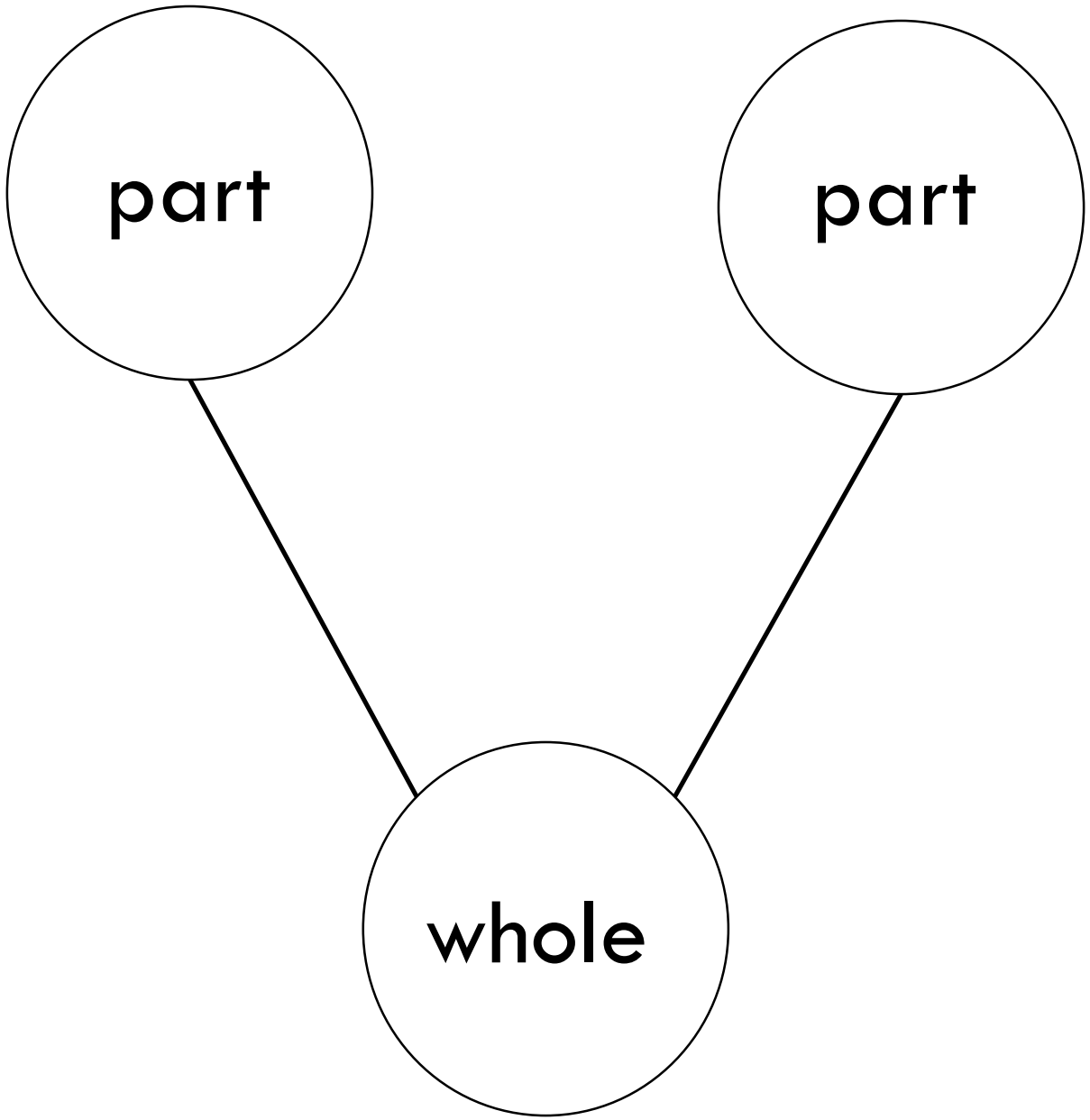


Abstract

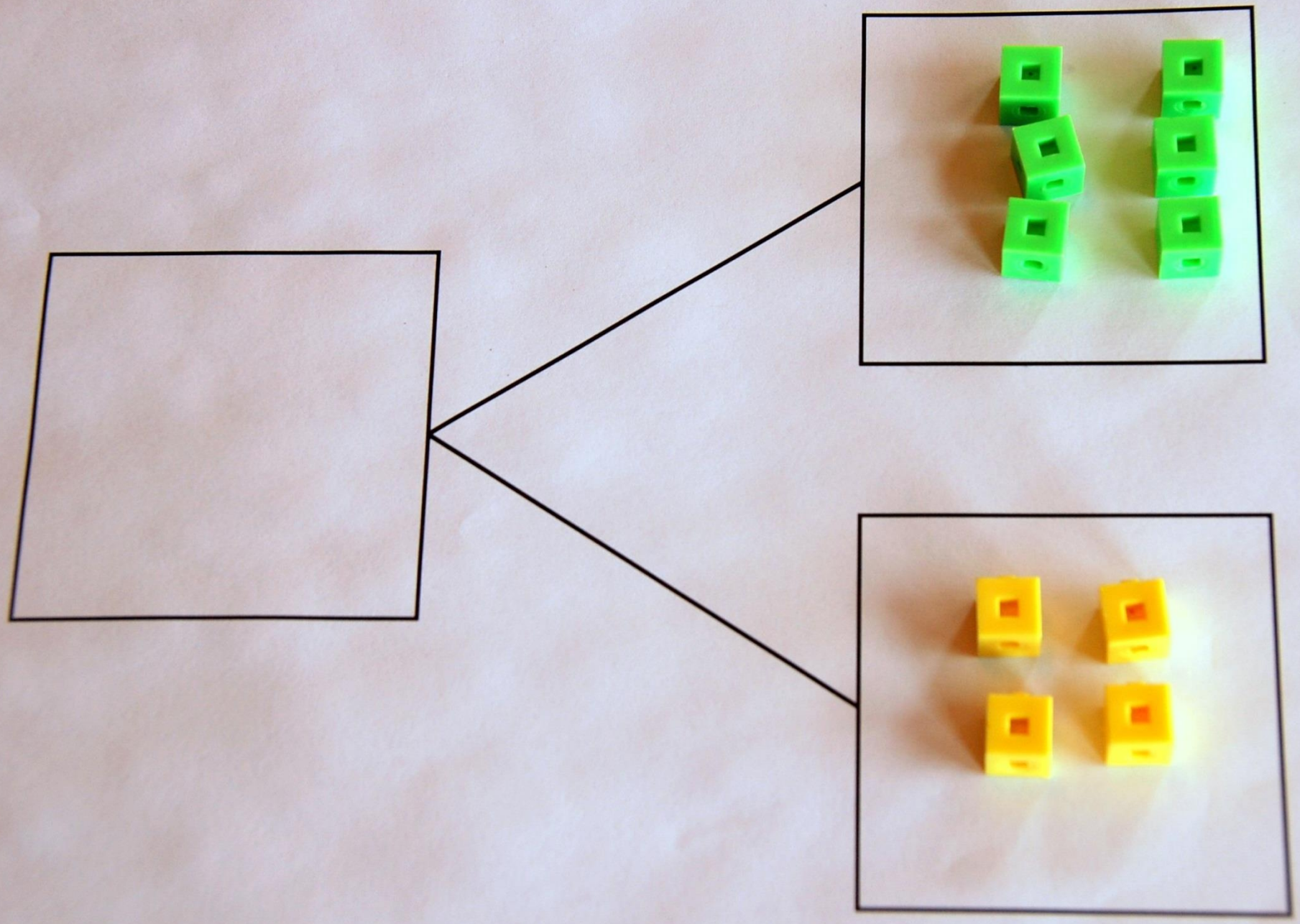
Goal?
Understanding
& applying
algorithms!

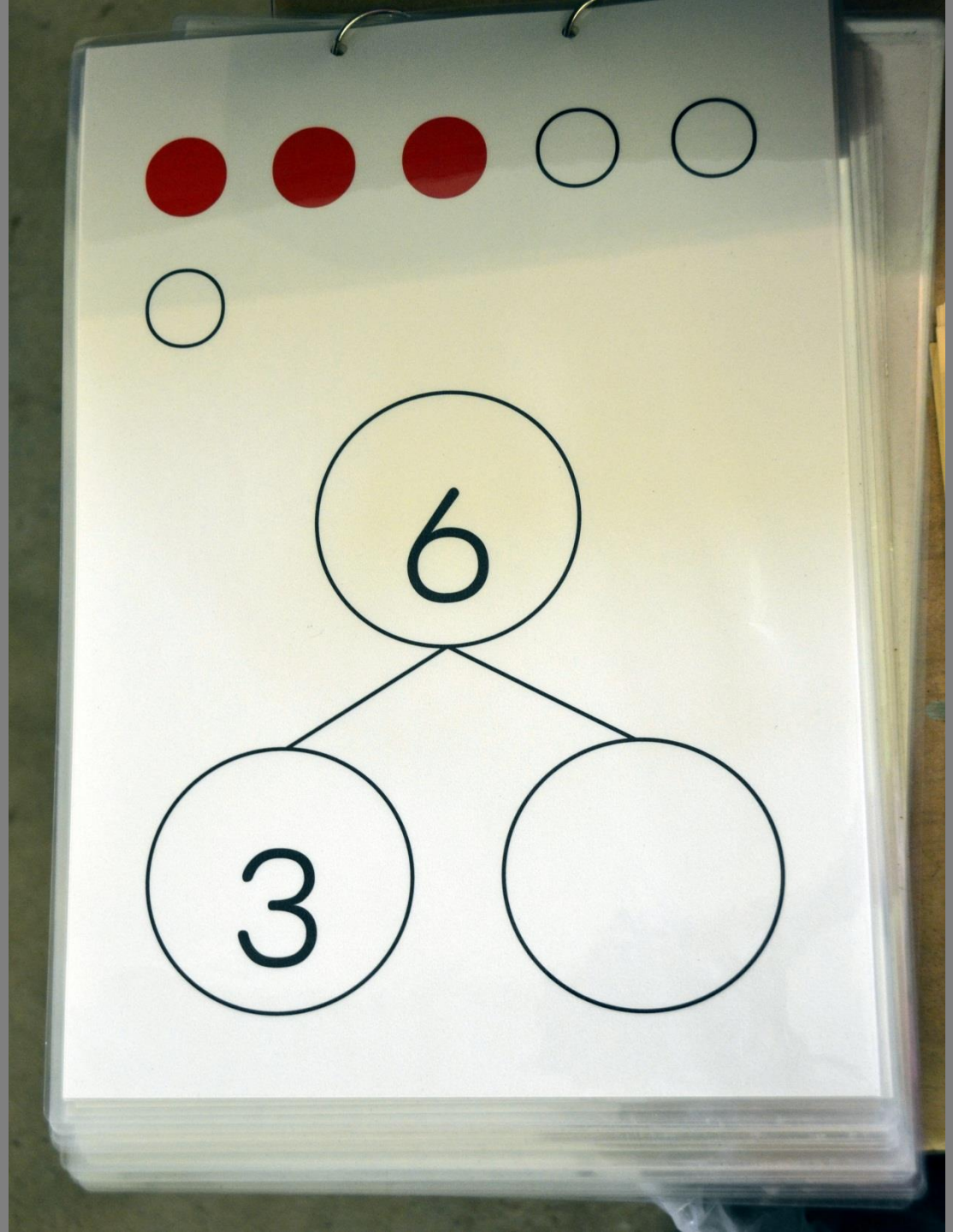
Number Bonds



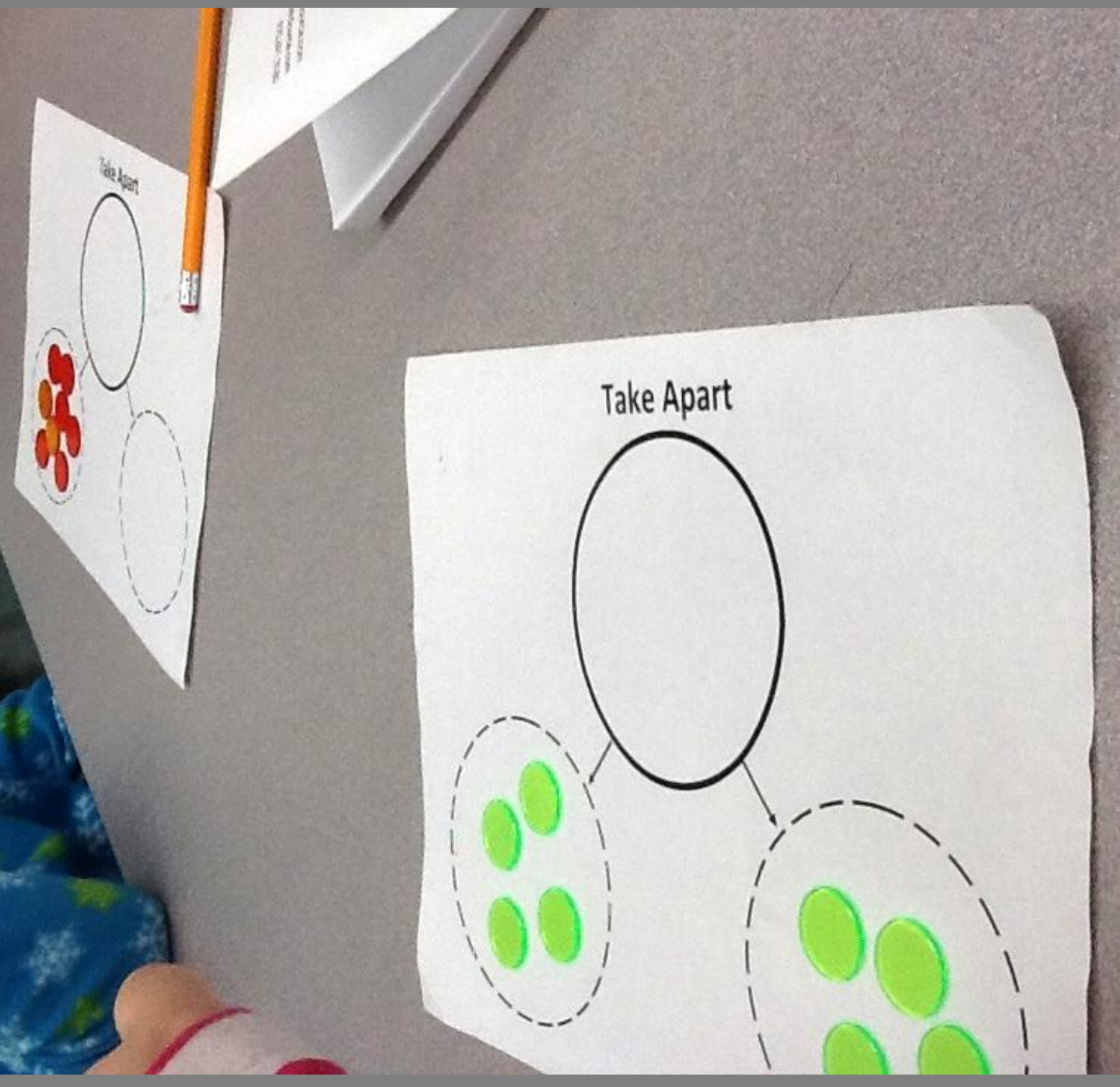

part
part
whole

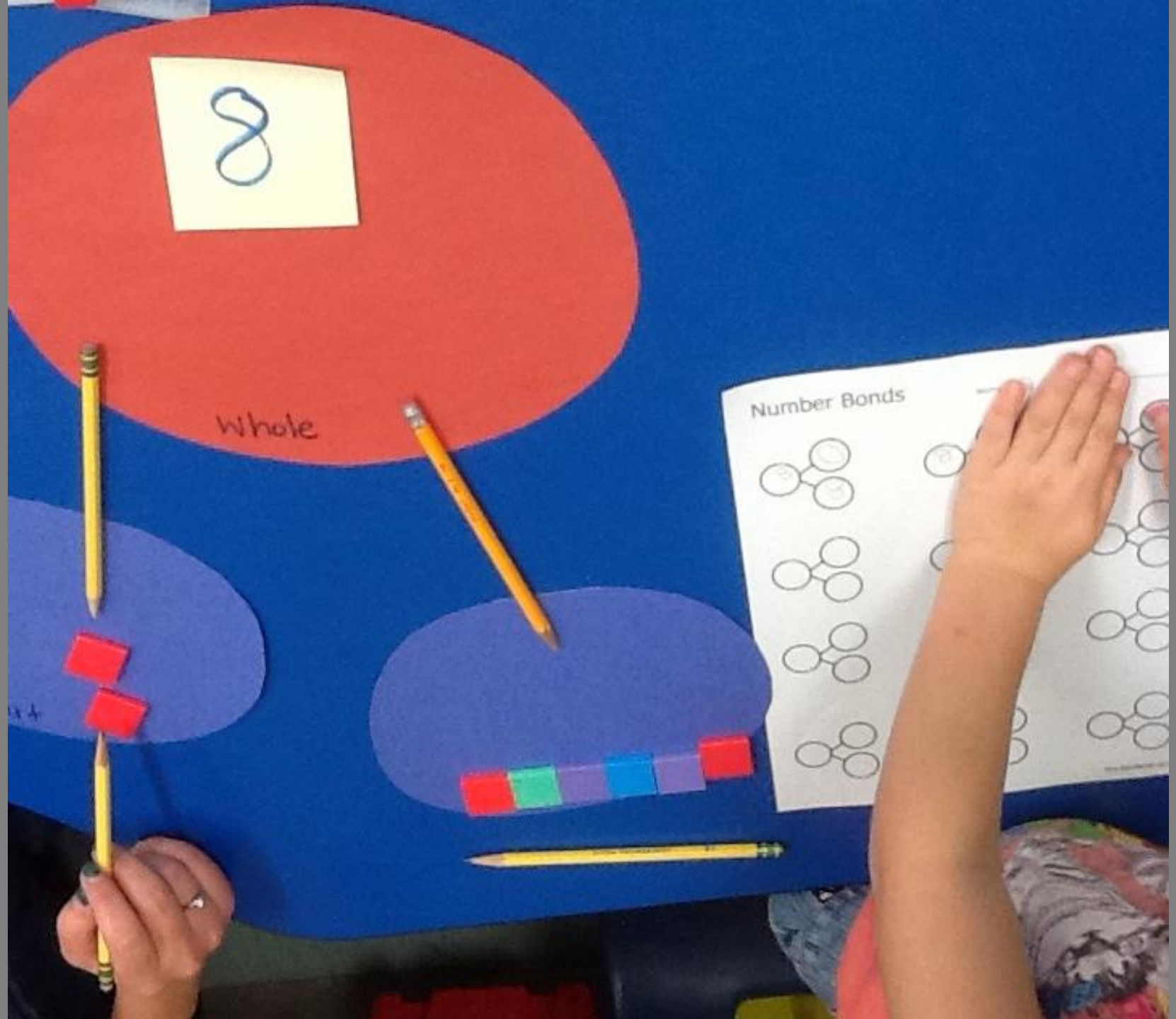
10
2
?

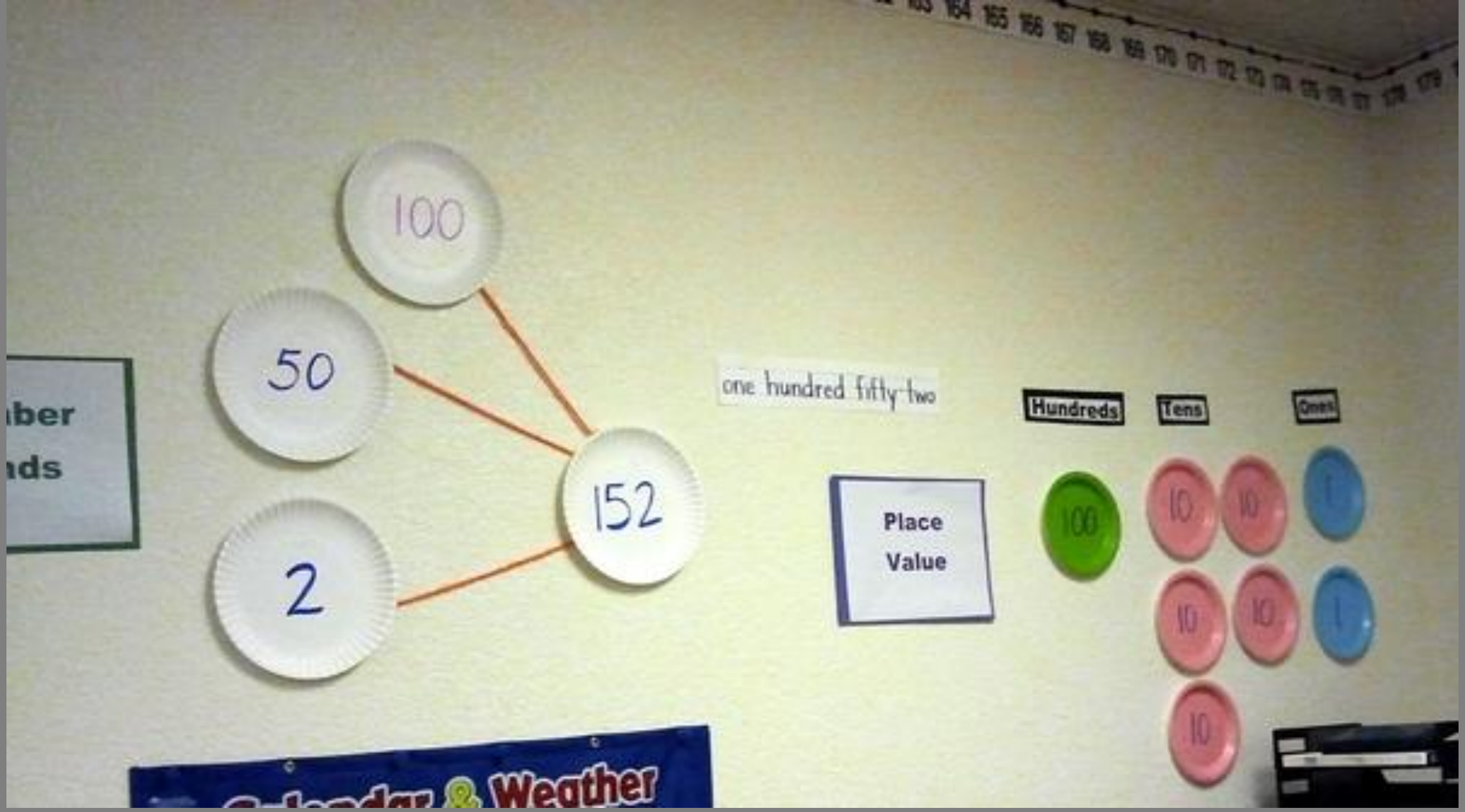


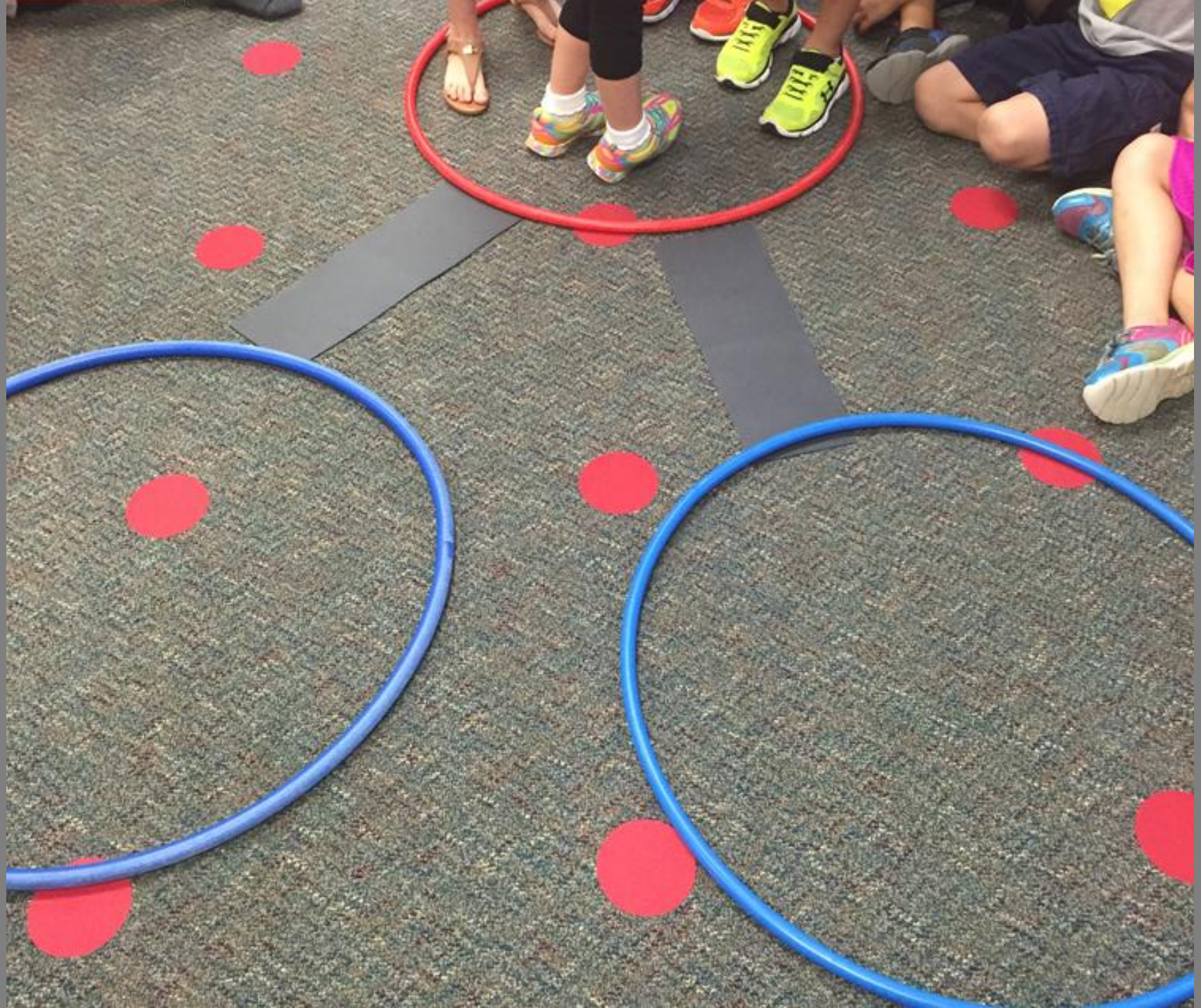

6
3

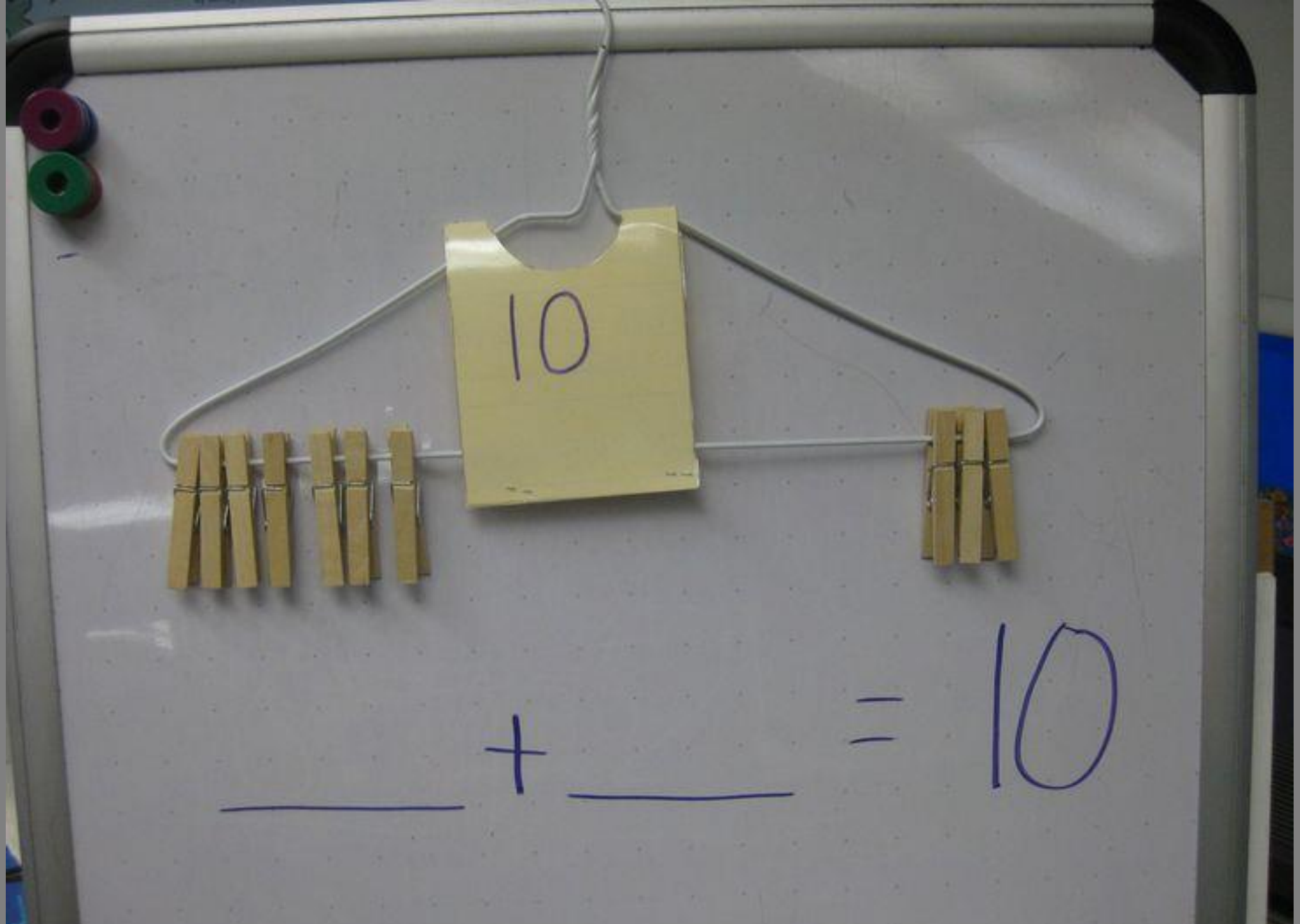
1
1
0
8
6
2
whole
part
part

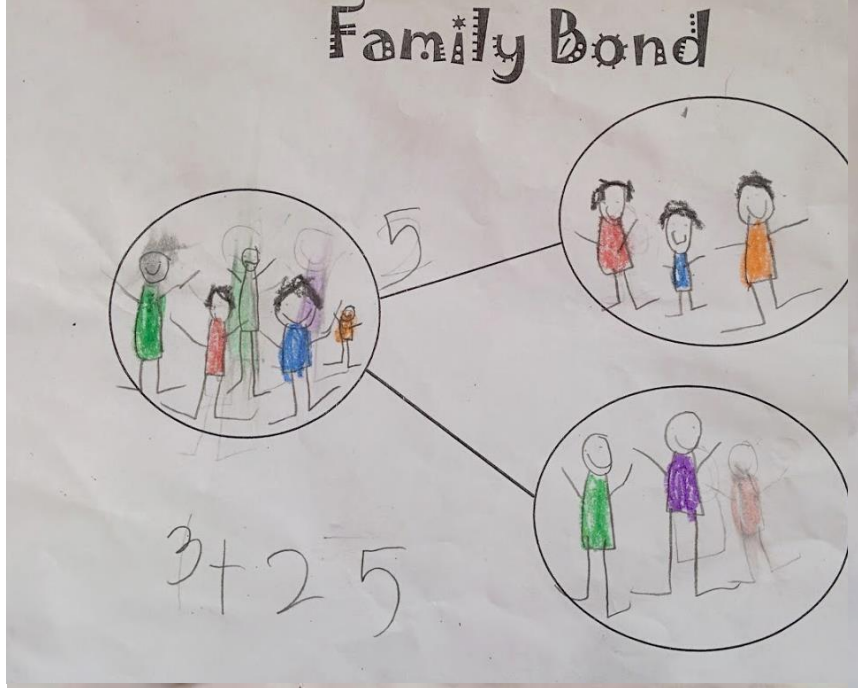

Take Apart
Take Apart

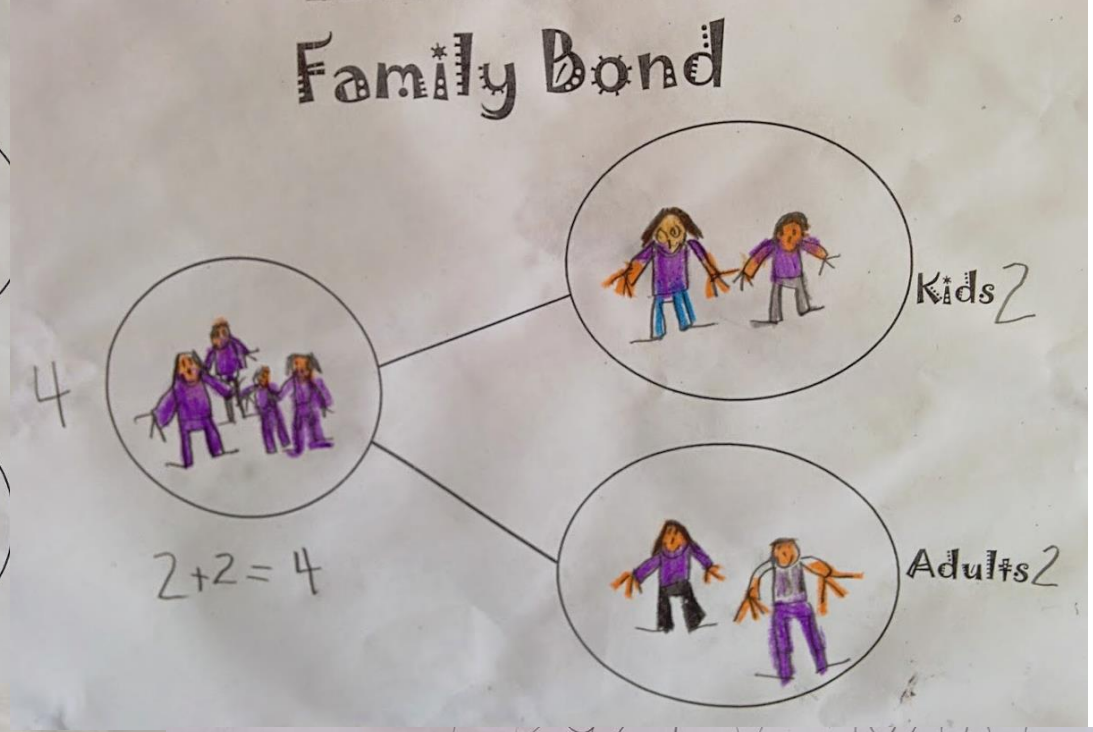

8
Whole
Number Bonds

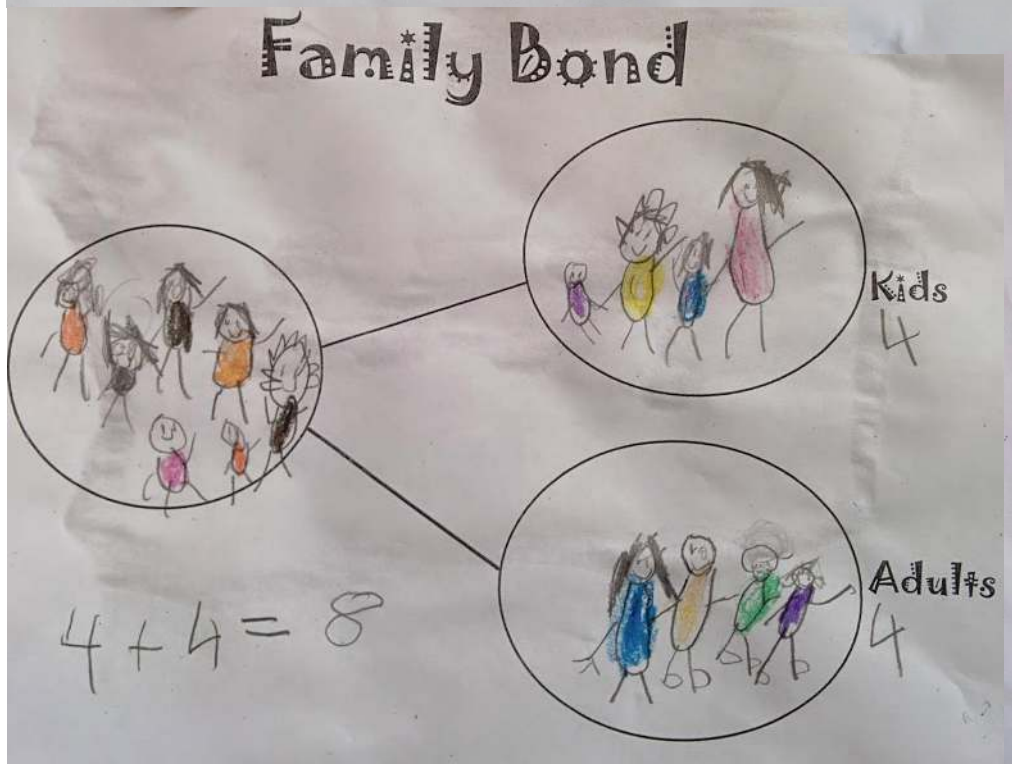

100
50
2
152
one hundred fifty-two
Place Value
Hundreds
Tens
Ones
100
10
10
10
10
10
1
1
Calendar & Weather




10
_____ + _____ = 10


Family Bond
5
3+2 5

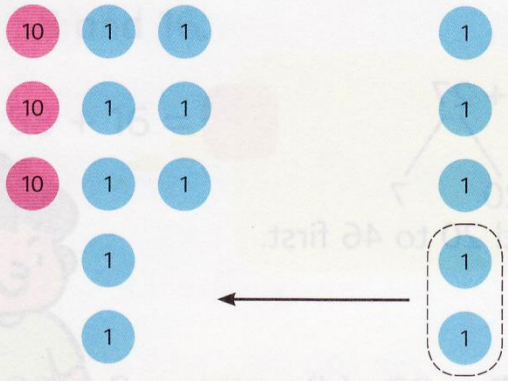

Family Bond
Kids 2
Adults 2
4
2+2=4


Family Bond
Kids
4
Adults
4
4+4=8

Family Bond
Kids
Adults

1 Mental Calculation

Add 38 and 5.



38 + 5 = ☐

2 3

Make a 10 first.
38 + 5

2 3

38 + 2 = 40
40 + 3 = 43

1. Add.

(a) 42 + 3

2 + 3 < 10

I can add the ones.

42 + 3

40 2

2 + 3 = 5
40 + 5 = 45

42 + 3 = ☐

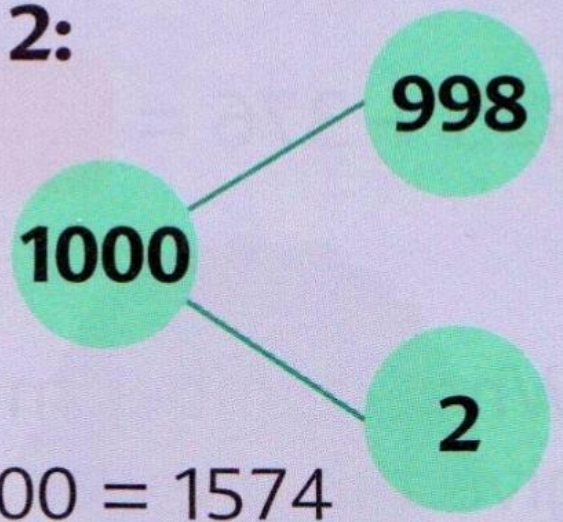
4A Four Operations of Whole Numbers

6. Add 574 and 998.

Method 1:

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 574 \\ +\quad 998 \\ \hline 1572 \end{array}$$

Method 2:



574 + 1000 = 1574

1574 − 2 = 1572

So, 574 + 998 = 1572

4B Decimals

8. (a) Add 0.42 and 0.9.

$0.42 + 0.9 = 0.02 + 1.3$

$= \square$

$0.42 + 0.9$

$0.4 \quad 0.02$

$0.4 + 0.9 = 1.3$

(b) Add 0.42 and 0.09.

$0.42 + 0.09 = 0.4 + 0.11$

$= \square$

$0.42 + 0.09$

$0.4 \quad 0.02$

$0.02 + 0.09 = 0.11$

$$49 + 32 = ?$$

1 31

$$50 + 31 = ?$$

49 min. + 32 min. = ?

11 21

1 hr. 21 min.

16.20 ÷ 4 = ?

16 20 hundredths

Long Division Method

```
      4.05
4 | 16.20
    16
      0.20
      0.20
         0
```

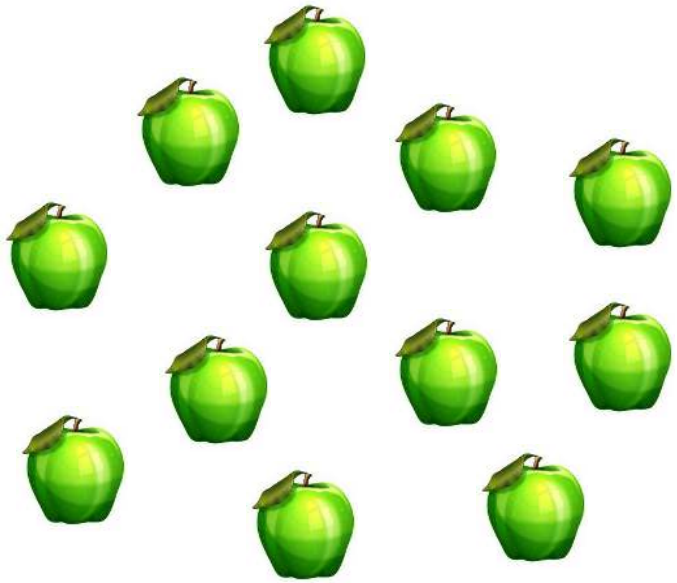
$$51 \div 3 =$$

30 21

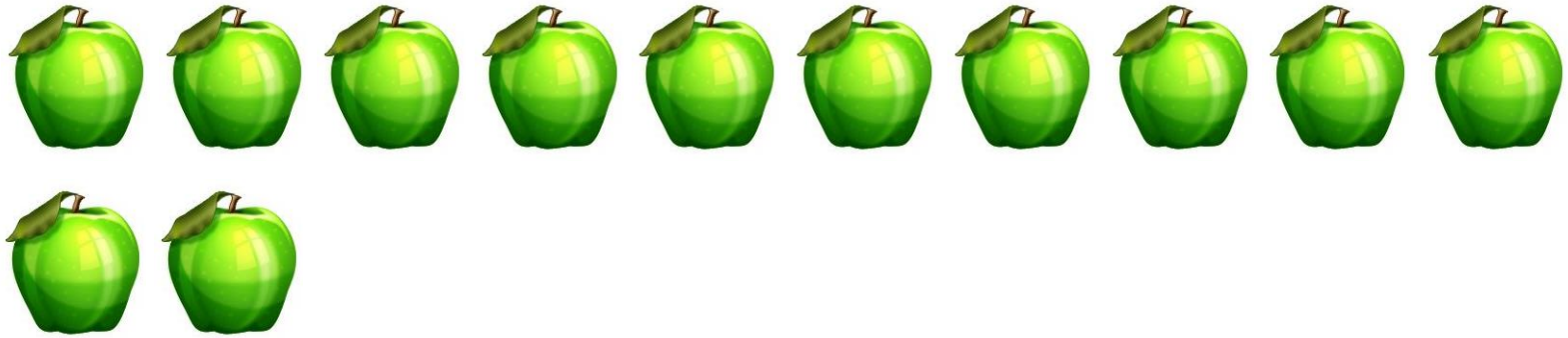
$$117 \div 3 = ?$$

Grade 1 – Make “Ten Strategy”

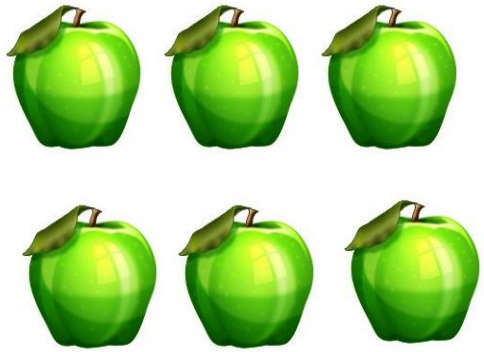
How many apples?

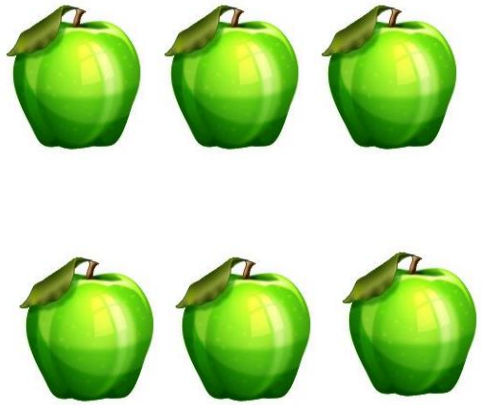


10 and 2 makes 12

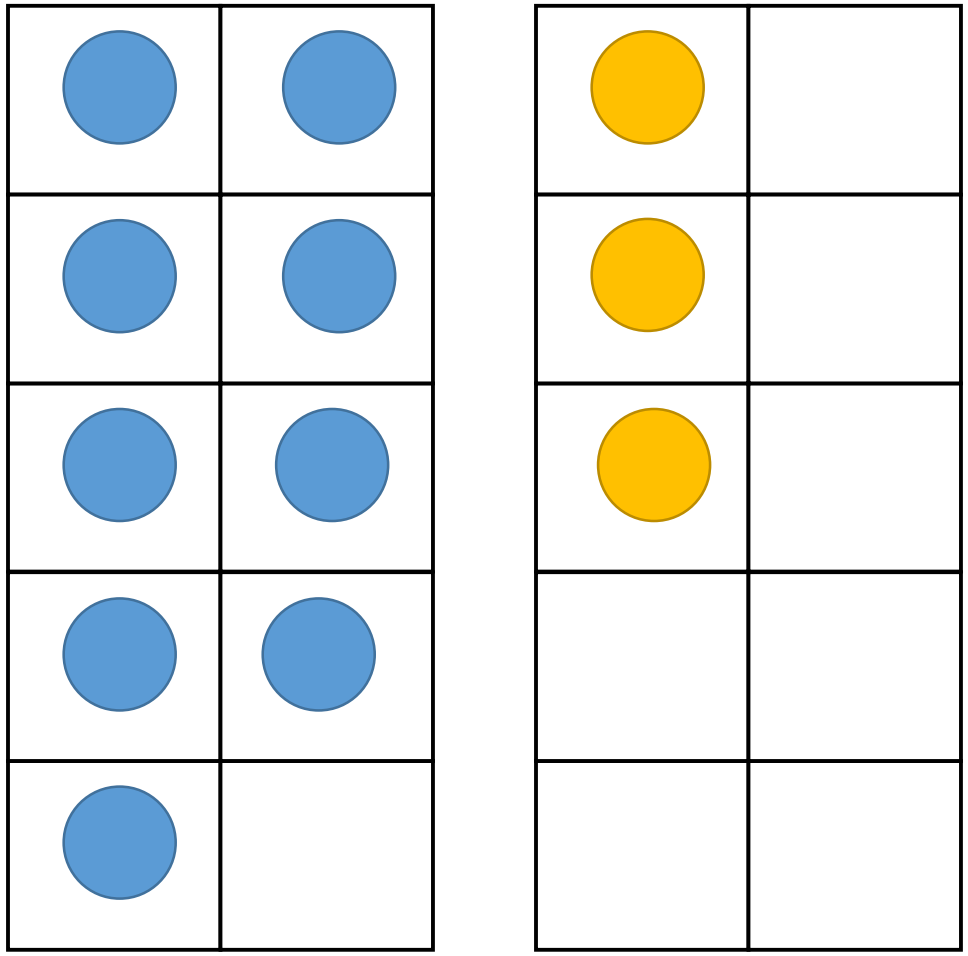


$$10 + 2 = ?$$



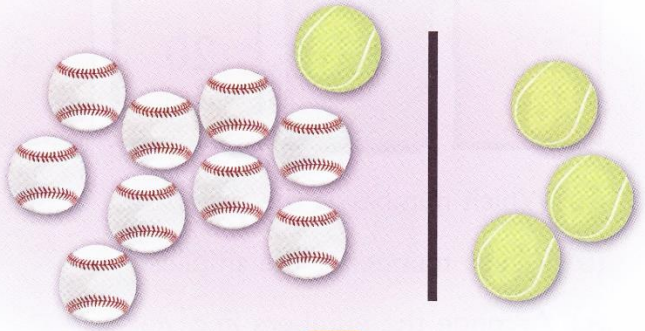


$$9 + 3 = 10 + 2$$



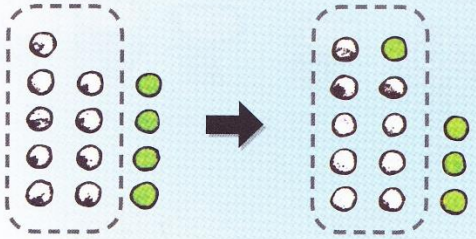
2 Addition and Subtraction

How many balls are there altogether?



9 + 4 = ☐

There are ☐ balls altogether.

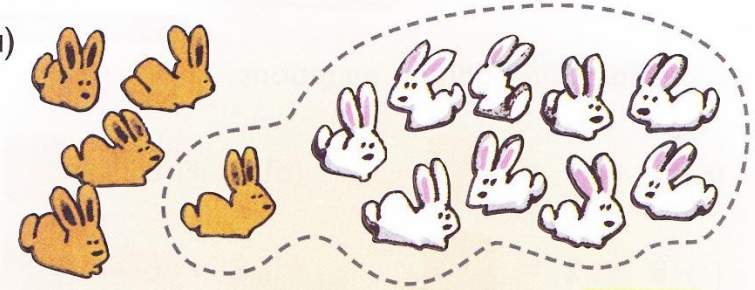


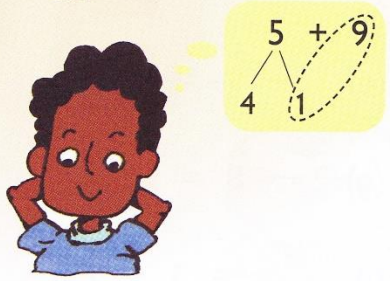
9 and 1 make 10.

10 + 3 = 13

1. Add by making 10 first.

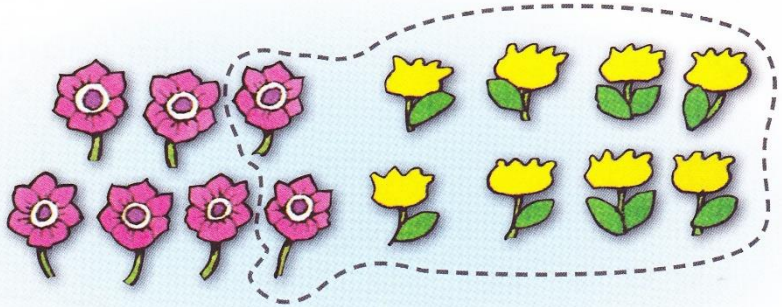
(a)

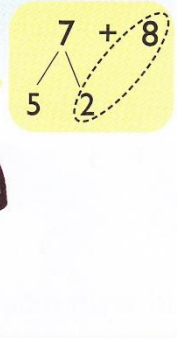




5 + 9 = ☐

(b)





7 + 8 = ☐

Exercise 6, pages 107–108

2. Complete the addition equations.

(a) 9 + 1 = ☐

(b) 9 + 8 = ☐

(c) 8 + 2 = ☐

(d) 8 + 6 = ☐

(e) 1 + 9 = ☐

(f) 3 + 9 = ☐

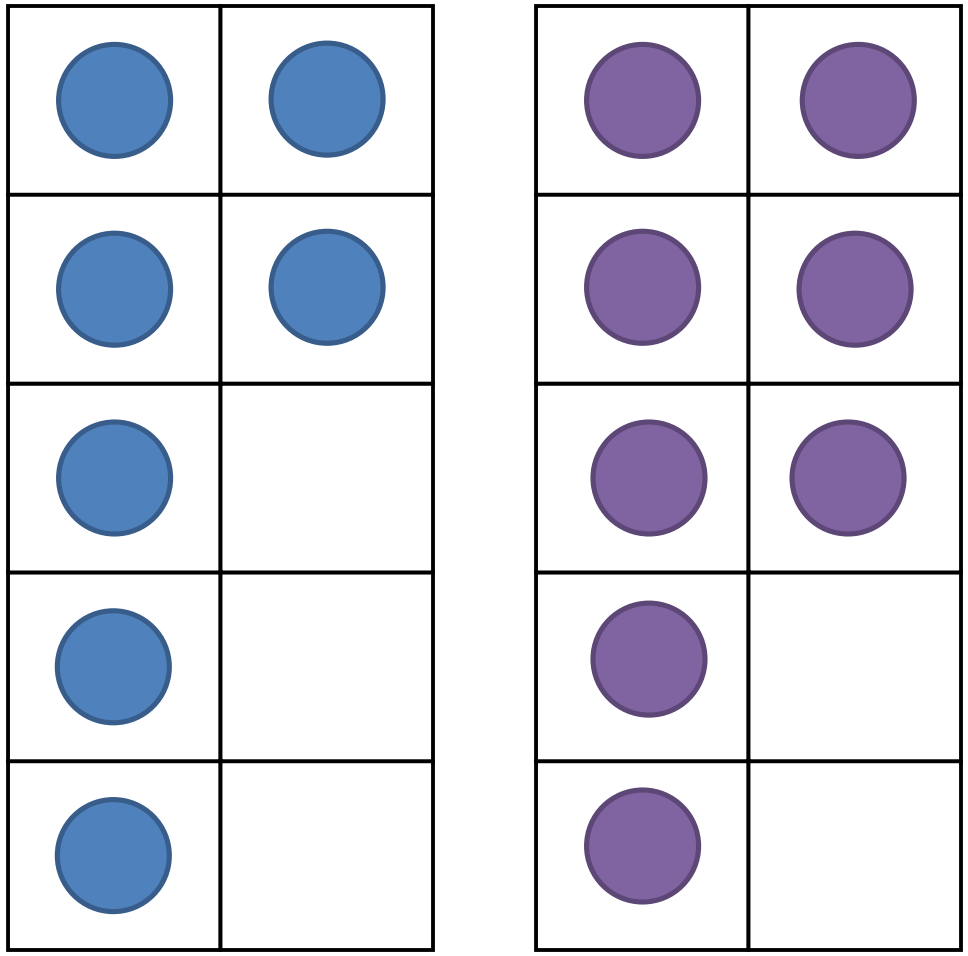
(g) 2 + 8 = ☐

(h) 5 + 8 = ☐

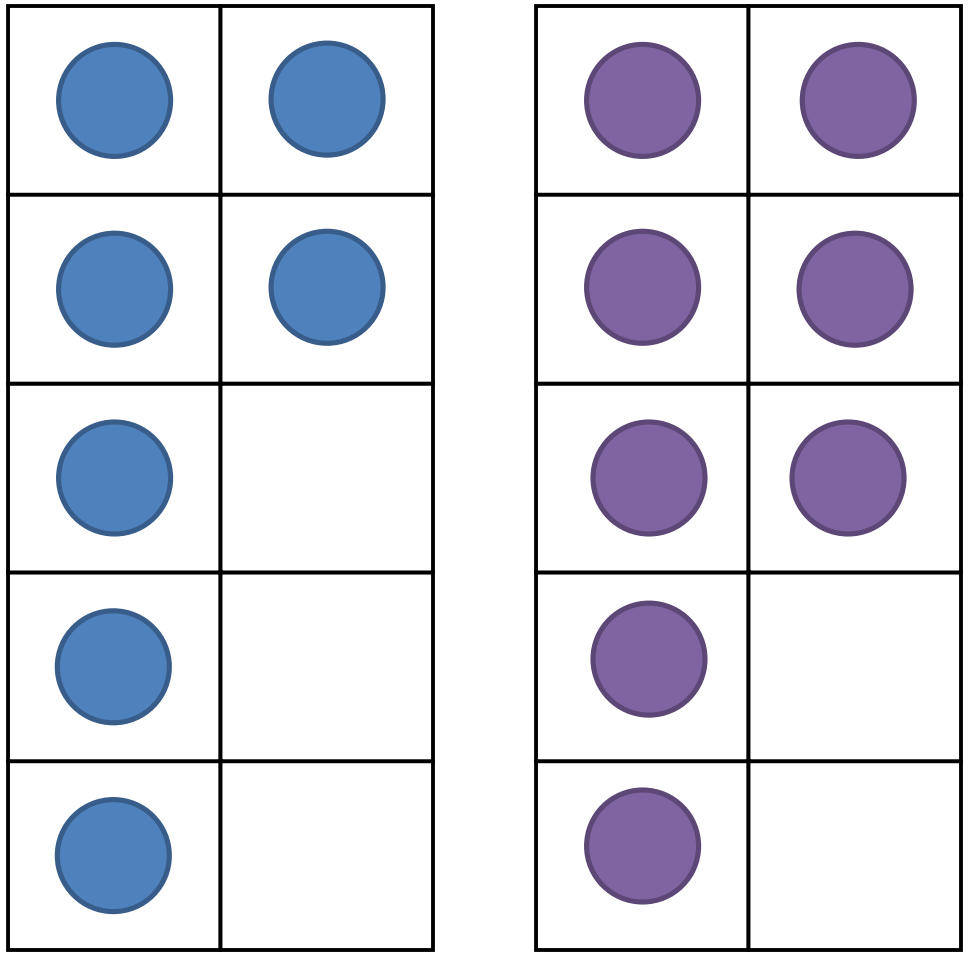
(i) ☐ = 3 + 7

(j) ☐ = 6 + 7

Exercise 7, pages 109–110



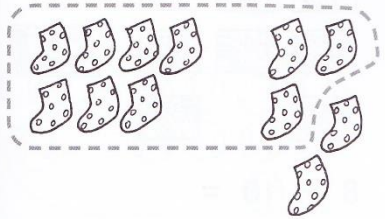
$7 + 8 = ?$



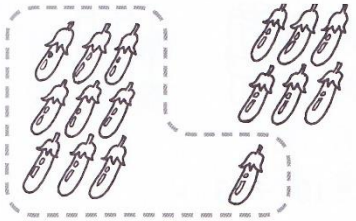
$7 + 8 = ?$

EXERCISE 6

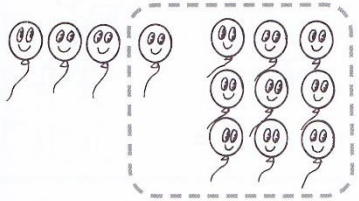
1. Add.



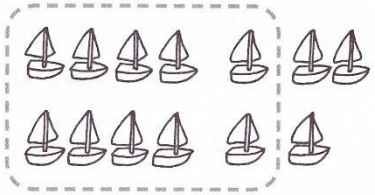
7 + 5 =



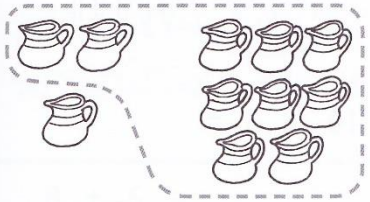
9 + 7 =



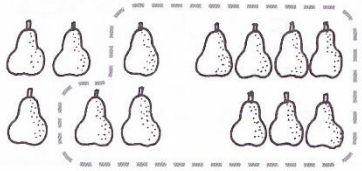
4 + 9 =



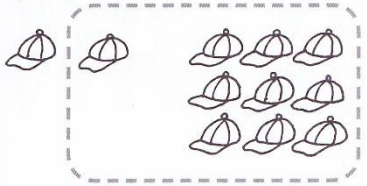
8 + 5 =



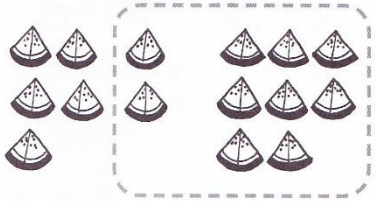
3 + 8 =



6 + 7 =

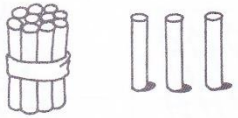


2 + 9 =

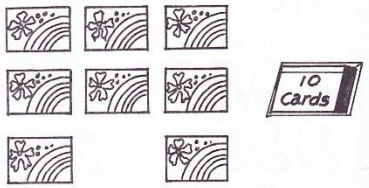


7 + 8 =

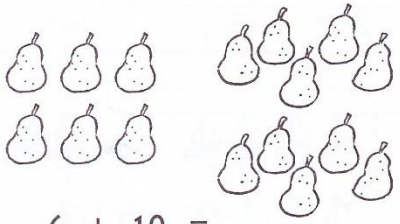
2. Add.



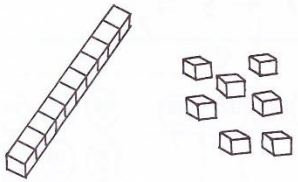
10 + 3 =



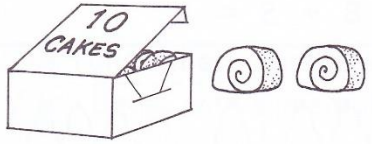
8 + 10 =



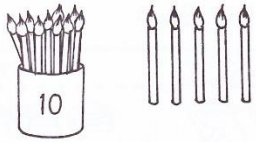
6 + 10 =



10 + 7 =



10 + 2 =

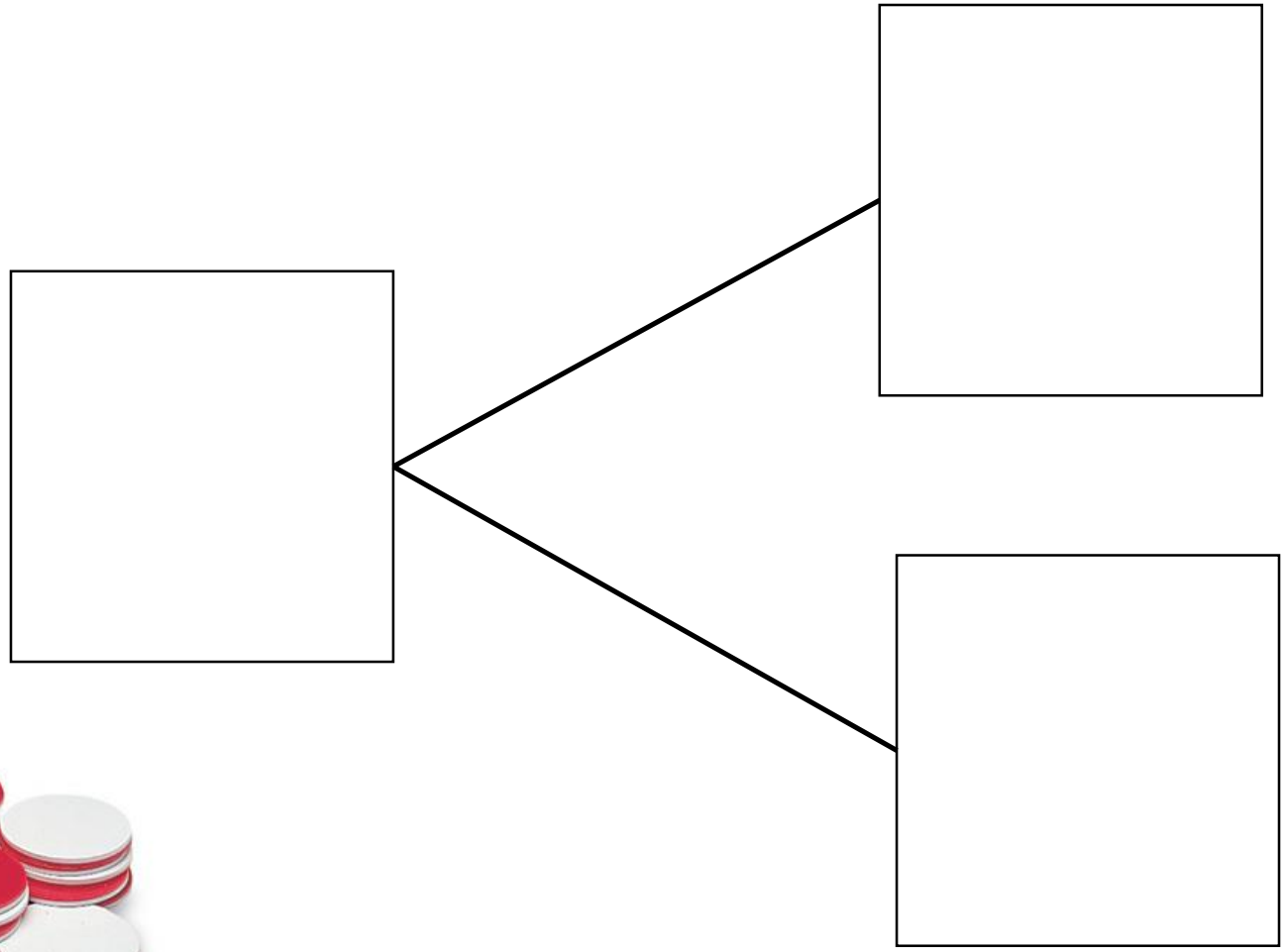


10 + 5 =

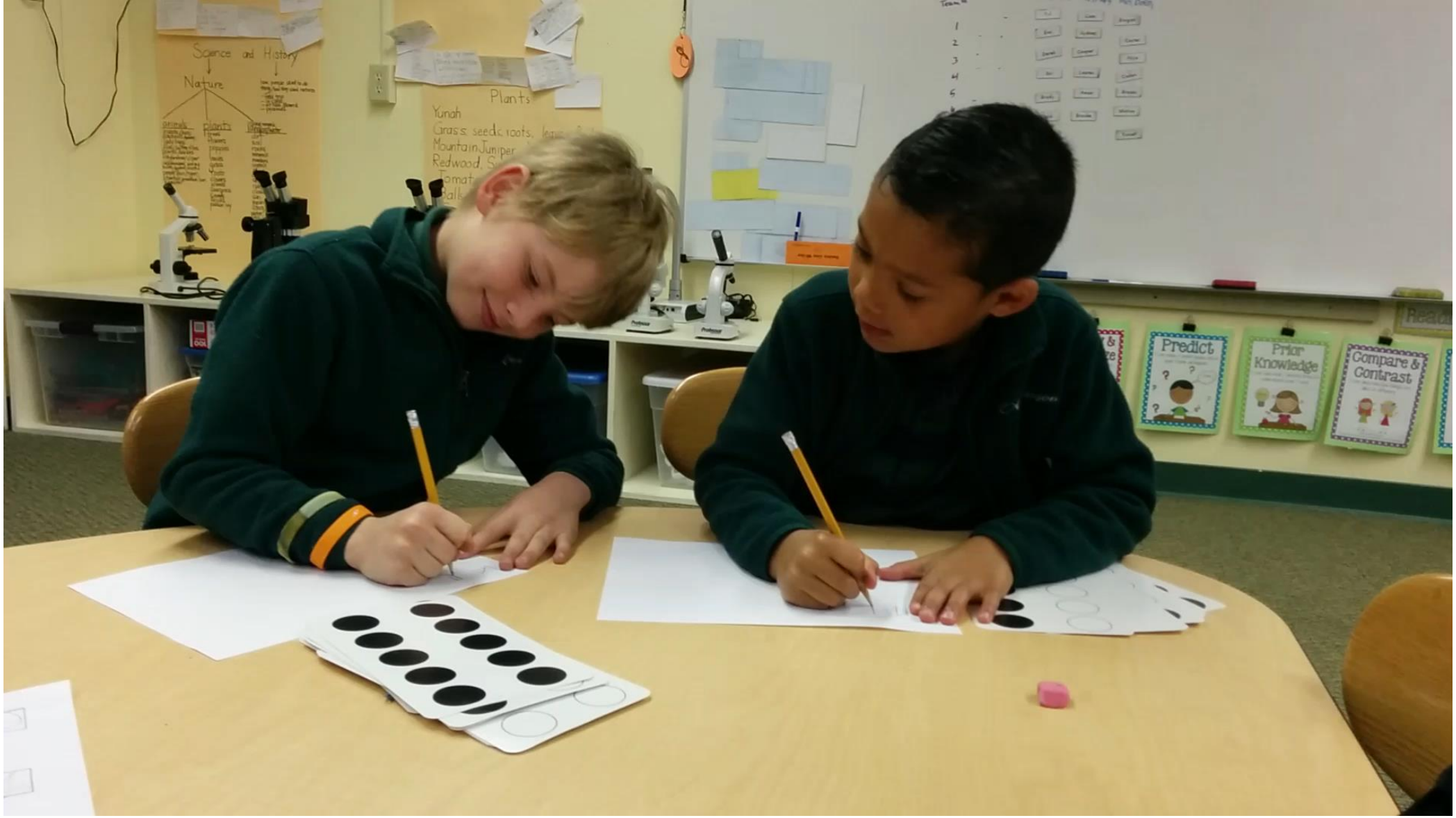
4 + 10 =

1 + 10 =

Number Bonds to Ten



Rock
Paper
Scissors
Math


Science and History
Nature
Plants
Predict
Prior Knowledge
Compare & Contrast

City Springs

Karen has 3 picture books and 6 storybooks.
How many books does she have in all?

cawnt on

She has 9 books in all.

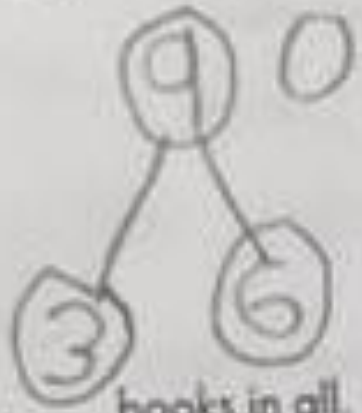
7

Fill in the blank.

Karen has 3 picture books and 6 storybooks.
How many books does she have in all?

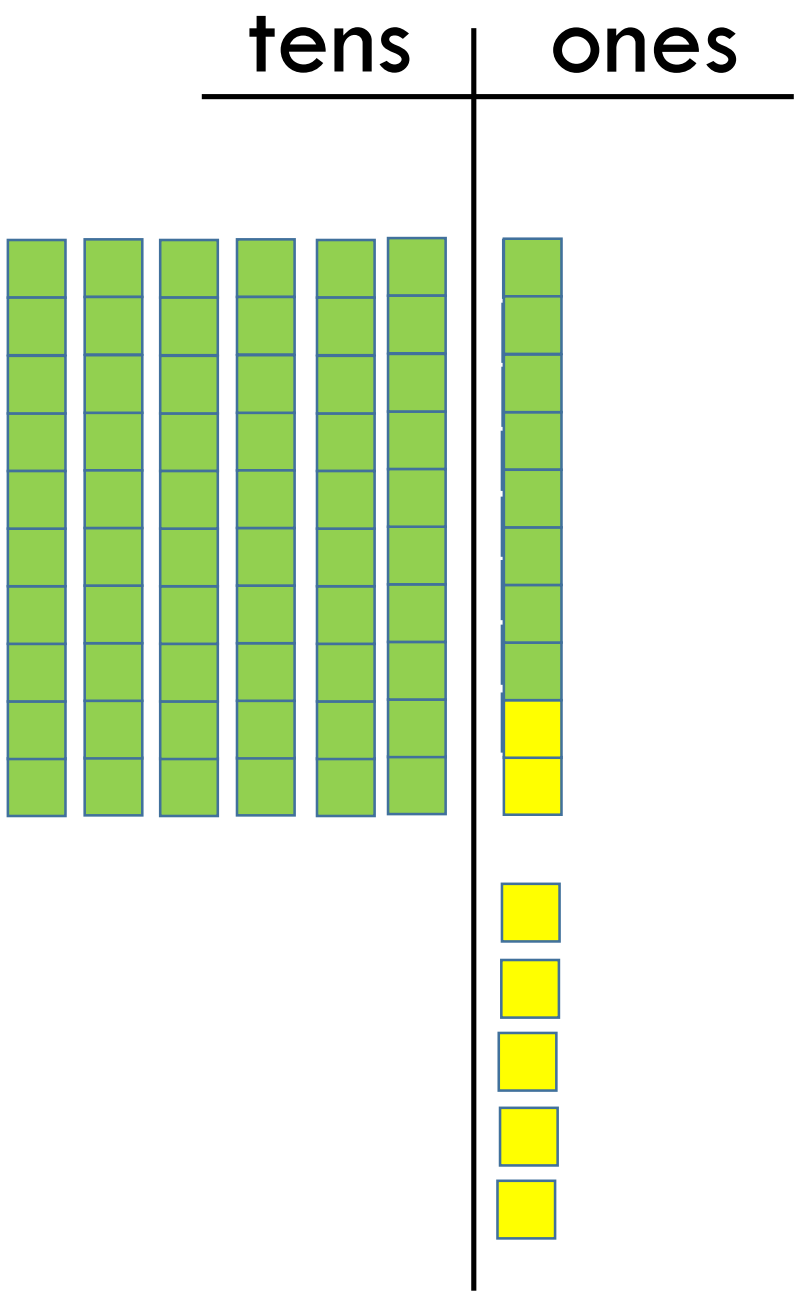
3+6=9

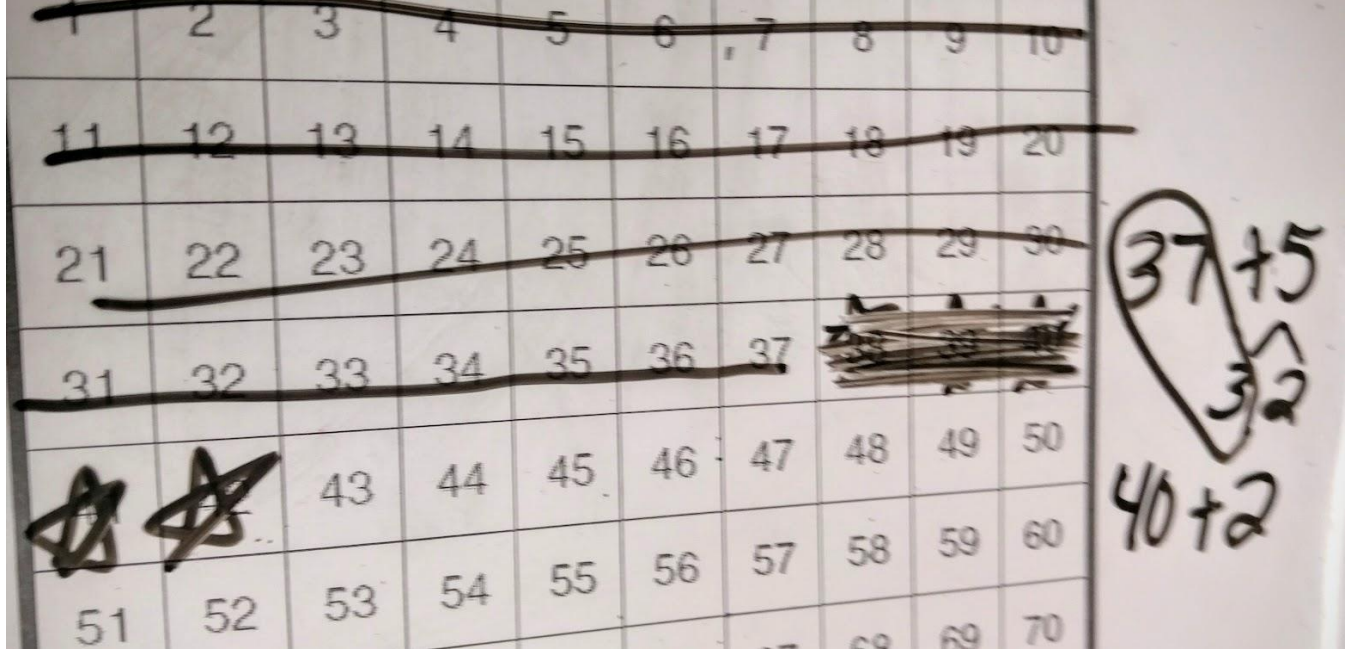
9 otomutisud.

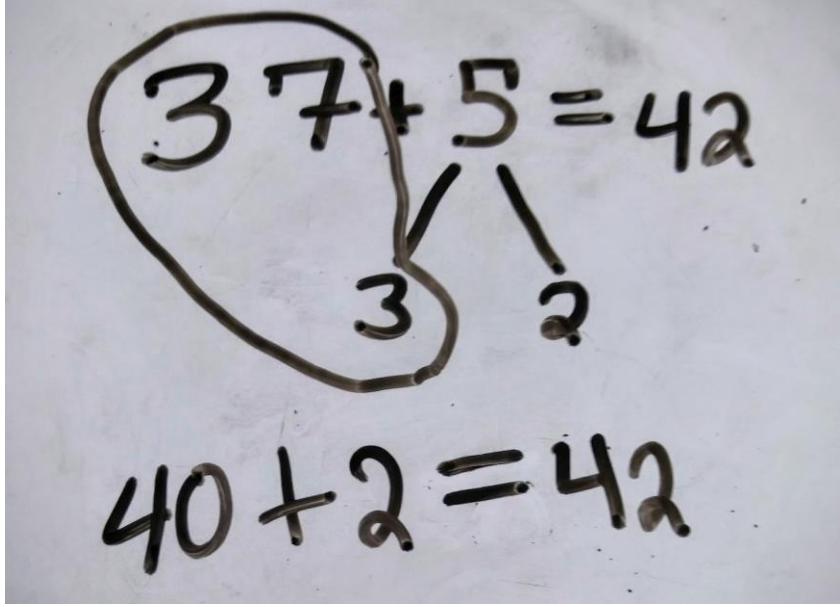


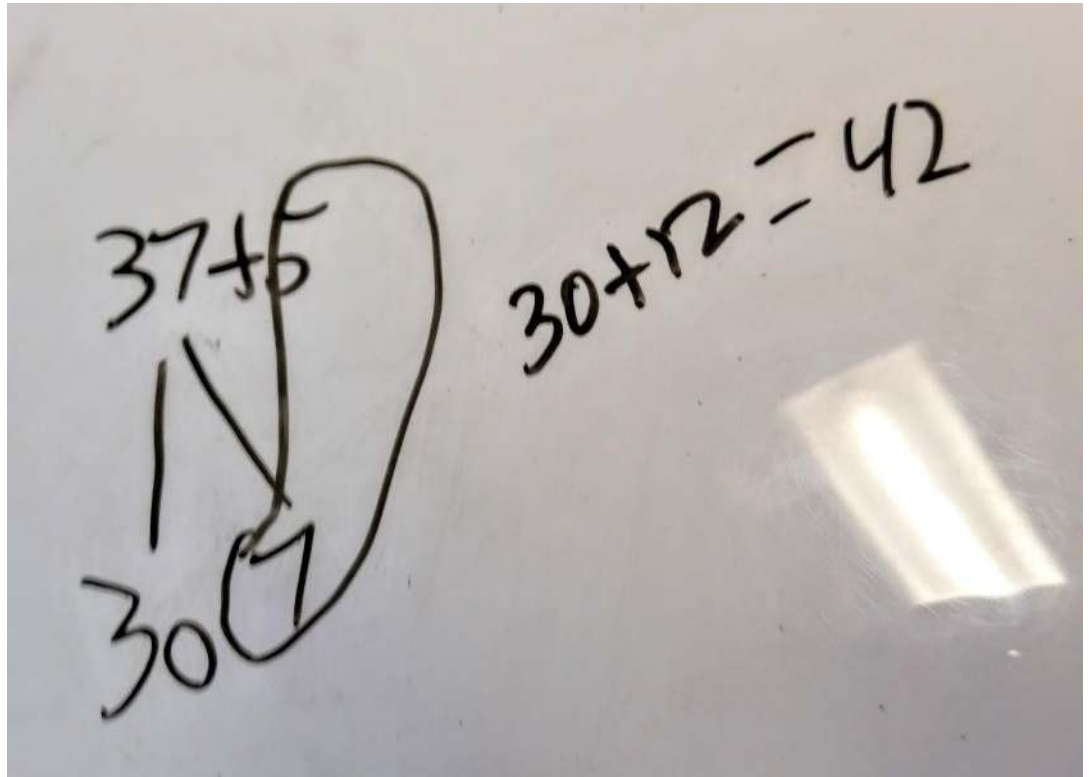
She has books in all.

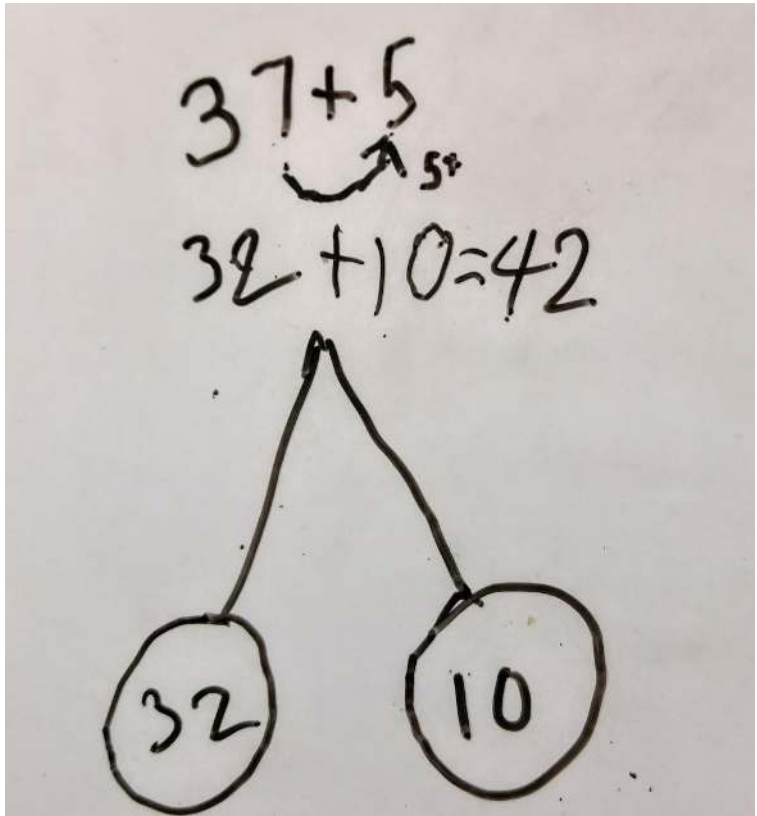
68 + 5 = ?



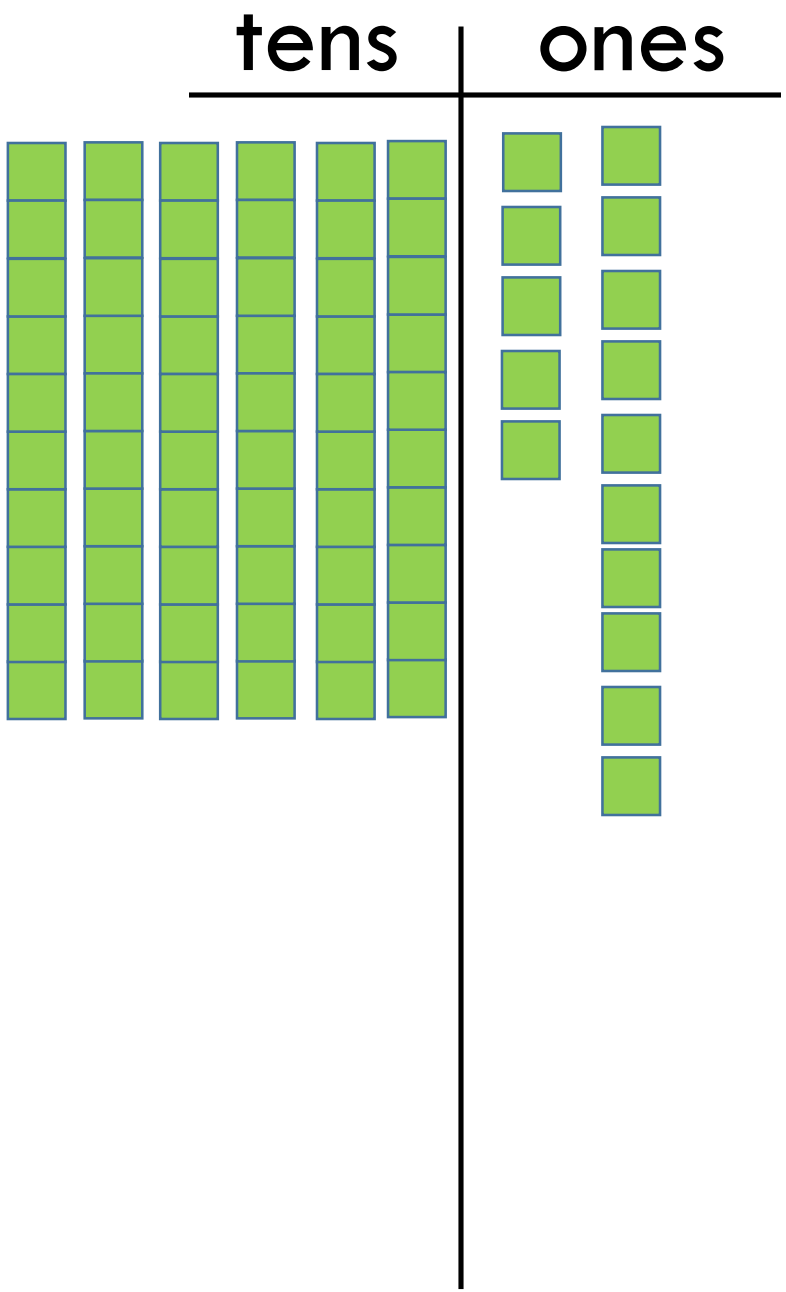

37+5
32
40+2


37+5=42
3
2
40+2=42


37+5
30+12=42


37+5
5+
32+10=42
32
10

65 - 8 = ?



Transitioning to Place Value Chips

10 FEDERAL RESERVE NOTE 10
DF 62655841 A
F6
THE UNITED STATES OF AMERICA
DF 62655841 A
TEN DOLLARS

FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
B 03542754 F
ONE DOLLAR

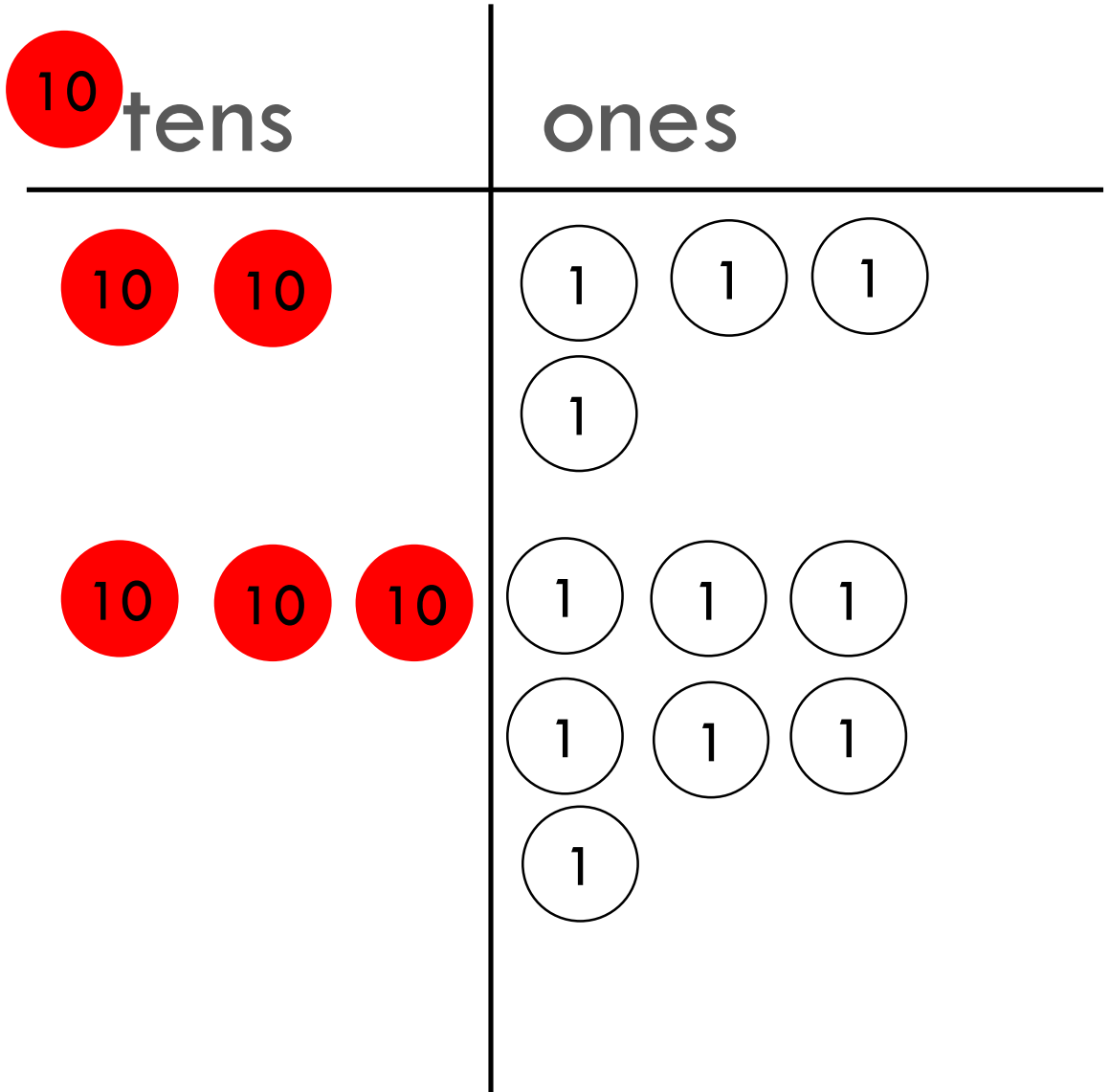
hundreds	tens	ones

Grade 2 – Unit 2
Addition & Subtraction

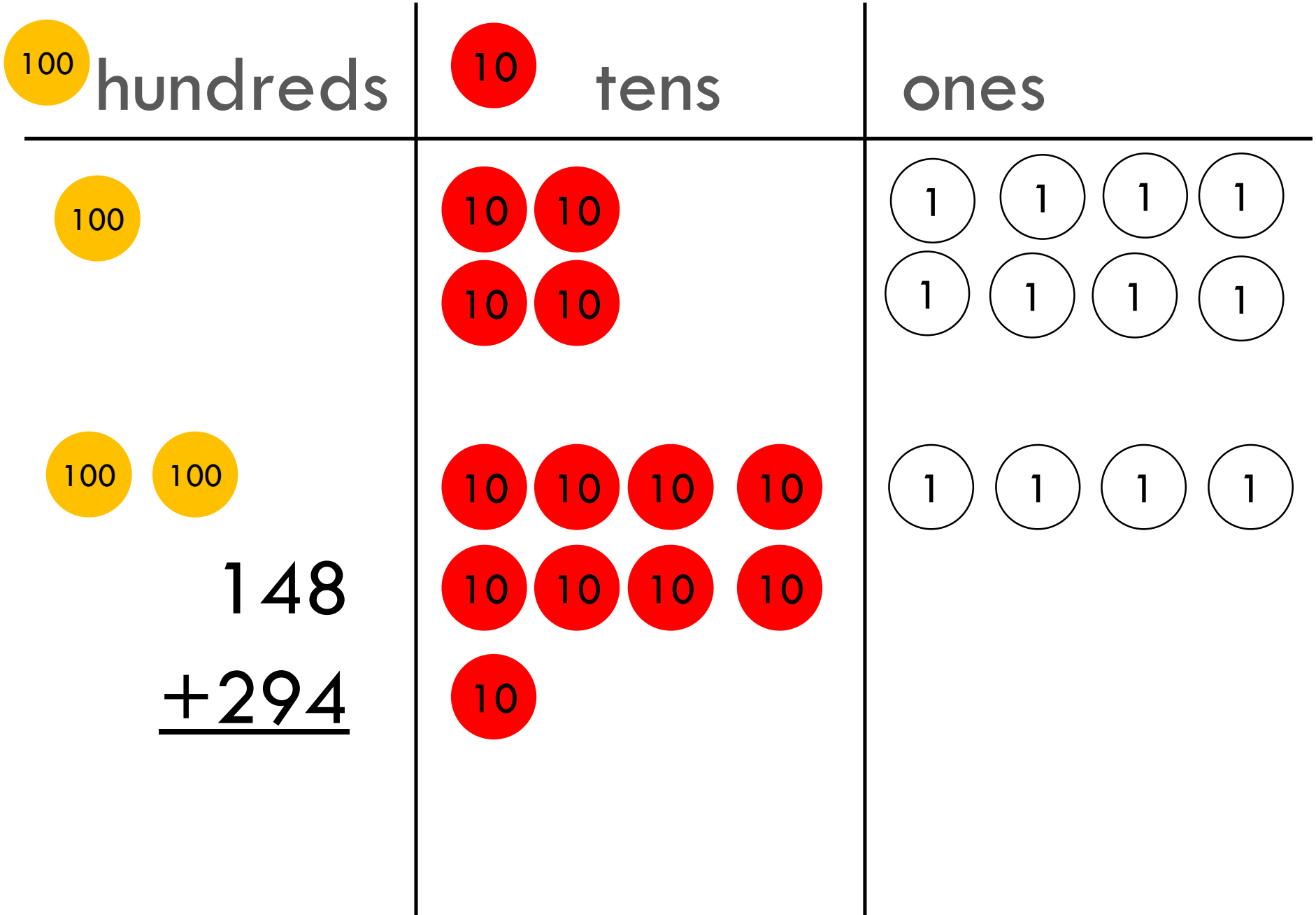
Renaming ones

24 + 37

$$\begin{array}{r} 24 \\ +\ 37 \\ \hline \end{array}$$



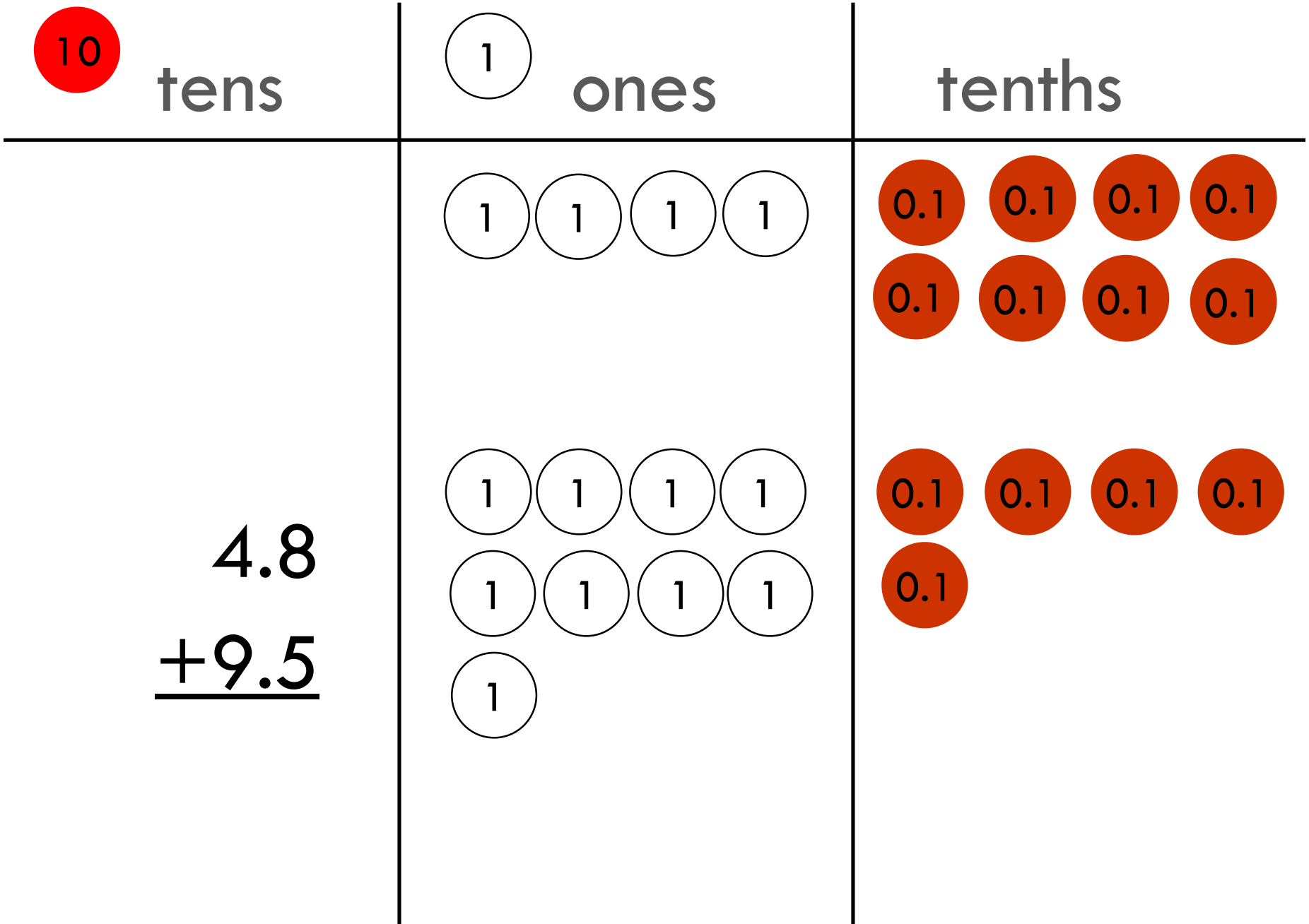
148 + 294



148

+294

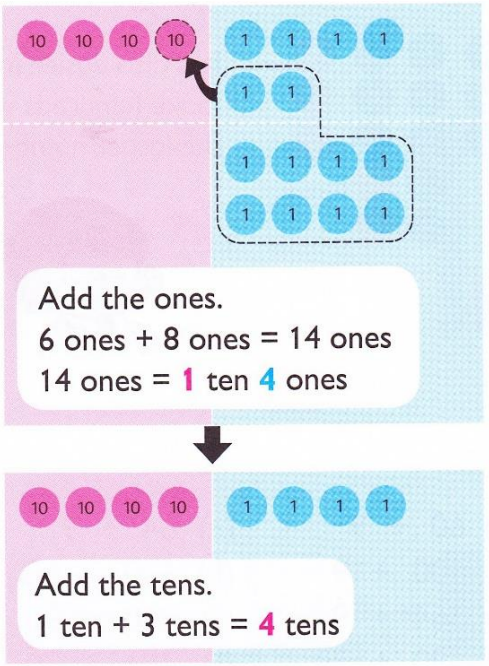
4.8 + 9.5


10
tens
1
ones
tenths
1 1 1 1
0.1 0.1 0.1 0.1
0.1 0.1 0.1 0.1
1 1 1 1
1 1 1 1
1
0.1 0.1 0.1 0.1
0.1
4.8
+9.5

2. Add 36 and 8.

$$\begin{array}{r} 36 \\ +\ \ 8 \\ \hline \square \end{array}$$

Here is another way to add.
Add the ones first.
When there are 10 ones or more, change 10 ones for 1 ten.



Add the ones.

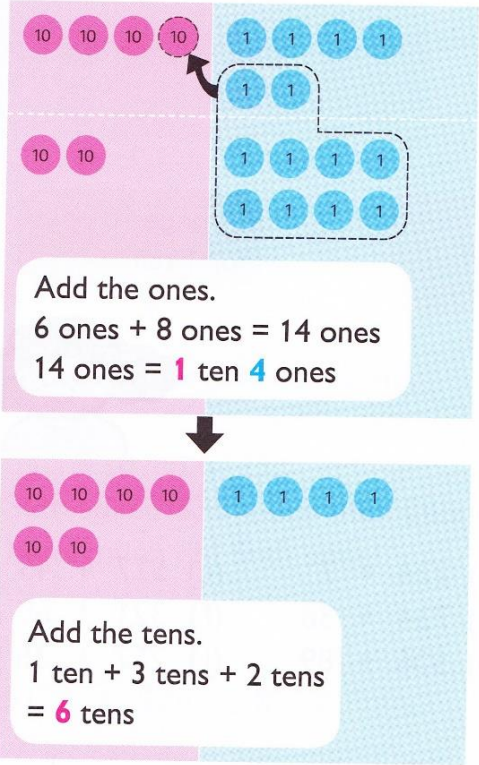
$$\begin{array}{r} {}^{1}3\,6 \\ +\ \ 8 \\ \hline 4 \end{array}$$

Add the tens.

$$\begin{array}{r} {}^{1}3\,6 \\ +\ \ 8 \\ \hline 4\,4 \end{array}$$

3. Add 36 and 28.

$$\begin{array}{r} 36 \\ +\ 28 \\ \hline \square \end{array}$$



Add the ones.

$$\begin{array}{r} {}^{1}3\,6 \\ +\ 28 \\ \hline 4 \end{array}$$

Add the tens.

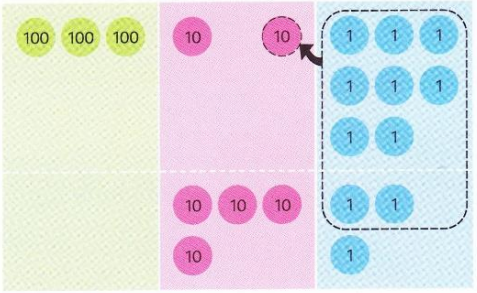
$$\begin{array}{r} {}^{1}3\,6 \\ +\ 28 \\ \hline 6\,4 \end{array}$$

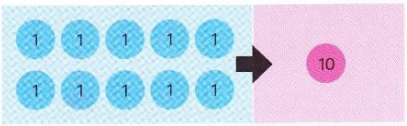
4. Find the value of

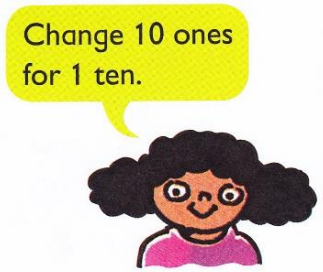
(a) 35 + 7 (b) 75 + 5 (c) 48 + 38
(d) 54 + 29 (e) 57 + 13 (f) 59 + 31
(g) 28 + 45 (h) 54 + 27 (i) 36 + 36

5. Add 318 and 43.
Do we have to change 10 ones for 1 ten?

$$\begin{array}{r} 318 \\ +\ \ 43 \\ \hline \square \end{array}$$





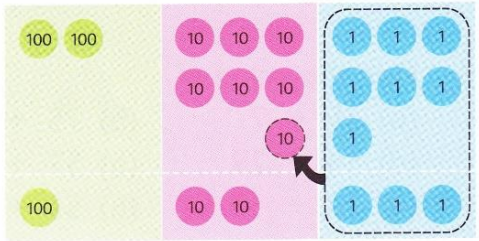


6. Find the value of

(a) 315 + 8	(b) 224 + 7	(c) 527 + 45
(d) 608 + 48	(e) 734 + 36	(f) 321 + 69
(g) 428 + 63	(h) 506 + 89	(i) 257 + 36

7. Add 267 and 123.

$$\begin{array}{r} 267 \\ +\ 123 \\ \hline \square \end{array}$$



8. Find the value of

(a) 127 + 365	(b) 452 + 219
(c) 639 + 124	(d) 745 + 136
(e) 506 + 104	(f) 828 + 162

9. Lily is 18 years old.
Her father is 26 years older than Lily.
How old is her father?

$$\begin{array}{r} 18 \\ +\ 26 \\ \hline \square \end{array}$$

18 + 26 = ☐

Her father is ☐ years old.

10. Diane saved $125.
Seth saved $36 more than Diane.
How much did Seth save?

$$\begin{array}{r} 125 \\ +\ 36 \\ \hline \square \end{array}$$

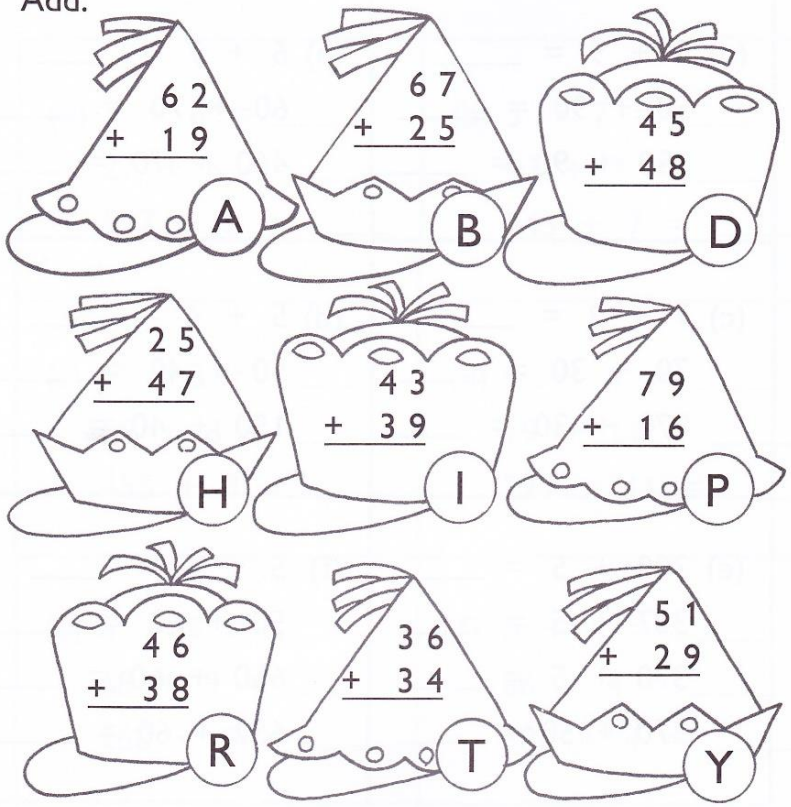
$125 + $36 = $☐

Seth saved $☐.

Exercise 11, pages 60–61

EXERCISE 11

1. Add.



Write the letters that match the answers.
You will find a message.

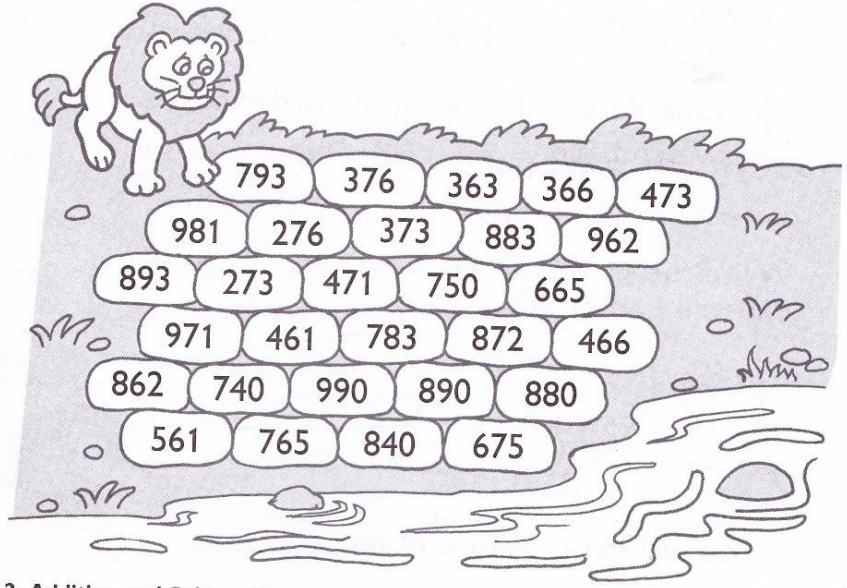
	A			
72	81	95	95	80

						A	
92	82	84	70	72	93	81	80

2. Add.

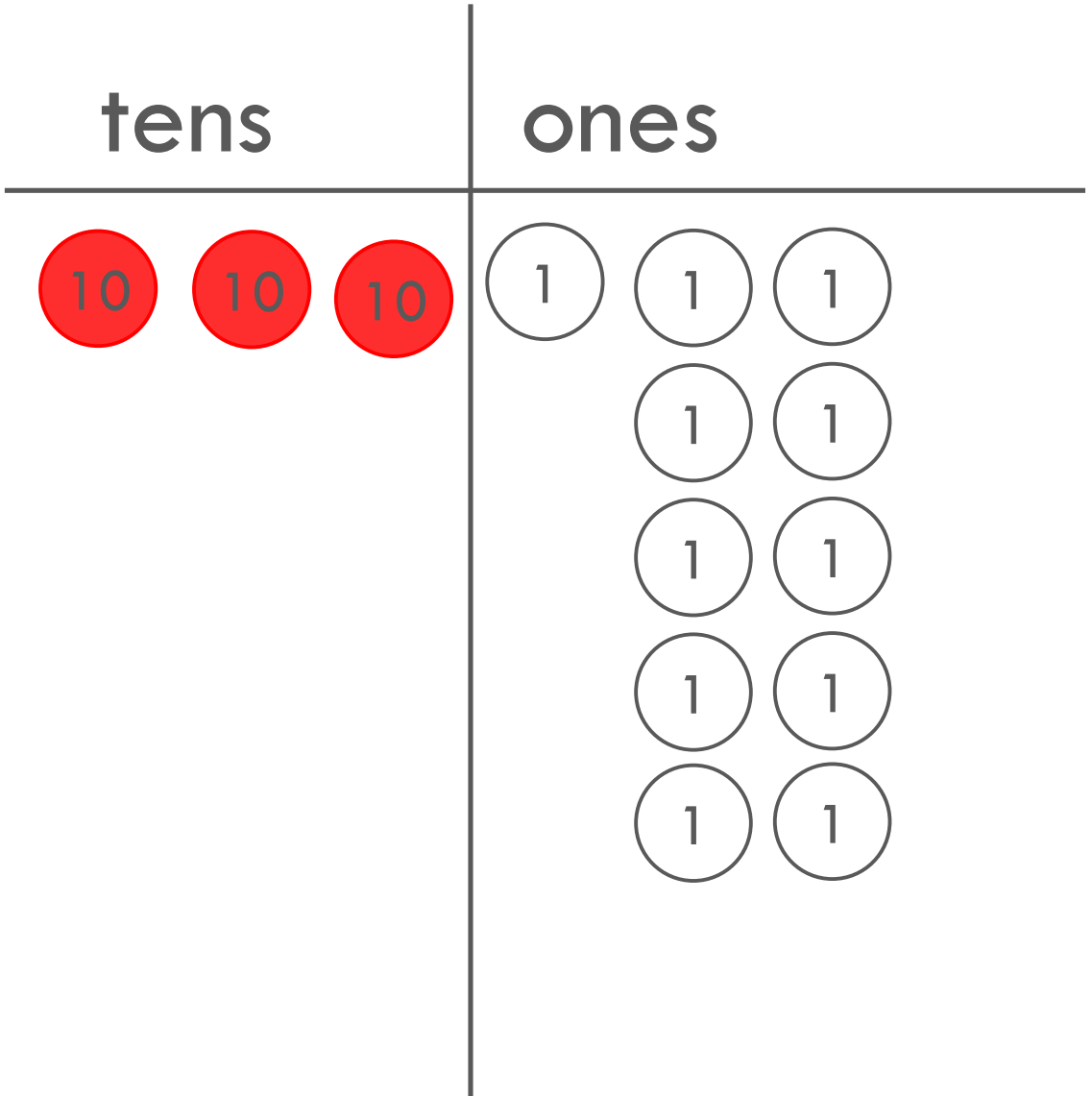
913 + 68	234 + 139	402 + 69
527 + 266	703 + 169	328 + 48
605 + 145	346 + 329	836 + 54

Color the spaces that contain the answers.
You will help Mr. Lion find his way to the river.



Regrouping/Renaming

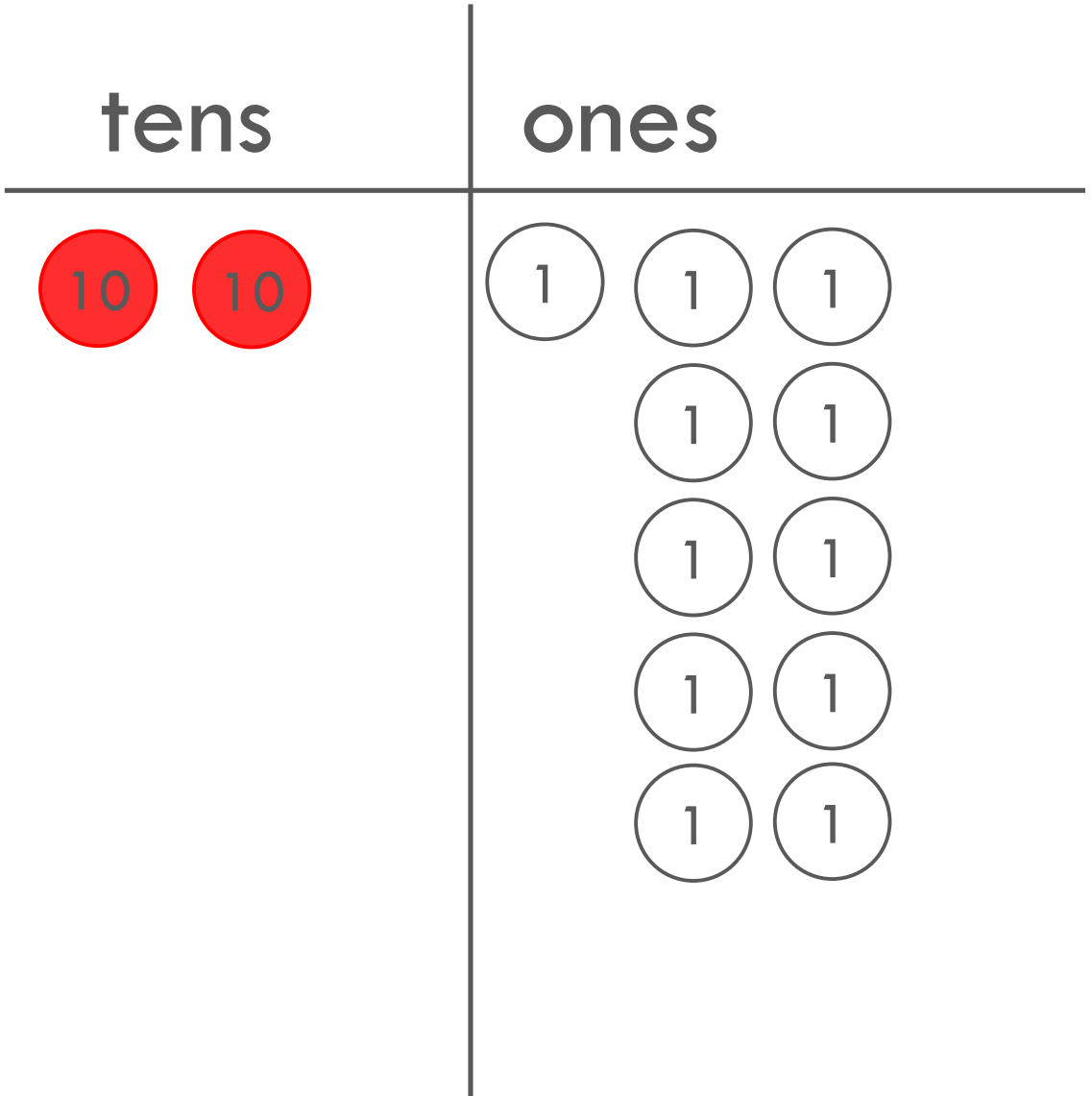
31 - 16



Regrouping/Renaming

31 - 16

$$\begin{array}{r} 31 \\ \underline{-\ 16} \end{array}$$



91
-73

4000 - 278

thousands	hundreds	tens	ones
1000 1000 1000 1000	100 100 100 100 100 100 100 100 100 100	10 10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

4000 - 278

thousands	hundreds	tens	ones
1000 1000 1000	100 100 100 100 100 100 100 100 100	10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

$$\begin{array}{r} 4000 \\ -\ 278 \\ \hline \end{array}$$

4000 - 278

thousands	hundreds	tens	ones
1000 1000 1000 1000	100 100 100 100 100 100 100 100 100 100	10 10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

$$\begin{array}{r} 4000 \\ -\ 278 \\ \hline \end{array}$$

4000 - 278

thousands	hundreds	tens	ones
1000 1000 1000	100 100 100 100 100 100 100 100 100	10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

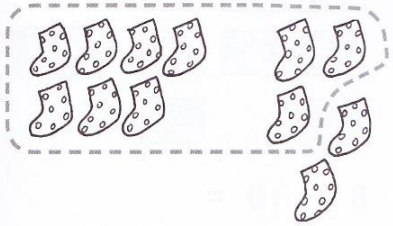
3 9 9 10

~~4000~~

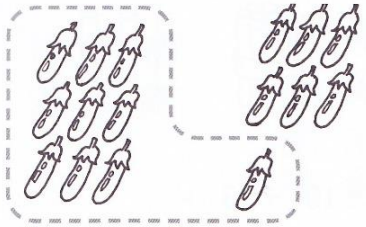
\- 278

EXERCISE 6

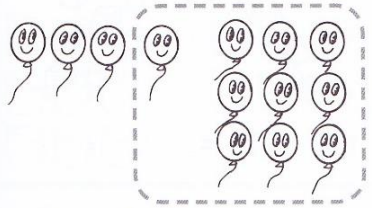
1. Add.



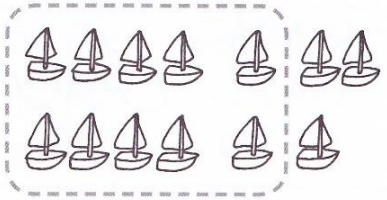
7 + 5 =



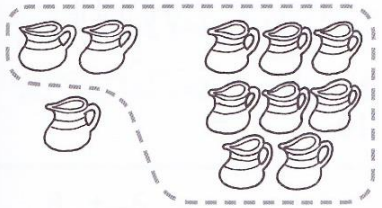
9 + 7 =



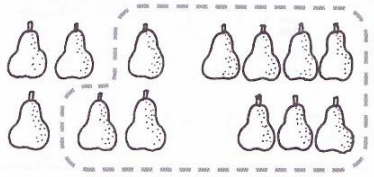
4 + 9 =



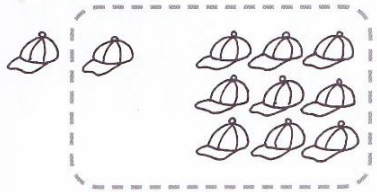
8 + 5 =



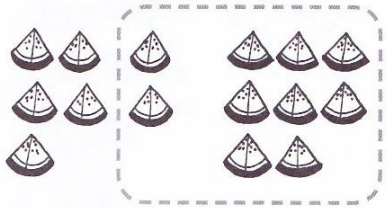
3 + 8 =



6 + 7 =

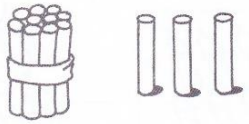


2 + 9 =

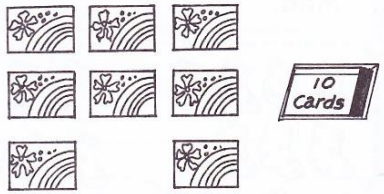


7 + 8 =

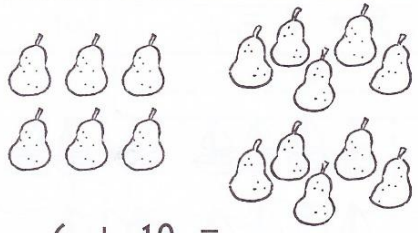
2. Add.



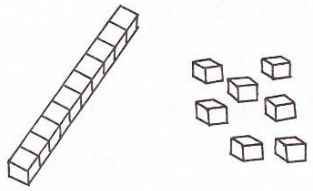
10 + 3 =



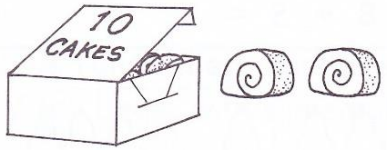
8 + 10 =



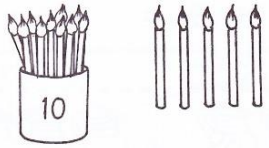
6 + 10 =



10 + 7 =

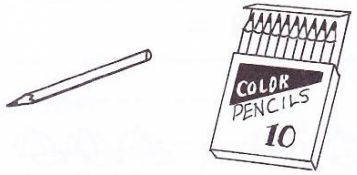


10 + 2 =



10 + 5 =

4 + 10 =



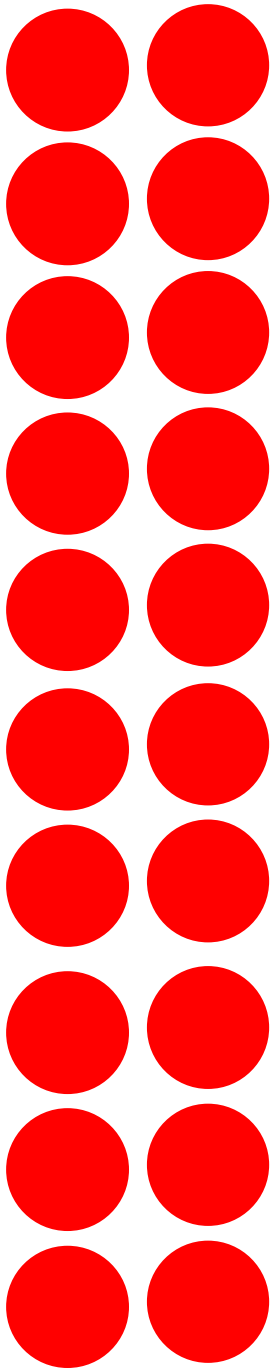
1 + 10 =

4.2 - 1.3

tens	ones	tenths
	1 1 1 1	0.1 0.1
4.2		0.1 0.1 0.1 0.1
- 1.3		0.1 0.1 0.1 0.1
		0.1 0.1

Grade 2 – Multiplication & Division

Build Multiplication table for 2



$2 \times 1 = 2$

$2 \times 2 = 4$

$2 \times 3 = 6$

$2 \times 4 = 8$

$2 \times 5 = 10$

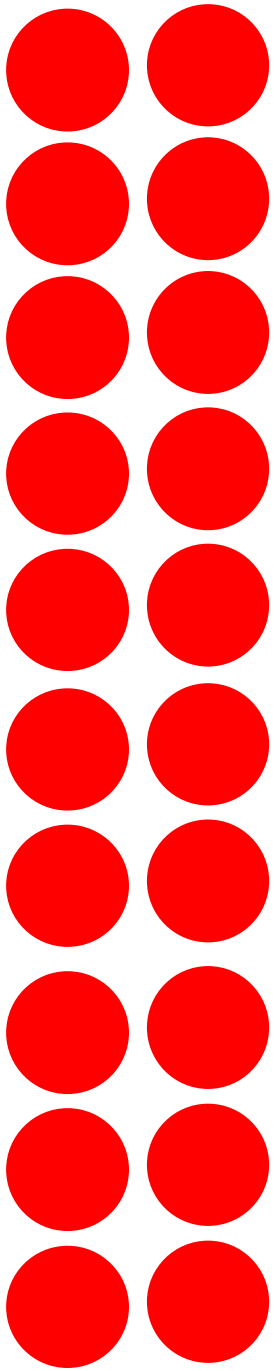
$2 \times 6 = 12$

$2 \times 7 = 14$

$2 \times 8 = 16$

$2 \times 9 = 18$

$2 \times 10 = 20$



$2 \times 1 = 2$

+2

$2 \times 2 = 4$

+2

$2 \times 3 = 6$

+2

$2 \times 4 = 8$

+2

$2 \times 5 = 10$

+2

$2 \times 6 = 12$

+2

$2 \times 7 = 14$

+2

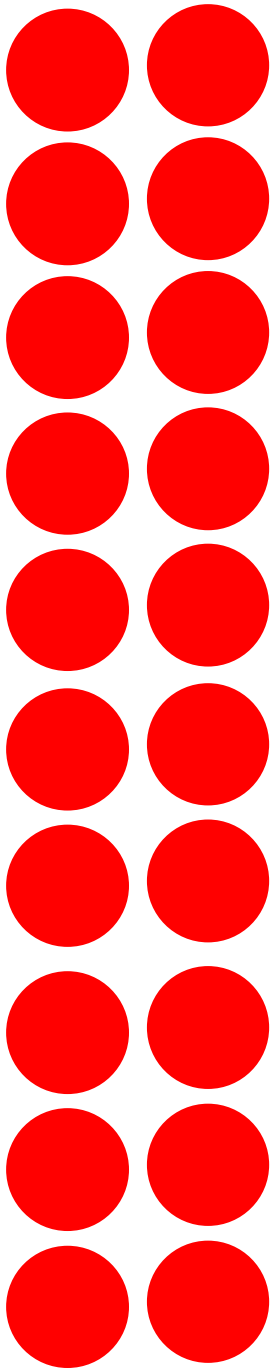
$2 \times 8 = 16$

+2

$2 \times 9 = 18$

+2

$2 \times 10 = 20$



$2 \times 1 = 2$

+2

$2 \times 2 = 4$

+2

$2 \times 3 = 6$

+2

$2 \times 4 = 8$

+2

$2 \times 5 = 10$

+2

$2 \times 6 = 12$

+2

$2 \times 7 = 14$

+2 **-2**

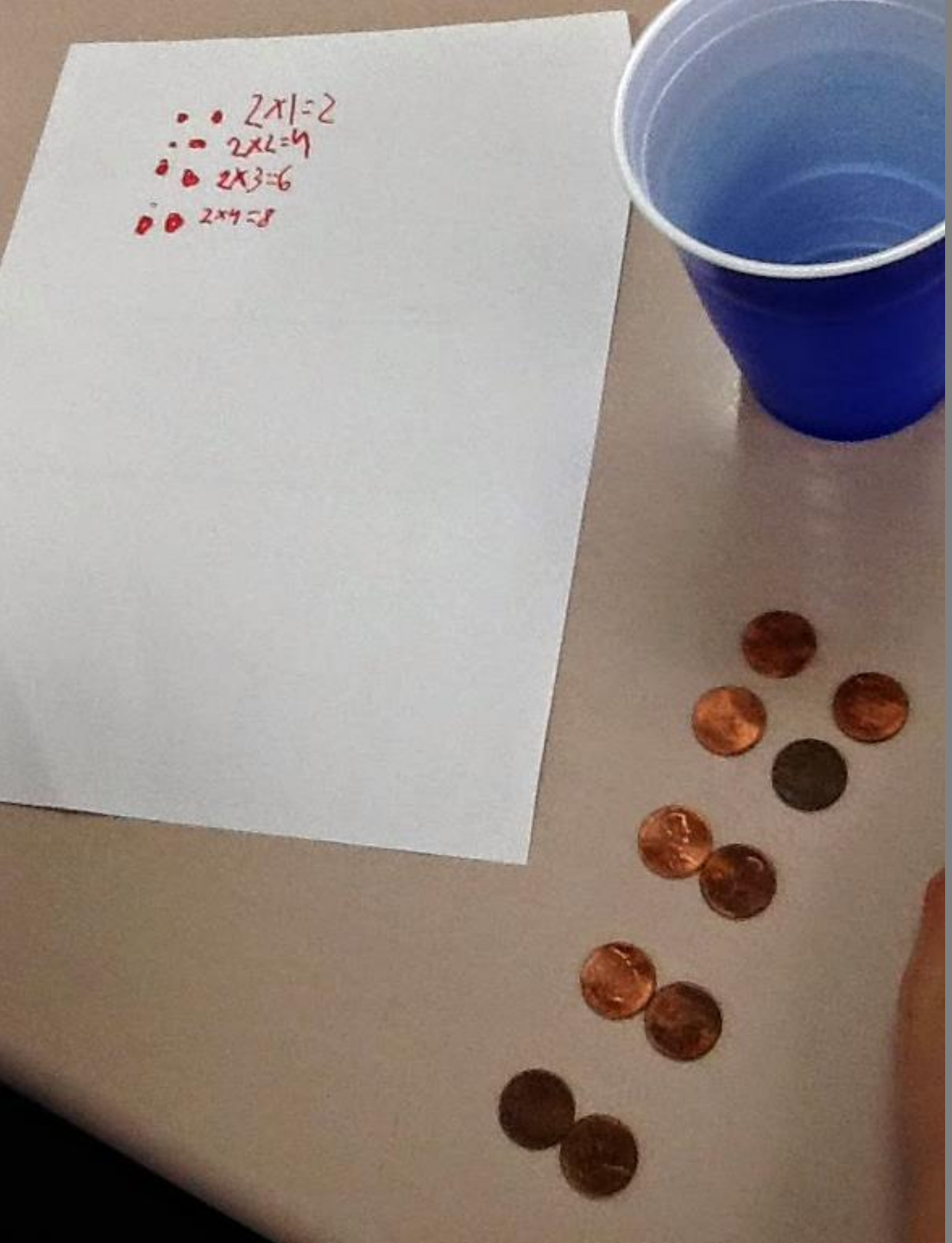
$2 \times 8 = 16$

+2 **-2**

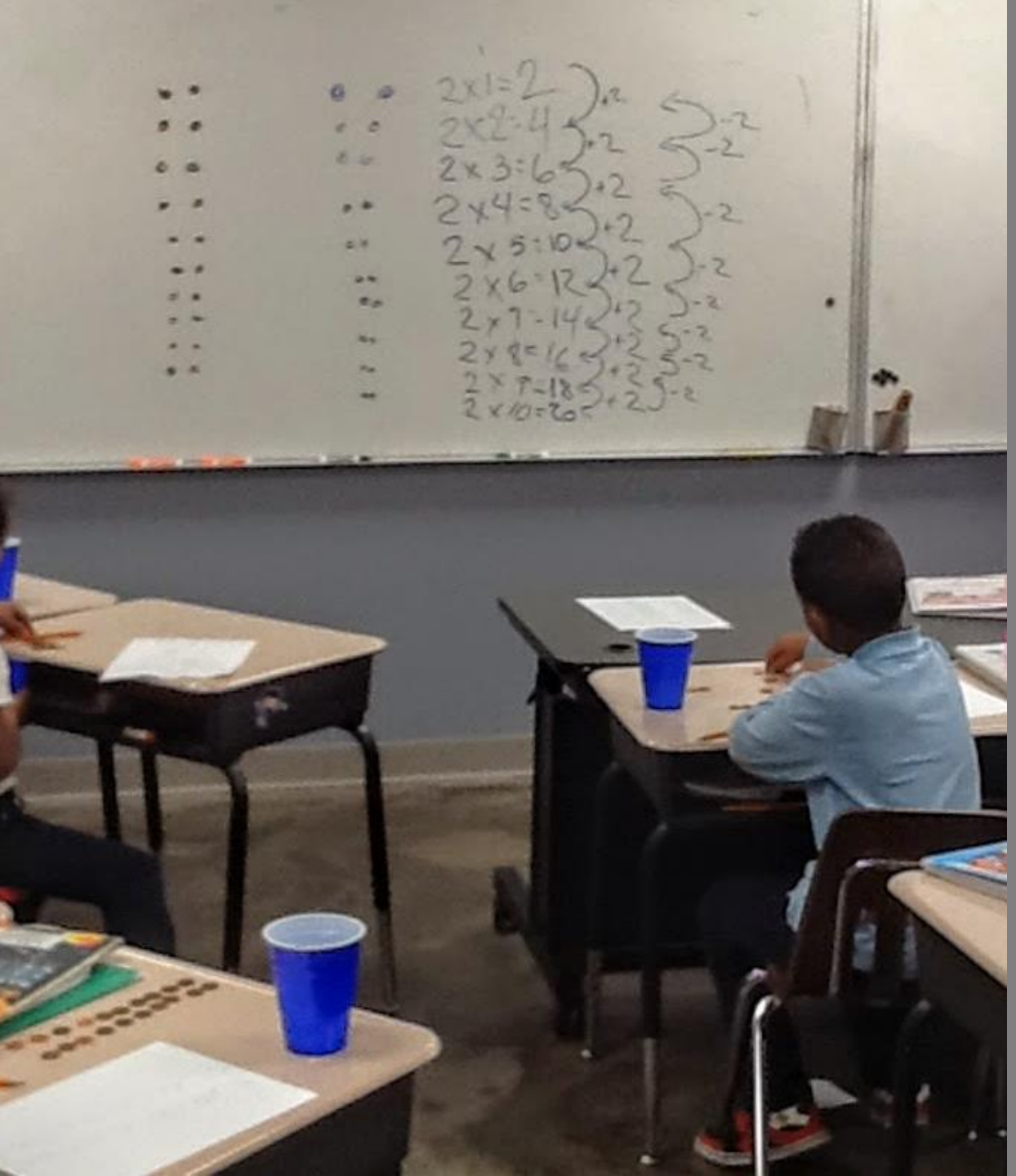
$2 \times 9 = 18$

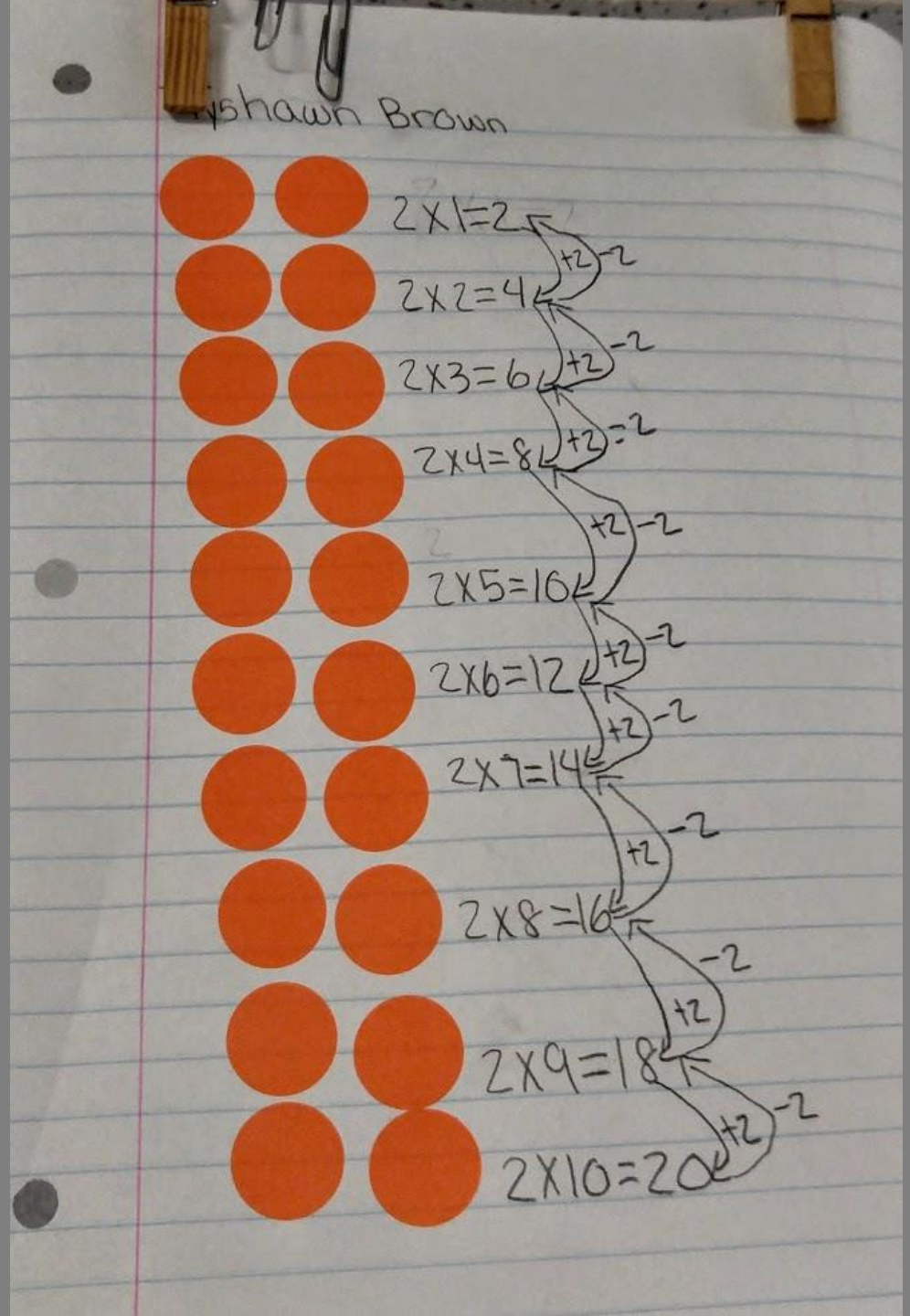
+2 **-2**

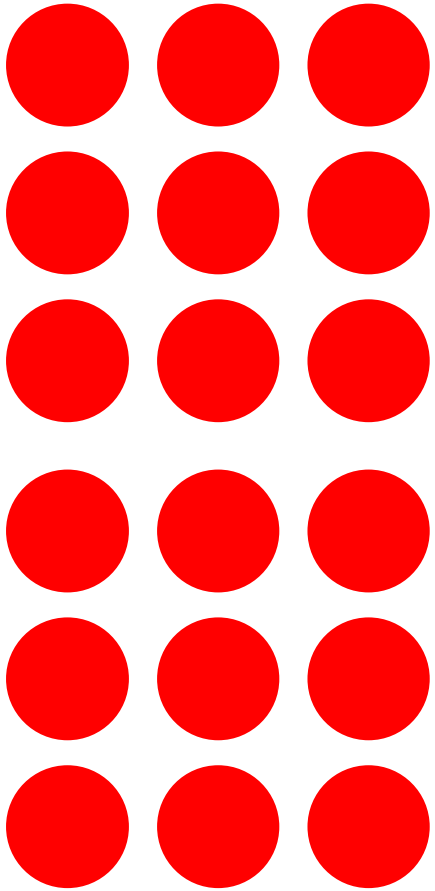
$2 \times 10 = 20$


2x1=2
2x2=4
2x3=6
2x4=8

1x2=2


2x1=2
2x2=4
2x3=6
2x4=8
2x5=10
2x6=12
2x7=14
2x8=16
2x9=18
2x10=20
+2
-2


shawn Brown
2x1=2
2x2=4
2x3=6
2x4=8
2x5=10
2x6=12
2x7=14
2x8=16
2x9=18
2x10=20
+2
-2



$3 \times 1 = 3 = 1 \times 3$

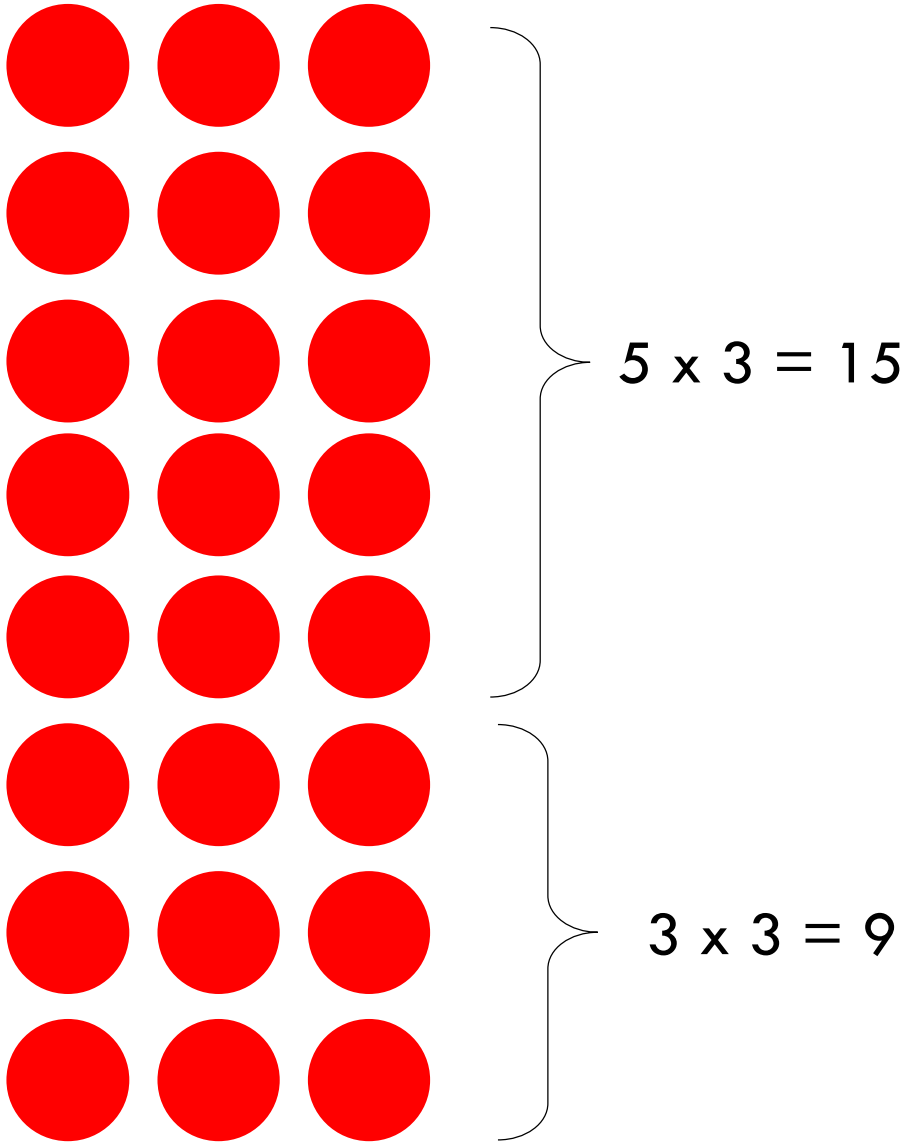
$3 \times 2 = 6 = 2 \times 3$

$3 \times 3 = 9 = 3 \times 3$

$3 \times 3 = 9$

2 groups of $3 \times 3 = 9 + 9$

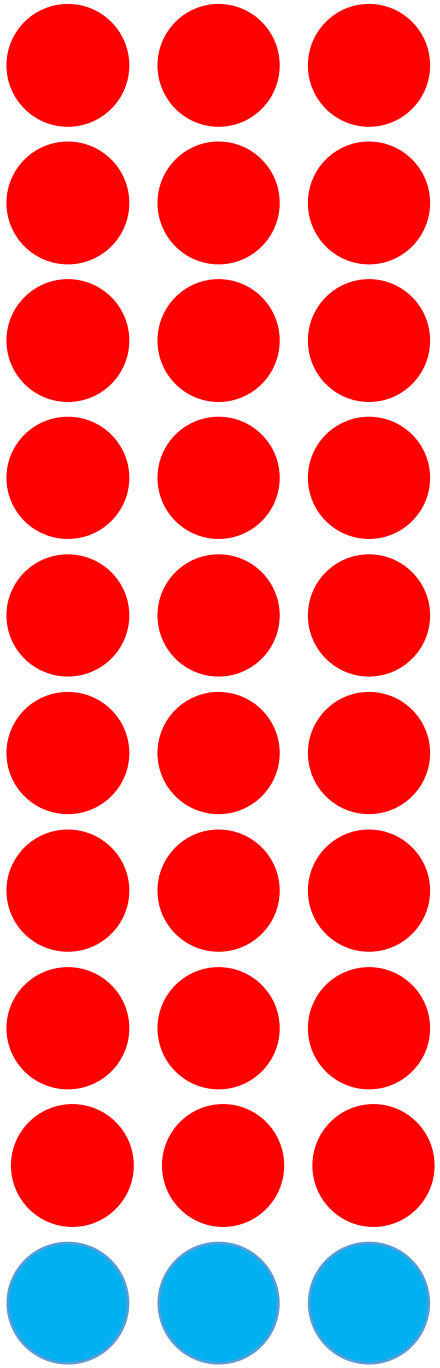
6 groups of $3 = 18$



$5 \times 3 = 15$

$3 \times 3 = 9$

8 groups of $3 = 15 + 9$

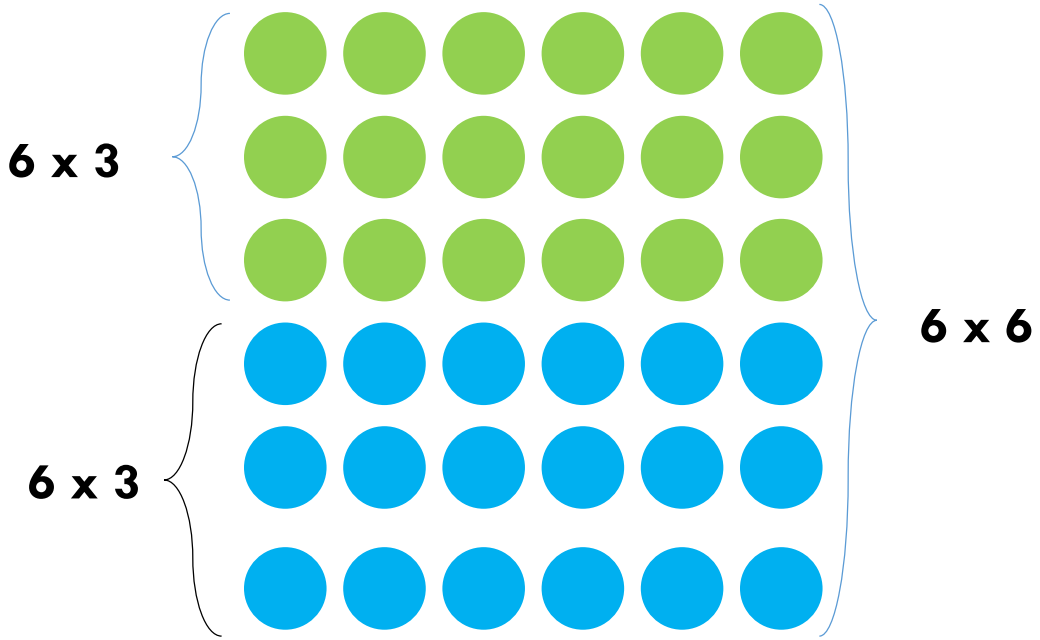


$9 \times 3 =$

$10 \times 3 - 1 \times 3 = 30 - 3$

Multiplication Facts

$$6 \times 3 = 18$$



$$6 \times 6 = 18 + 18$$

3 3

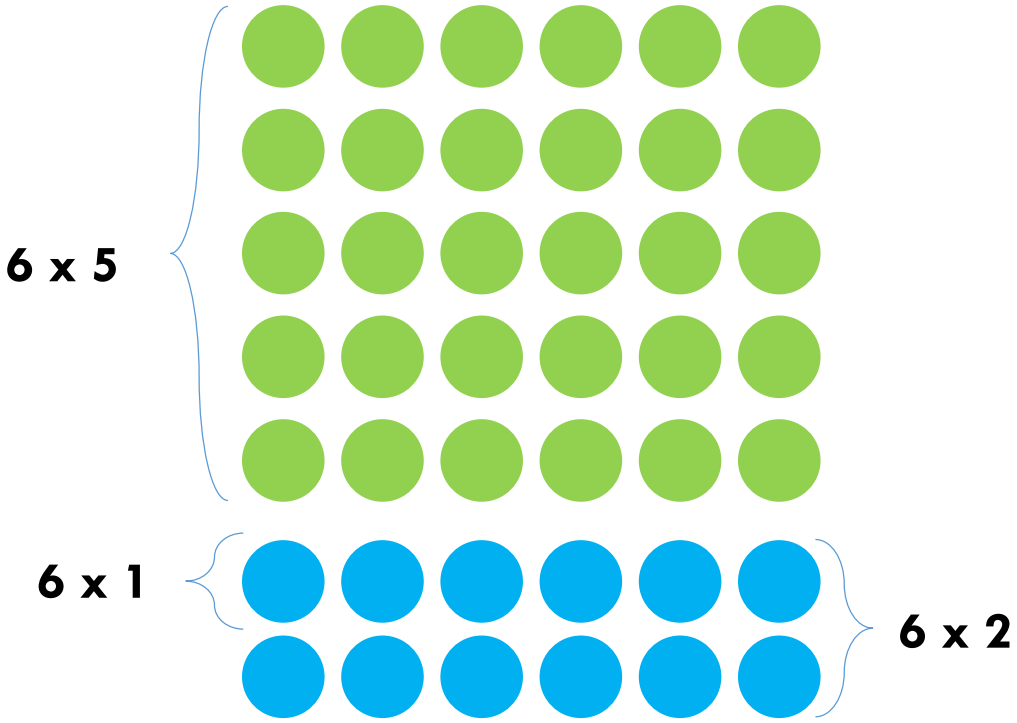
Multiplication Facts

$$6 \times 6 = 30 + 5$$

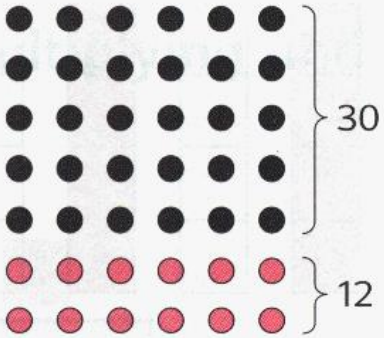
5 1

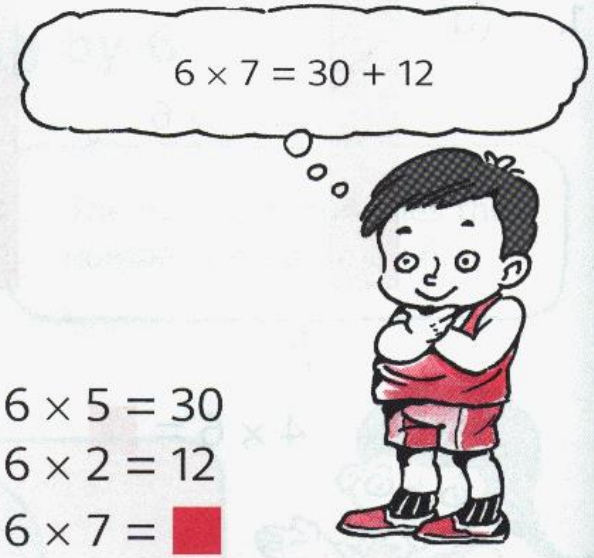
$$6 \times 7 = 30 + 12$$

5 2



(b)



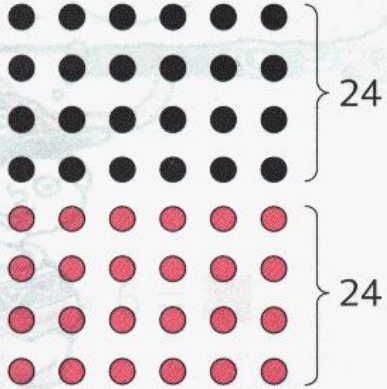


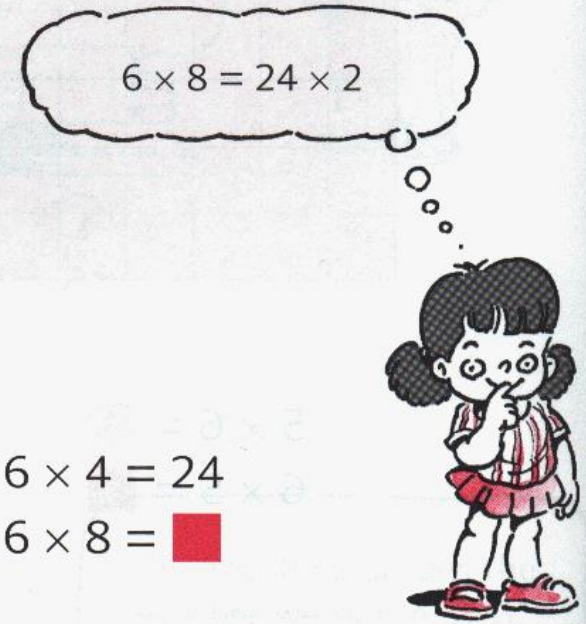
$6 \times 5 = 30$

$6 \times 2 = 12$

$6 \times 7 = \square$

(c)





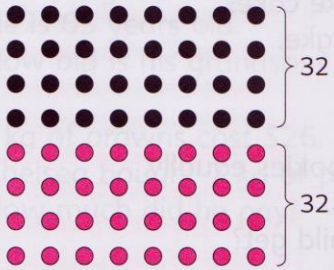
$6 \times 4 = 24$

$6 \times 8 = \square$

4 Multiplying and Dividing by 8

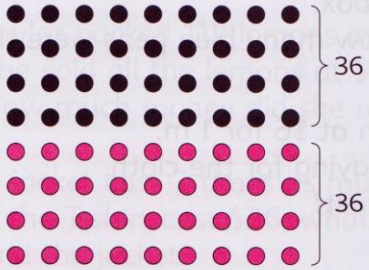
× 2 × 2

1 × 2 = 2	1 × 4 = 4	1 × 8 = 8
2 × 2 = 4	2 × 4 = 8	2 × 8 = 16
3 × 2 = 6	3 × 4 = 12	3 × 8 = 24
4 × 2 = 8	4 × 4 = 16	4 × 8 = 32
5 × 2 = 10	5 × 4 = 20	5 × 8 = 40
6 × 2 = 12	6 × 4 = 24	6 × 8 = 48
7 × 2 = 14	7 × 4 = 28	7 × 8 = 56
8 × 2 = 16	8 × 4 = 32	8 × 8 = ■
9 × 2 = 18	9 × 4 = 36	9 × 8 = ■
10 × 2 = 20	10 × 4 = 40	10 × 8 = 80



8 × 4 = 32

8 × 8 = ■



9 × 4 = 36

9 × 8 = ■

Grade 3 – Multiplication & Division

Multiply a 2-digit number by a 1-digit number

42 x 2

40 2

hundreds	tens	ones
	10 10 10 10	1 1

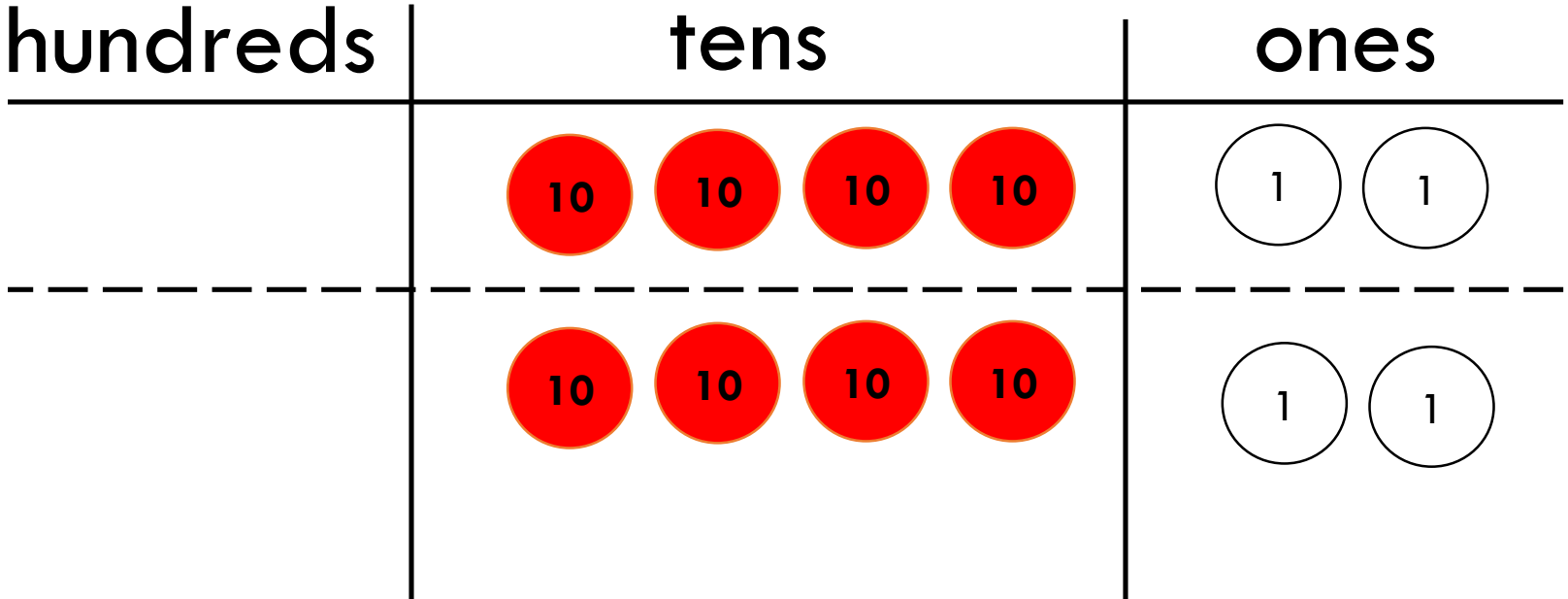
$$42 \times 2$$

$$40 \quad 2$$

$$40 \times 2 = 80$$

$$2 \times 2 = 4$$

$$80 + 4 = 84$$



$$
\begin{array}{r}
42 \\
\times \quad 2 \\
\hline
4 \\
80 \\
\hline
84
\end{array}
\qquad
\begin{array}{l}
2 \times 2 \\
40 \times 2
\end{array}
$$

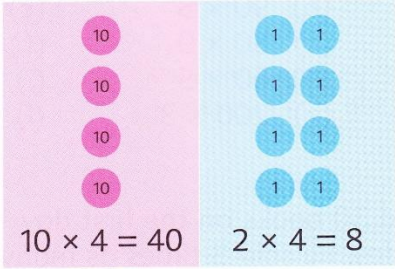
42 x 4

hundreds	tens	ones
	10 10 10 10	1 1
	10 10 10 10	1 1
	10 10 10 10	1 1
	10 10 10 10	1 1

$$
\begin{array}{r}
42 \\
\times \quad 4 \\
\hline
8 \\
160 \\
\hline
168
\end{array}
\qquad
\begin{array}{l}
2 \times 4 \\
40 \times 4
\end{array}
$$

4. Multiply 12 by 4.

$$\begin{array}{r} 12 \\ \times \quad 4 \\ \hline \square \end{array}$$



Multiply the ones by 4.

$$\begin{array}{r} 12 \\ \times \quad 4 \\ \hline 8 \end{array}$$

Multiply the tens by 4.

$$\begin{array}{r} 12 \\ \times \quad 4 \\ \hline 48 \end{array}$$

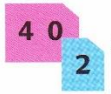
12 × 4 = (**10** × 4) + (**2** × 4)
= 40 + 8

When we multiply 12 by 4, the **product** is 48.

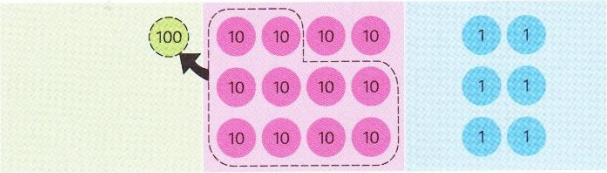
5. Find the product.
(a) 23 × 3 (b) 32 × 2 (c) 2 × 42 (d) 4 × 21

6. Multiply 42 by 3.

$$\begin{array}{r} 42 \\ \times \quad 3 \\ \hline \square \end{array}$$



42 × 3 = (**40** × 3) + (**2** × 3)
= 120 + 6



Multiply the ones by 3.

$$\begin{array}{r} 42 \\ \times \quad 3 \\ \hline 6 \end{array}$$

Multiply the tens by 3.

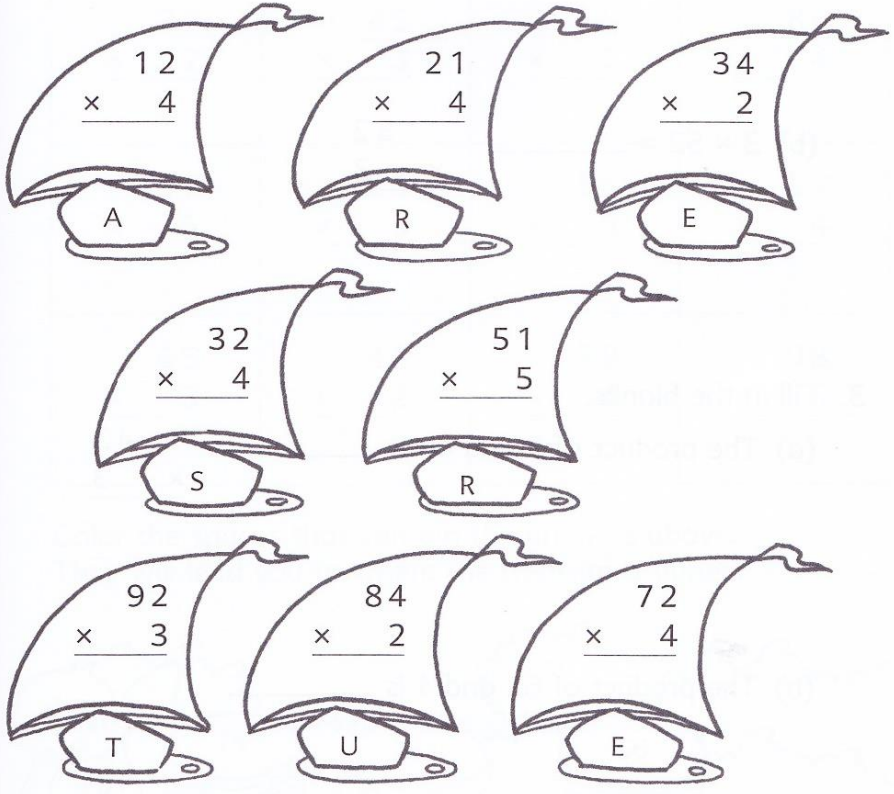
$$\begin{array}{r} 42 \\ \times \quad 3 \\ \hline 126 \end{array}$$

7. Find the product.
(a) 52 × 3 (b) 64 × 2 (c) 91 × 4 (d) 73 × 2

Exercise 9, pages 99–100

EXERCISE 9

1. Multiply.



Where are the boats sailing to?

Write the letters that match the answers to find out.

			A				
276	84	68	48	128	168	255	288

ISLAND

2. Find the product of each of the following.

(a) 41 × 5 = _____

$$\begin{array}{r} 41 \\ \times \quad 5 \\ \hline ___ \end{array}$$

(b) 3 × 52 = _____

$$\begin{array}{r} 52 \\ \times \quad 3 \\ \hline ___ \end{array}$$

3. Fill in the blanks.

(a) The product of 3 and 43 is _____.

$$\begin{array}{r} 43 \\ \times \quad 3 \\ \hline ___ \end{array}$$

(b) The product of 62 and 4 is _____.

(c) The product of 5 and 71 is _____.

36 x 3

30 6

(30 x 3) + (6 x 3)

hundreds	tens	ones
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1

hundreds	tens	ones
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1

hundreds	10 tens	ones
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1
	10 10 10	1 1 1 1 1 1

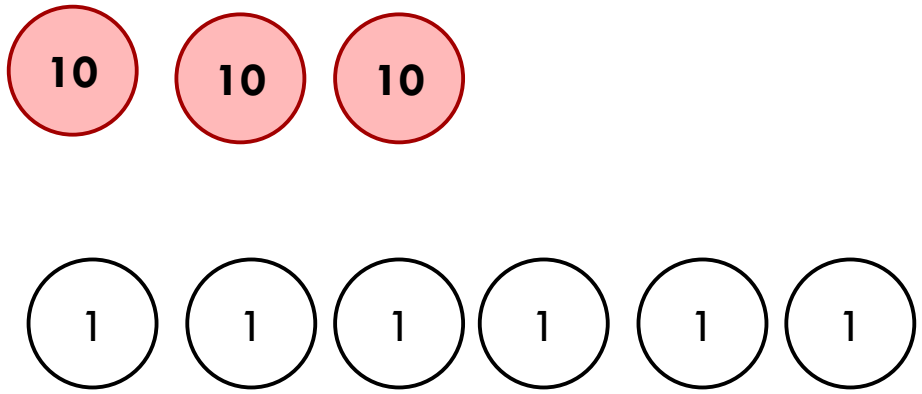
hundreds	10 tens	ones
100	10 10 10	1 1 1
		1 1 1
	10 10 10	1
		1
	10 10 10	

hundreds	tens	ones
100		1 1 1 1 1 1 1 1
1	0	8

Grade 3 – Long Division

36 ÷ 3= ?

tens	ones

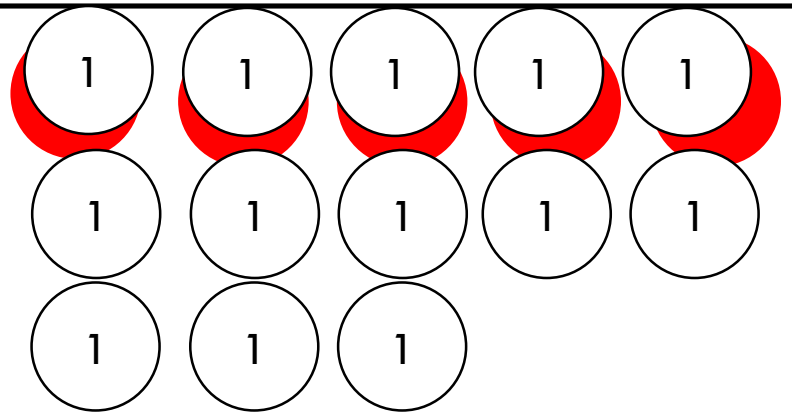


tens

$$
\begin{array}{r|l}
 & 1\ 2 \\
\hline
3 & 3\ 6 \\
 & -\ 3 \\
\hline
 & 0\ 6 \\
 & -\ 6 \\
\hline
 & 0
\end{array}
$$

53 ÷ 2= ?

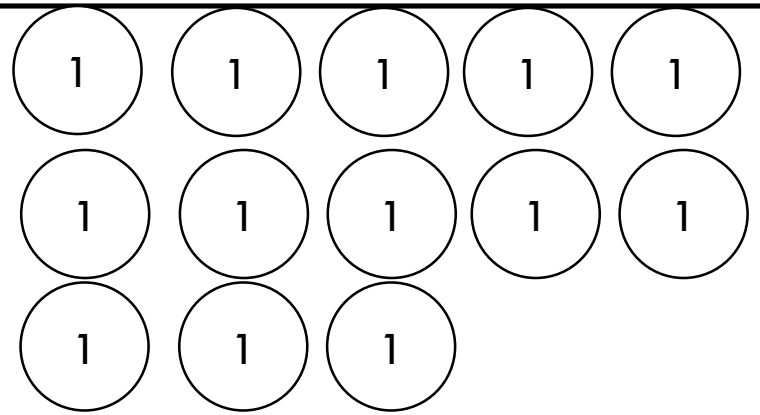
tens	ones



tens

$$\begin{array}{r} 2 \\ 2\overline{)53} \\ \underline{-4} \\ 1 \end{array}$$

53 ÷ 2= ?



tens	ones
10	
10	
10	
10	

tens ones

$$
\begin{array}{r|l}
 & 26 \quad R\ 1 \\
2 & \overline{53} \\
 & \underline{-4} \\
 & 13 \\
 & \underline{-12} \\
 & 1
\end{array}
$$

Division Algorithm

Divide (Does)

Multiply (McDonalds)

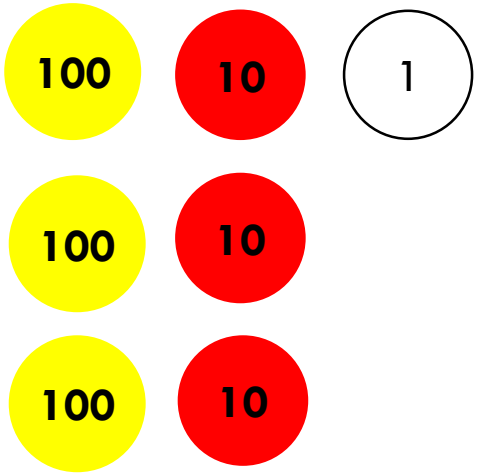
Subtract (Serve)

Bring down (Burgers)

```
      tens  ones
        2 6 R 1
2 ⟌ 5 3
    -4
      1 3
     -1 2
        1
```

331 ÷ 2 = ?

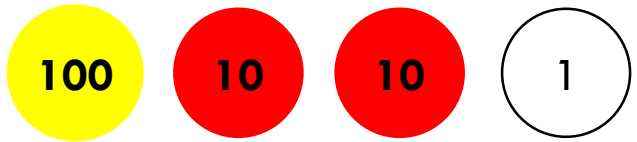
hundreds	tens	ones



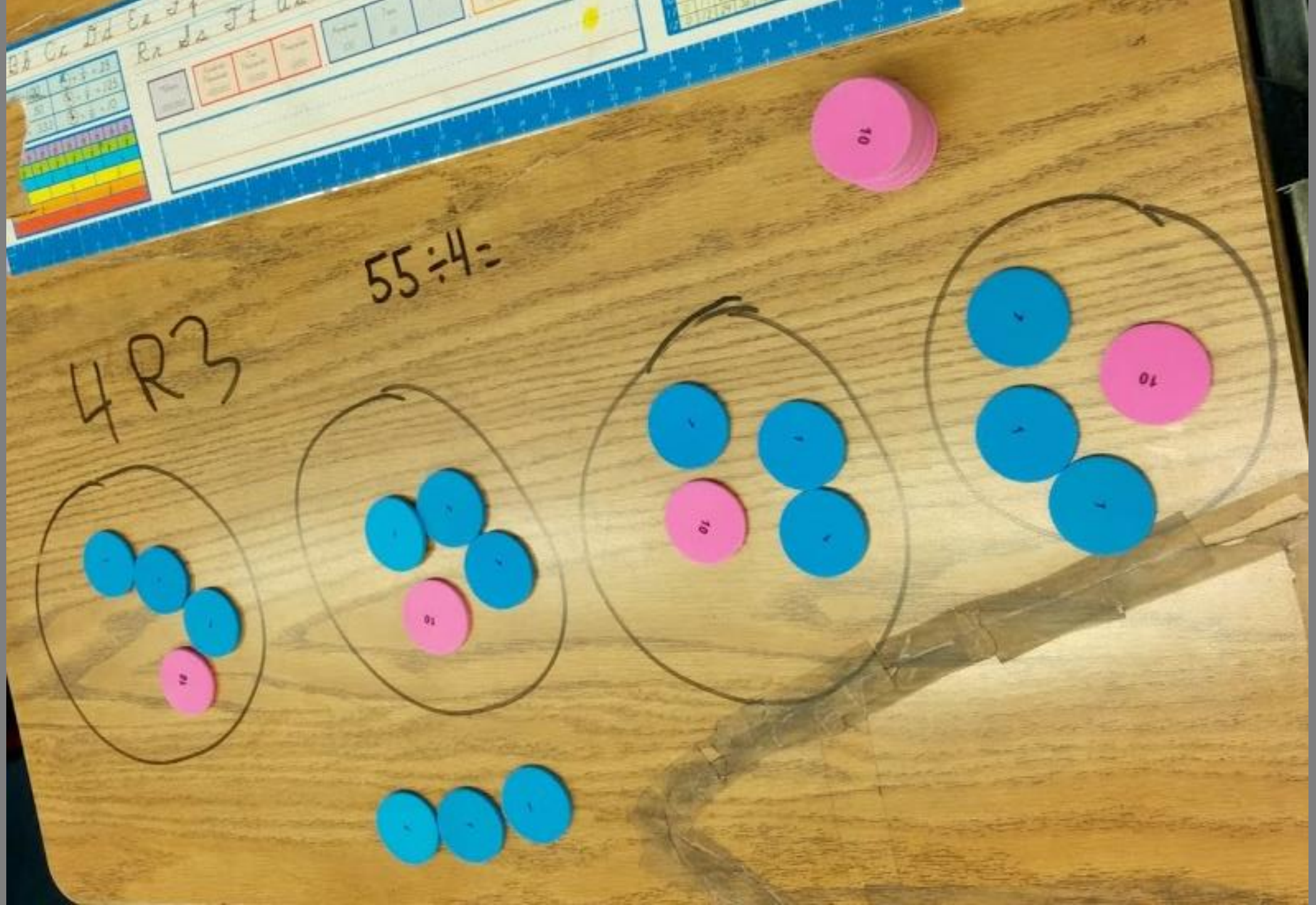
$$2\overline{)331}$$

121 ÷ 2= ?

tens	ones



55÷4=13R3
THINK Ticket
"I was caught thinking!"
10
1
10
1
10
1
1
10
1
1
1


55÷4=
4R3

$5560 \div 5 = ?$

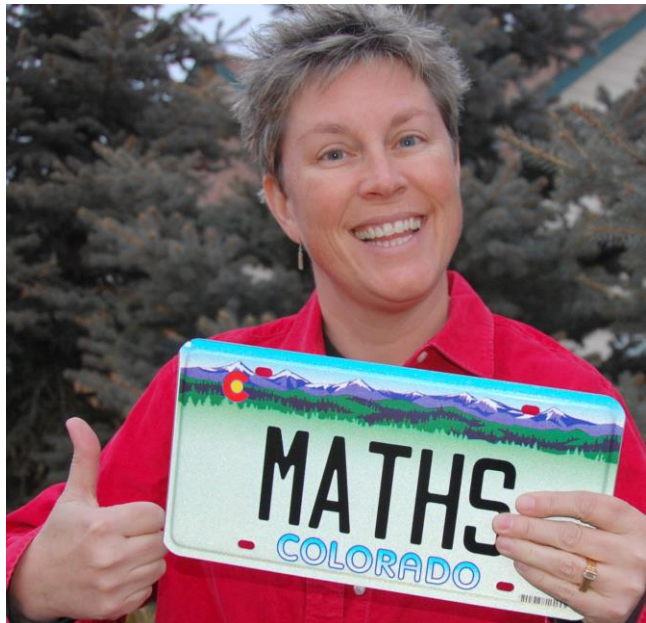
Questions?

Thanks for coming!

Cassy Turner

Cassy@SingaporeMathSource.com

Handout: https://goo.gl/j4aBY



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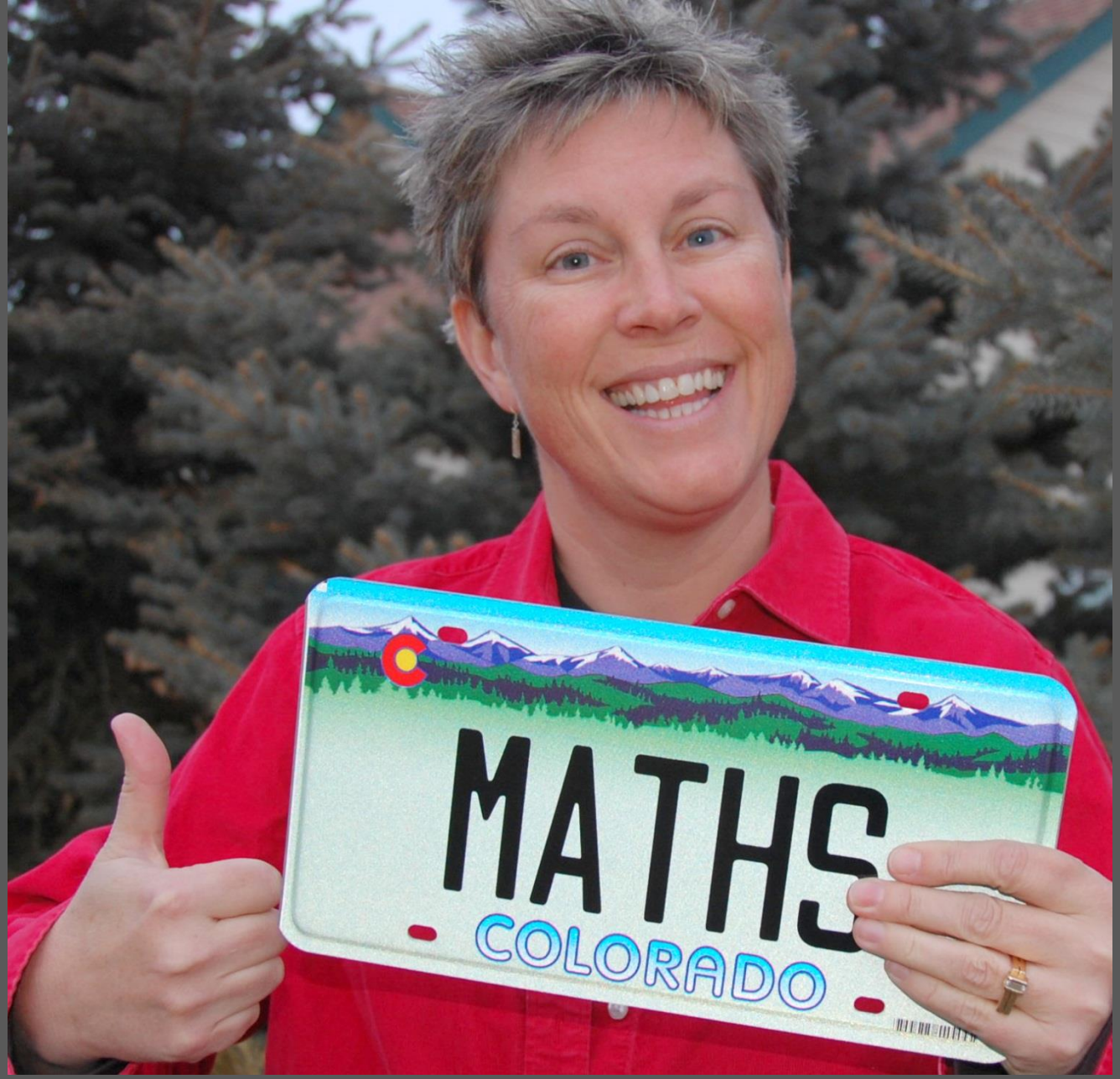

Thanks for coming!