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Singapore Math Demystified! Can Solving Problems Unravel Our Fear Of Math?

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*The Daily Riff is featuring a four-part series **Singapore Math Demystified!** by Bill Jackson, Math Helping Teacher, Scarsdale, NY Public Schools, one of the highest performing districts in the country, due to popular interest in this topic from parents and schools. If you missed Part 1, Why I Became Interested in Singapore Math, check it out HERE.*

For non-math types, don't let the pentagon below "throw you" - just skip it - Bill explains Singapore so clearly and simply for parents AND for educators. Math-phobia is one of the more challenging issues in this country's educational make-up and what better way to learn than from countries such as Singapore, which ranks top internationally. By the way, "pedagogy" is synonymous with "teaching methods" so nothing mysterious about that! Include the word in one of your next parent-teacher conferences and your teacher may find the conversation more intriguing and certainly more challenging. - Ed. Note

Philosophy and Pedagogical Approach of Singapore Math
Part 2

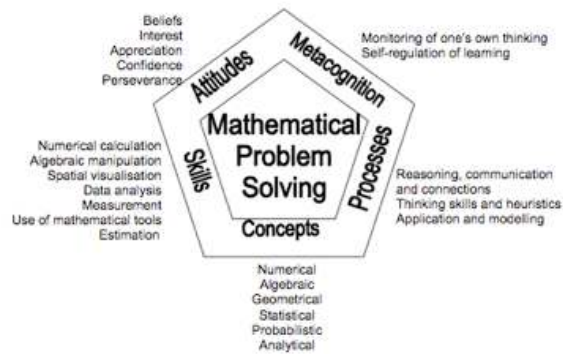by Bill Jackson

In 1989, the National Council of Teachers of Mathematics (NCTM) published the Principles and Standards, which advocated a problem-solving approach for the teaching and learning of mathematics. Thoughtful application of this approach, however is still relatively uncommon in U.S. classrooms. Analysis of the TIMSS videotape study revealed that Japanese math lessons embodied the spirit of the NCTM Standards more than American lessons. How could this be, given that NCTM is a U.S. organization?

It turns out that while American teachers were still trying to make sense of the standards, Japanese teachers were studying how to make them come alive in their classrooms through lesson study.

Another nation that took the NCTM Standards and other research on problem-based approaches seriously is Singapore. After its independence in 1965, Singapore realized
that without any natural resources it would have to rely on human capital for success,
so they embarked on an effort to develop a highly educated citizenry. Various education
reforms were initiated and in 1980 the Curriculum Development Institute was established,
which developed the Primary Mathematics program. This program was based on the
concrete, pictorial, abstract approach. This approach, founded on the work of renowned cognitive American psychologist Jerome Bruner, encourages
mathematical problem solving, thinking and communication.

The fact that problem solving is the central idea in Singapore math can be seen in the
pentagon from Singapore's Mathematics Framework below.

Beliefs
Interest
Appreciation
Confidence
Perseverance

Attitudes

Metacognition

Monitoring of one's own thinking
Self-regulation of learning

Mathematical Problem Solving

Skills

Numerical calculation
Algebraic manipulation
Spatial visualisation
Data analysis
Measurement
Use of mathematical tools
Estimation

Processes

Reasoning, communication and connections
Thinking skills and heuristics
Application and modelling

Concepts

Numerical
Algebraic
Geometrical
Statistical
Probabilistic
Analytical

The Framework states, "***Mathematical problem solving is central to mathematics learning***. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems. The development of mathematical problem solving ability is dependent on five inter-related components, namely, Concepts, Skills, Processes, Attitudes and Metacognition."

Problem solving and mathematical thinking are two big ideas behind Singapore math. To understand this better, let's look at an example that many American elementary students struggle with, long division. ***As noted in part one of this blog, word problems are often the last thing on the page in U.S. mathematics textbooks. Often times teachers never even get to these problems*** or if they do, usually only the advanced students have the opportunity to tackle them while struggling students continue to practice procedures. The third grade Primary Mathematics textbook, however, introduces long division with a word problem. The description below is one way how the concept of long division might be introduced in Singapore math.

After a brief warm up with multiplication and division flash cards, the teacher introduces a problem by saying, "Our friend Meihua has some toy soldiers. She wants to put them equally in some tents." (Note that no numbers are mentioned and there is no question asked yet.) The teacher then asks the students to try to imagine the situation and discuss what it means to put the soldiers in tents equally. Students share examples such as, "If she has 15 soldiers and 3 tents, she could put 5 soldiers in each tent," and "If there were 10 soldiers and 5 tents she could put 2 soldiers in each tent."

Next, the teacher gives the students 14 counters (chips) to represent the soldiers and 4 cups to represent the tents. Then she poses the problem, "Meihua has 14 toy soldiers. She puts the toy soldiers equally into 4 tents. How many soldiers are there in each tent? How many soldiers are left?"

Students begin solving the problem by dividing the counters among the cups as well as any other methods they come up with (e.g. using the multiplication table of 4). Then, *several students share their solution methods with the class, the methods as well as errors are discussed and evaluated, and important ideas are highlighted*. The teacher then uses these ideas to introduce the division algorithm for 14÷4, relating each step to the soldiers and the tents. Lastly, vocabulary words such as dividend, divisor and quotient are introduced and related to the problem.

Solving and discussing this "anchor problem" has taken a little more than half of the one-hour math period. For the rest of the class, students solve and discuss a few carefully selected problems from the textbook, gradually moving from simple to more complex numbers, and using pictures from the textbook to help them form a mental picture of the process. Over the course of the unit, students will learn to calculate long division problems using only numbers but they will have frequent opportunities to move through this cycle of concrete manipulation to pictorial representation to abstract calculation. ***Since there are fewer topics in the Singapore math textbook, the students have sufficient time to study the concept in depth to understand and master it.***

I would like to point out a few reasons why this approach is helpful.

1)   The problem situation provides a familiar context for students to think about an abstract concept.

Although 14÷4 might seem easy to adults, for a third grader, especially one who is already struggling in mathematics, it may seem quite abstract. The use of a familiar situation makes it easier to understand. Every child can think about dividing toy soldiers equally into tents. Since no numbers are mentioned initially, no child is lost. Every child can enter the lesson successfully at some level.

The problem setting also makes mathematical terms like divisor, quotient and remainder easier to understand. ***All too often, abstract mathematical concepts and vocabulary are presented in a void.*** *But the problem situation gives the terms meaning.* The toy soldiers become the dividend, the tents the divisor, the number of soldiers in each tent the quotient, and the soldiers who don't have a tent the remainder. When used in this way, instead creating difficulty, ***the word problem makes learning more concrete by presenting abstract ideas in a familiar context.*** As a result, students' attitudes towards problem

solving and mathematics in general improve.

2) By sharing and discussing their solution methods, students develop metacognitive
processes (ed. note: metacognition - awareness & understanding about one's own
thinking).

Sharing and discussing their methods requires students to think about their metacognition, as well as the thinking of their classmates.
When students share their own solution methods they are required to make their thinking clear and explicit so
their classmates will understand them. When they have to listen to their classmates' methods and restate their friends' thinking in their
own words they learn how to listen
to and learn from each other. The pentagon from the Singapore Mathematics Framework shows how important reasoning,
communication, thinking and metacognition are for students to become good mathematical problem solvers.

3) By spending time on concrete manipulation and pictorial representation, students are able
to internalize and visualize mathematical concepts.

The concept of division can be very abstract to children. Thinking about division using counters and cups is much easier than thinking
about it with abstract numbers. By manipulating concrete objects, students internalize the division process. **U.S. math programs often
also begin with concrete activities but students usually go from using concrete objects right to abstract calculation.
Singapore math textbooks include an intermediary pictorial stage.** By looking at pictures of concrete objects being divided
equally, students form a mental image of what long division looks like. When they finally get to abstract calculation, they have already
internalized and visualized the process.

*The above example shows a little of what Singapore math is. But I would also like to discuss what it is not, in order to dispel a couple of common
misconceptions:*

1) Singapore math is **not** the way we learned math as kids (that is, unless you went to school
in a developed East Asian nation).

In Singapore math, traditional algorithms are learned but conceptual understanding is taught before procedural fluency. For example,
the long division algorithm taught in Singapore math textbooks is the same one we traditionally learn in the U.S., but it is presented and
explained quite differently than the way most of us learned it (see Power Point presentation below). In Singapore math, students not only
learn how to do an algorithm but also why and how every step of the algorithm works.

2) Singapore math is not "drill and kill," facts memorization, or rote learning of procedures.

In Singapore math, students learn their addition, subtraction and multiplication facts and eventually memorize them. But they also learn
the structure and patterns behind the facts so if they forget them, they are able to reconstruct them in their minds. From early grades,
children learn how to compose and decompose numbers and manipulate them in useful ways **in order to calculate mentally.**

Singapore math is about students solving problems, thinking deeply, sharing their ideas, and learning from one another. Conceptual, procedural and
factual understanding is developed through problem solving and carefully structured practice and as a result, students learn how to think deeply and
appreciate mathematics.

We cannot to go into sufficient detail in this blog about how mathematical concepts are presented in Singapore math so I am attaching a
SingMathPPT.pdf presentation that will give you an idea of how basic algorithms and mental calculation are taught. (large file so download may take a
couple minutes).   In future posts, I will discuss the use of bar models for problem solving, and tips for successful implementation for schools that are
interested in using Singapore math.

Until next time,

Bill Jackson
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Part 1: Singapore Math: Can It Solve Our Country's Math-phobia?

Part 3: "The Famous Bar Modeling: Is Singapore Math The Most Visual Math?"

Part 4: Bringing Singapore Math to Your School + Tips For Teachers

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